Unit - 5

Trusses

Static Indeterminacy of Trusses:

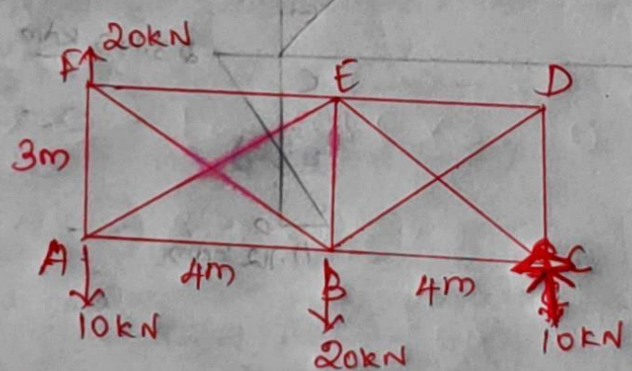
$$D_s = m + R - 2j$$

M = No of members

R = No of Reactions

j = No. of Joints

1. Determine the forces in the members of the truss as shown in figure. The diagonals are to be designed to support both tensile and compressive forces. Use cantilever method.



Soln:

Static Indeterminacy:

$$D_s = m + R - 2j$$

$m = 11, \quad R = 1, \quad j = 6$

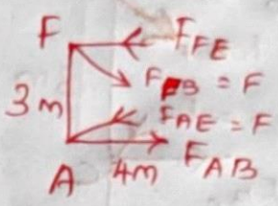
$$D_s = (11+1) - (2 \times 6)$$

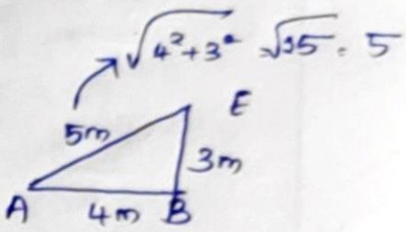
$$= 0.$$

Statically Determinate Truss.

Unknown forces:

Consider Joints A & F:





$\cos\theta = 4/5$

$\sin\theta = 3/5$

$\Sigma V = 0,$

$20 - 10 - 2F\sin\theta = 0$

$10 - 2F \times 3/5 = 0$

$F = 8.33\,kN$

$F_{AB} = 8.33\,kN\,(T)$

$F_{AE} = 8.33\,kN\,(C)$

$\Sigma M_A = 0,$

$F_{FB} \times \cos\theta \times 3 - F_{FE} \times 3 = 0$

$8.33 \times 4/5 \times 3 - F_{FE} \times 3 = 0$

$F_{FE} = 6.67\,kN\,(C)$

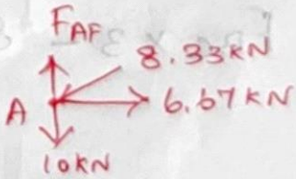
$\Sigma M_F = 0,$

$-F_{AE} \times \cos\theta \times 3 + F_{AB} \times 3 = 0$

$-8.33 \times 4/5 \times 3 + F_{AB} \times 3 = 0$

$F_{AB} = 6.67\,kN\,(T)$

At Joint A:



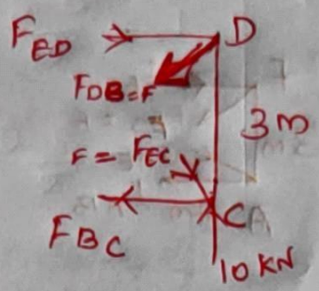
$\Sigma y = 0,$

$F_{AF} - 8.33\sin\theta - 10 = 0$

$F_{AF} - 8.33 \times 3/5 - 10 = 0$

$F_{AF} = 15\,kN\,(T)$

Consider a joints C & D :



$\Sigma F_y = 0$,

$10 - 2F \sin\theta = 0$

$10 - 2 \times F \times 3/5 = 0$

$F = 8.33$ kN

$F_{DB} = 8.33$ kN (T)

$F_{EC} = 8.33$ kN (C)

