SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

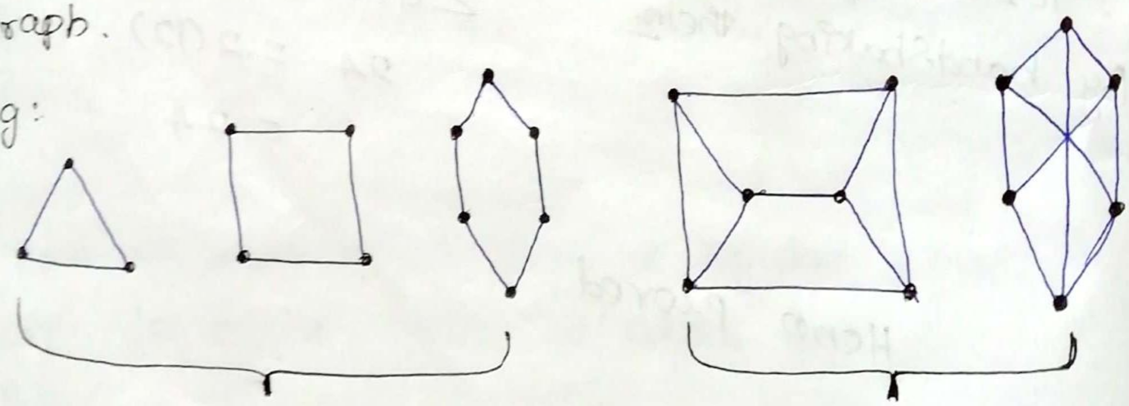
Coimbatore-641035.

Special types of Graphs

I. Regular graph:

A graph in which all the vertices are of the same degree is called a regular graph.

Eg:



2-Regular graph 3-Regular graph

2]. Complete graph:

In a graph, if there exist an edge between every pair of vertices, then such a graph is called complete graph.

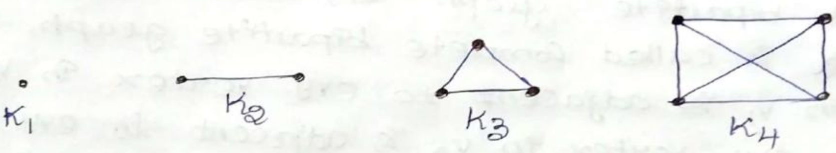
Note:

1]. Every complete graph is a regular graph.

2]. Every complete graph with n vertices is a $n-1$ regular graph.

3]. The complete graph on 'n' vertices is denoted by K_n.

Eg:



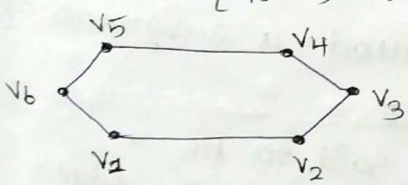
3]. Bipartite graph:

A graph G is said to be bipartite if its vertex set $V(G)$ can be partitioned into two disjoint non-empty sets V_1 and V_2, $V_1 \cup V_2 = V(G)$, such that evy. edge in $E(G)$ has one end vertex in V_1 and another end vertex in V_2

Eg:

1]. Is C_6 is Bipartite graph?

$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

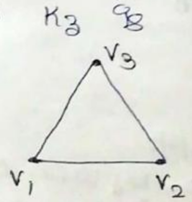


$V_1 = \{v_1, v_3, v_5\}$

$V_2 = \{v_2, v_4, v_6\}$

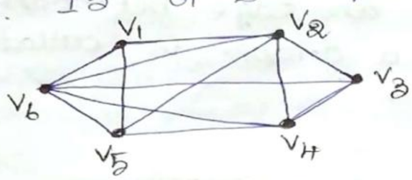
$\therefore$ C_6 is Bipartite.

2]. Is K_3 is Bipartite?



It can't partition into 2 disjoint nonempty subsets.
Since any 2 vertices are adjacent
Hence K_3 is not Bipartite.

3). Is G is Bipartite?



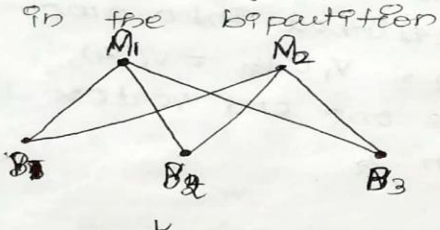
We cannot partition V into 2 disjoint nonempty subsets. Becos V_2 is adjacent to evy. other vertex and V_6 also adjacent to every other vertex.

Hence G is not Bipartite.

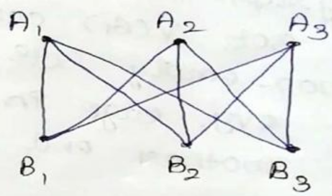
4). Complete Bipartite graph:

A bipartite graph G, with bipartition V_1 and V_2 is called complete bipartite graph, if every vertex in V_1 is adjacent to evy. vertex in V_2. clearly, evy. vertex in V_2 is adjacent to evy. vertex in V_1.

A graph with 'm' & 'n' vertices in the bipartition is denoted by $K_{m,n}$



$K_{2,3}$



$K_{3,3}$

Subgraph:

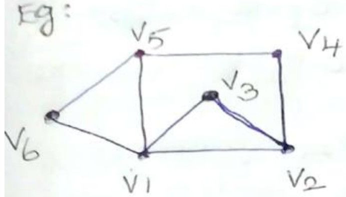
A graph $H = (V', E')$ is called a subgraph of $G = (V, E)$, if $V' \subseteq V$ and $E' \subseteq E$

In otherwords, the graph H is said to be a subgraph of G if all the vertices and all the edges of H are in G and if the adjacency is preserved in H exactly as in G.