$\Sigma M_C = 0$,

$F_{ED} \times 3 - F_{DB} \cos\theta \times 3 = 0$

$F_{ED} \times 3 - 8.33 \times 4/5 \times 3 = 0$

$F_{ED} = 6.67$ kN (C)

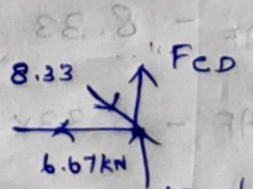
$\Sigma M_D = 0$,

$F_{BC} \times 3 - F_{EC} \cos\theta \times 3 = 0$

$F_{BC} \times 3 - 8.33 \times 4/5 \times 3 = 0$

$F_{BC} = 6.67$ kN (T)

At joint D,



$\Sigma F_y = 0$,

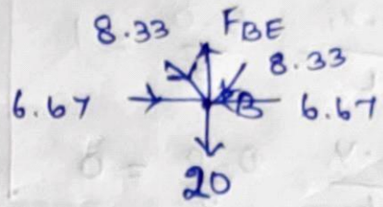
$10 + F_{CD} - 8.33 \times 3/5 = 0$

$F_{CD} = 5$ kN (T)

∴ Consider joint B!

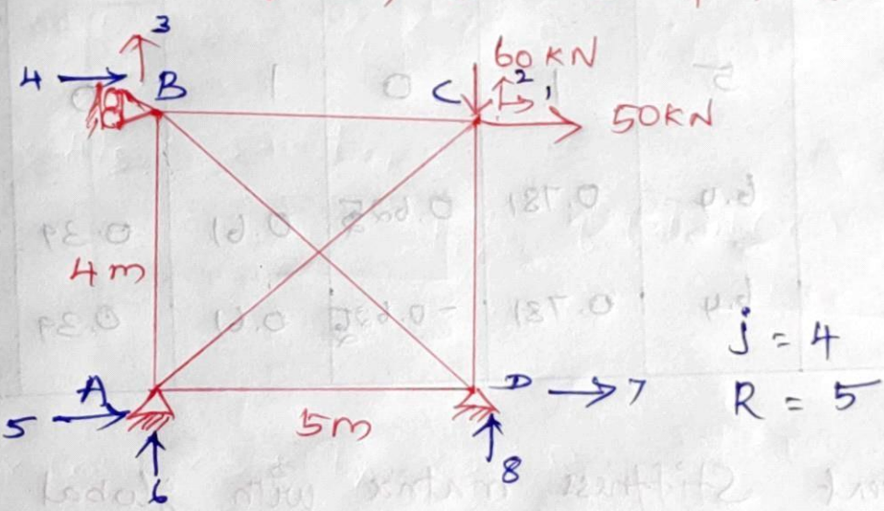
$\Sigma V = 0$,

$-20 + F_{BE} - 8.33 \times 3/5 - 8.33 \times 3/5 = 0$



$F_{BE} = 10\,KN\,(T)$

2. Analyse the truss as shown in figure if temperature of all the members is increased by 30°C, also length of member AC is found to be 5mm too short. $\alpha = 12 \times 10^{-6}/°C$, $E = 210$ GPa, $A = 750mm^2$.



$j = 4$

$R = 5$

Soln:

Degree of kinematic Indeterminacy:

$D_k = 2j - R$

$= (2 \times 4) - 5$

$= 3$

Joint Data and Member data:

Joint	x	y
A	0	0
B	0	4
C	5	4
D	5	0

A (0,0), B (0,4), D (5,0), C (5,4)

$c = \frac{x_2 - x_1}{L} = \frac{0-0}{4} = 0$

$s = \frac{y_2 - y_1}{L} = \frac{4-0}{4} = 1$

$c = \frac{x_2 - x_1}{L} = \frac{5-5}{4} = 0$

$s = \frac{y_2 - y_1}{L} = \frac{4-0}{4} = 1$

A (0,0) D (5,0)

$C = \frac{x_2 - x_1}{L} = \frac{5-0}{5} = 1$

$S = \frac{y_2 - y_1}{L} = \frac{0-0}{5} = 0$

A (0,0) C (5,4)

$C = \frac{x_2 - x_1}{L} = \frac{5-0}{6.4}$

$S = \frac{y_2 - y_1}{L} = \frac{4-0}{6.4}$

B (0,4) D (5,0)

$C = \frac{x_2 - x_1}{L} = \frac{5-0}{6.4-0}$

$S = \frac{0-4}{6.4-0}$

Member Data :

Member	L (cm)	C	S	C^2	S^2	CS
AB	4	0	1	0	1	0
DC	4	0	1	0	1	0
BC	5	1	0	1	0	0
AD	5	1	0	1	0	0
AC	6.4	0.781	0.625	0.61	0.39	0.238
BD	6.4	0.781	-0.625	0.61	0.39	-0.238

Element Stiffness matrix with global Co-ordinates :

$$K_{AB} = \frac{AE}{4} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 4 \\ 3 \end{matrix}$$

(5) (6) (4) (3)

C^2 CS $-C^2$ $-CS$

CS S^2 $-CS$ $-S^2$

$-C^2$ $-CS$ C^2 CS

$-CS$ $-S^2$ CS S^2

$$K_{BC} = \frac{AE}{4} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 4 \\ 3 \\ 1 \\ 2 \end{matrix}$$

(4) (3) (1) (2)

$$K_{AD} = \frac{AE}{5} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 5 \\ 6 \\ 7 \\ 8 \end{matrix}$$

(5) (6) (7) (8)

$$K_{DC} = \frac{AE}{4}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}\begin{matrix}7\\8\\1\\2\end{matrix}$$

$$K_{AC} = \frac{AE}{6.4}\begin{bmatrix} 0.61 & 0.238 & -0.61 & -0.238 \\ 0.238 & 0.39 & -0.238 & -0.39 \\ -0.61 & -0.238 & +0.61 & 0.238 \\ -0.238 & -0.39 & 0.238 & 0.39 \end{bmatrix}\begin{matrix}5\\6\\1\\2\end{matrix}$$

$$K_{BD} = \frac{AE}{6.4}\begin{bmatrix} 0.61 & -0.238 & -0.61 & 0.238 \\ -0.238 & 0.39 & 0.238 & -0.39 \\ -0.61 & 0.238 & 0.61 & -0.238 \\ 0.238 & -0.39 & -0.238 & 0.39 \end{bmatrix}\begin{matrix}4\\3\\7\\8\end{matrix}$$

Assembly of Structure Stiffness matrix:

$$K_S = AE\begin{bmatrix} 0.25+0+0.095 & 0+0+0.037 & 0 \\ 0+0+0.037 & 0+0.25+0.061 & 0 \\ 0 & 0 & 0.25+0+0.061 \end{bmatrix}\begin{matrix}1\\2\\3\end{matrix}$$

$$K_S = \begin{bmatrix} 0.295 & 0.037 & 0 \\ 0.037 & 0.311 & 0 \\ 0 & 0 & 0.311 \end{bmatrix}$$

Temperature Stresses or Lack of fit:

$\alpha = 12 \times 10^{-6}$

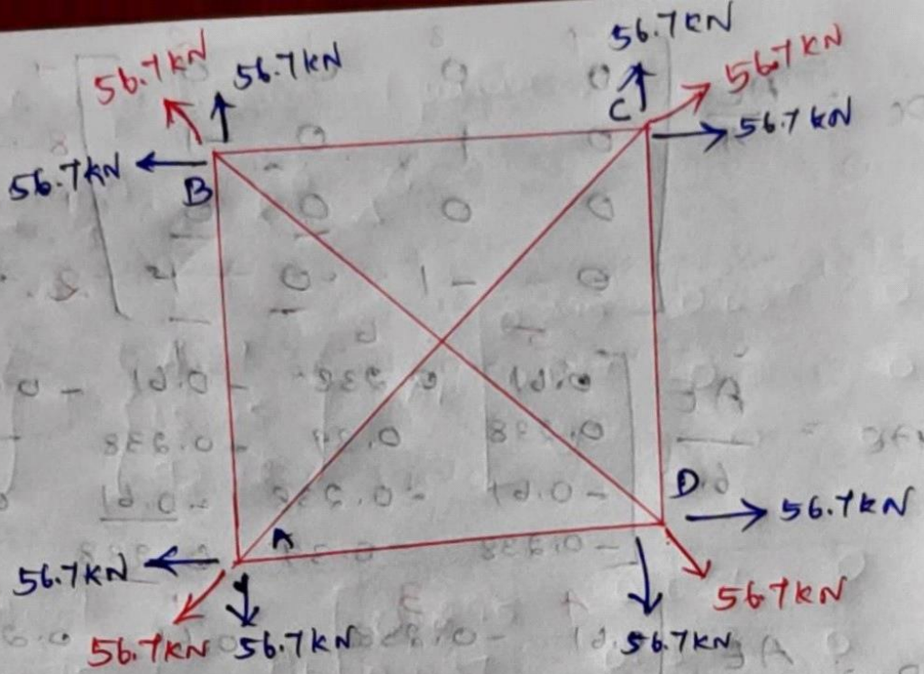
$A = 750\ mm^2$

$E = 210\ Gipa = 210\ KN/mm^2$

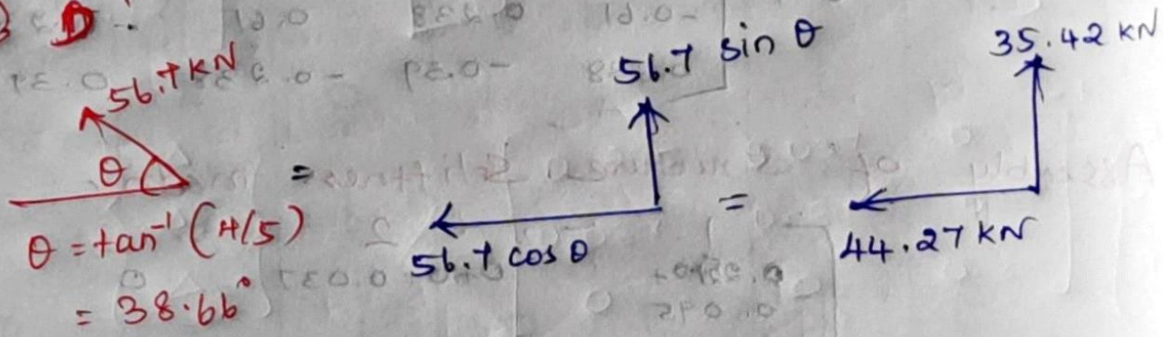
$\Delta t = 30°$

$$\alpha A E \cdot \Delta t = 12 \times 10^{-6} \times 750 \times 210 \times 30°$$

$$= 56.7\ KN$$



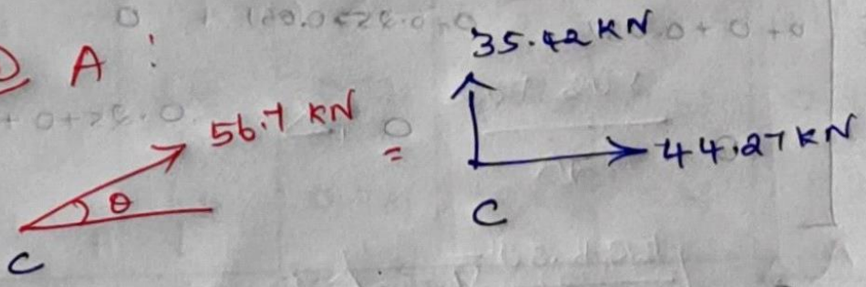
@ D :