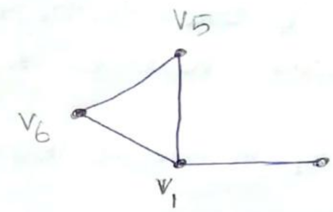
Note:

1). Each graph has its own subgraph.
2). A single vertex in a graph G is a subgraph of G
3). A single edge in G, together with its end vertices is also a subgraph of G.
4). A subgraph of a subgraph of G is also subgraph of G

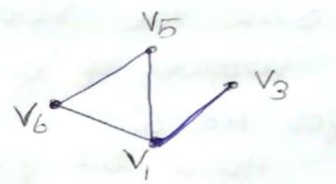
Eg:



Graph



Subgraph H_1



Not subgraph H_2

Here H_1 is a subgraph of G

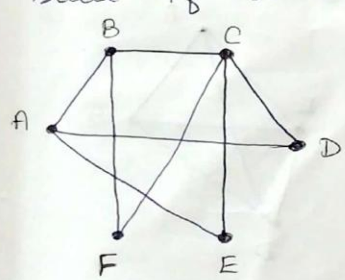
H_2 is not a subgraph of G since V_3 and V_4 are adjacent vertices in G

Note:

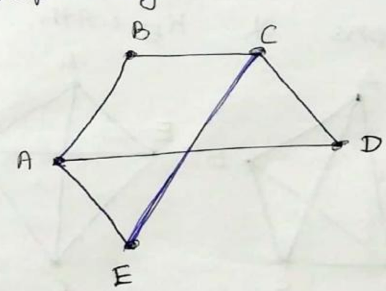
Any subgraph of a graph G can be obtained by removing certain vertices and edges from G. It is to be noted that the removal of an edge does not go with the removal of its adjacent vertices, where as the removal of a vertex goes with the removal of any edge incident on it.

Problems:

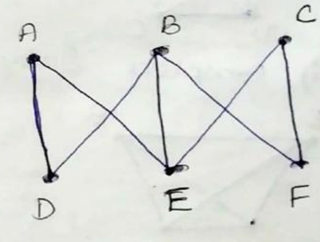
1. Determine which of the following graphs are bipartite and which are not. If a graph is bipartite, state if it is completely bipartite.



G_1



G_2



G_3

For G_1, since the vertices D, E, F are not connected by edges, they may be considered as one subset V_1 and $A, B, C \in V_2$

$\therefore V_1 = \{D, E, F\}$ and $V_2 = \{A, B, C\}$

Since the vertices V_1 are connected by edges to the vertices of V_2 but the vertices A, B, C of V_2 are to edges AB, BC.

Hence the graph G_1 is not a Bipartite graph.

For G_2,

$V_1 = \{A, C\}$ $\quad V_2 = \{B, D, E\}$

Hence the graph G_2 is Bipartite.

Both A and C are adjacent to B, D, E. Hence the graph is complete Bipartite graph

For G_3,

$V_1 = \{A, B, C\}$ $\quad V_2 = \{D, E, F\}$

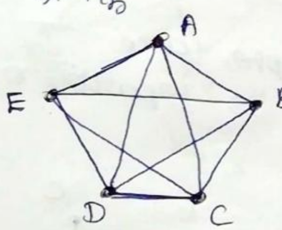
Hence the graph G_3 is Bipartite.

Here the vertices A, F and C, D are not connected by edges.

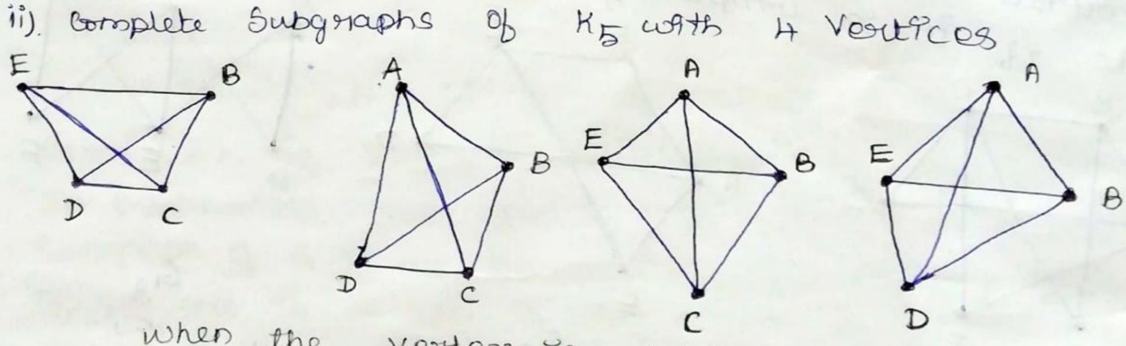
$\therefore$ It is not a complete bipartite graph.

2. Draw the complete graph K_5 with vertices A, B, C, D, E. Draw all complete subgraphs of K_5 with 4 vertices.

i). K_5 ii).



ii). Complete Subgraphs of K_5 with 4 vertices



When the vertex is deleted, the edges adjacent with it are also deleted.