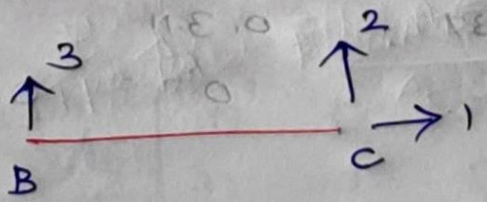
$\theta = \tan^{-1}(4/5)$

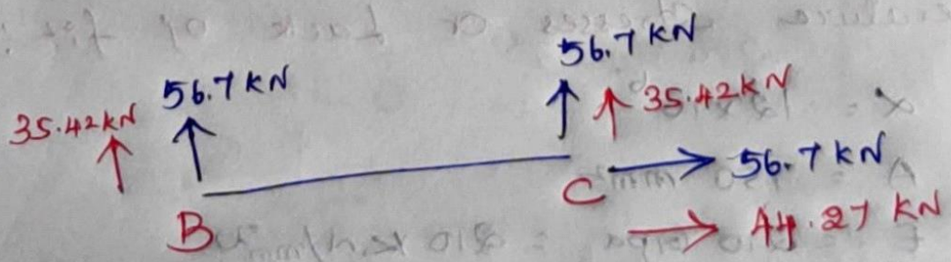
$= 38.66°$

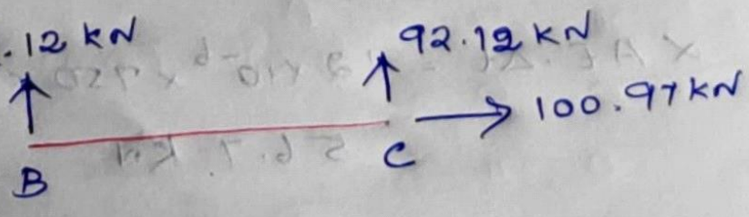
@ A :



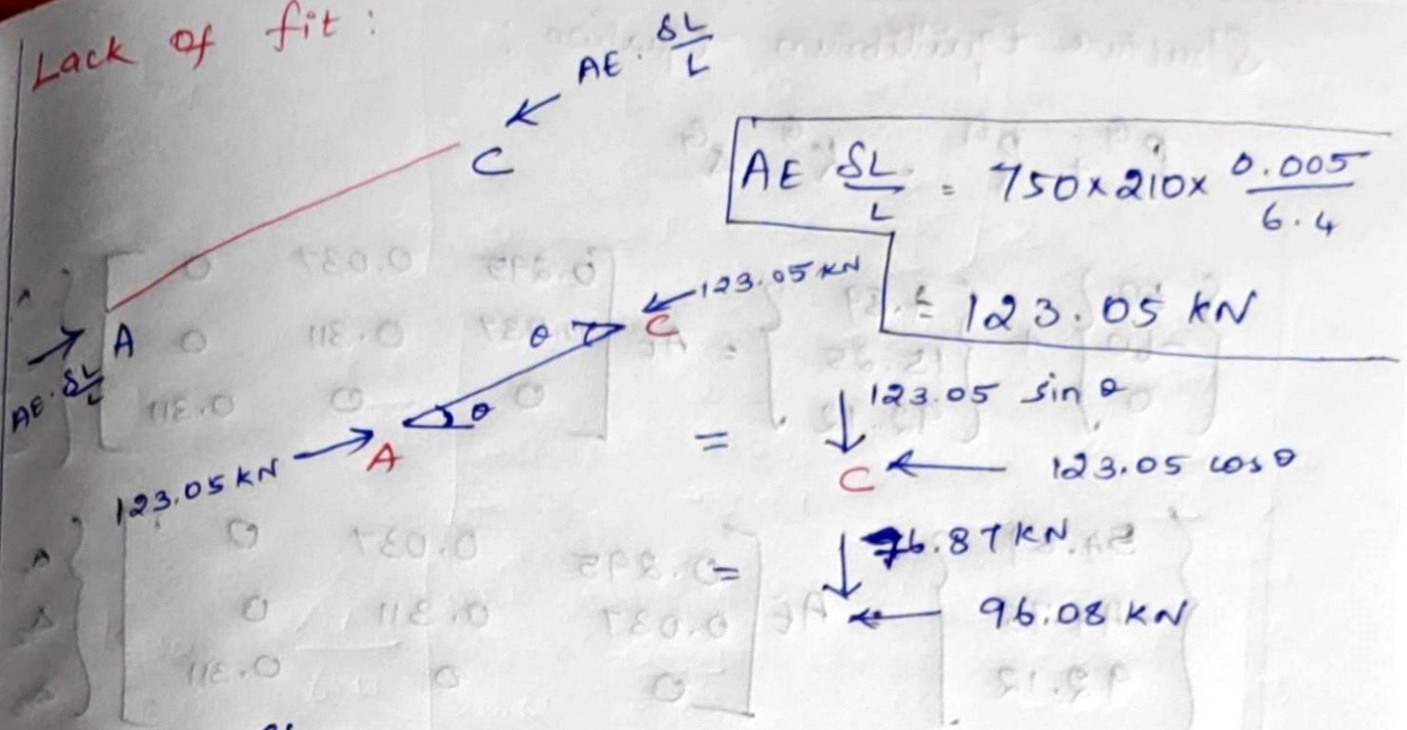
Member BC :







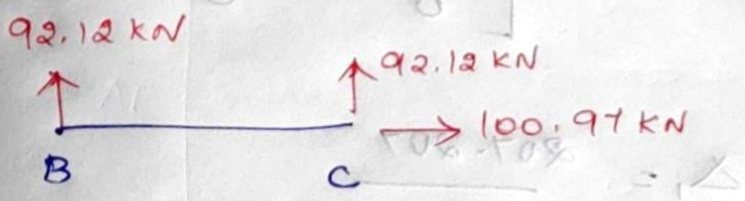
Lack of fit :



Total Stress :

Stress due to temperature + Lack of fit

Temperature Effect :-



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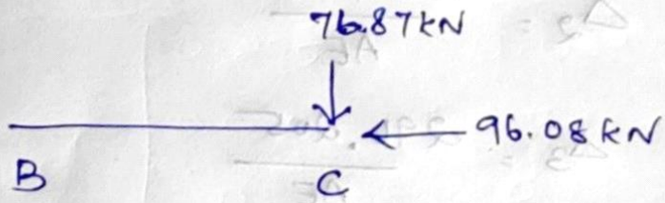
Lack of fit :



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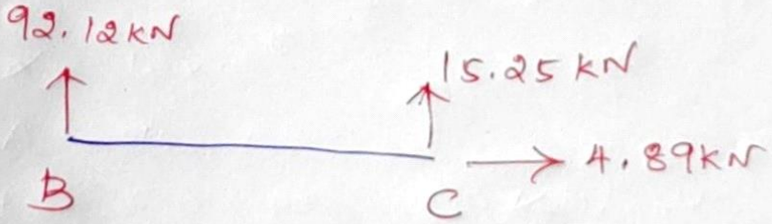

$$P_s^{eq} = \begin{Bmatrix} 4.89 \\ 15.25 \\ 92.12 \end{Bmatrix}$$

$$P_s^{G} = \begin{Bmatrix} 50 \\ -60 \\ 0 \end{Bmatrix}$$

Structure Equilibrium Equation :

$$P_s^G + P_s^{FA} = K_s^G \Delta^G$$

$$\begin{Bmatrix} 50 \\ -60 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 4.89 \\ 15.25 \\ 92.12 \end{Bmatrix} = AE \begin{bmatrix} 0.295 & 0.037 & 0 \\ 0.037 & 0.311 & 0 \\ 0 & 0 & 0.311 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{Bmatrix}$$

$$\begin{Bmatrix} 54.89 \\ -44.75 \\ 92.12 \end{Bmatrix} = AE \begin{bmatrix} 0.295 & 0.037 & 0 \\ 0.037 & 0.311 & 0 \\ 0 & 0 & 0.311 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{Bmatrix} = \frac{1}{AE} \begin{bmatrix} 0.295 & 0.037 & 0 \\ 0.037 & 0.311 & 0 \\ 0 & 0 & 0.311 \end{bmatrix}^{-1} \begin{Bmatrix} 54.89 \\ -44.75 \\ 92.12 \end{Bmatrix}$$

$$\Delta_1 = \frac{207.207}{AE}$$

$$\Delta_2 = \frac{-168.452}{AE}$$

$$\Delta_3 = \frac{296.205}{AE}$$