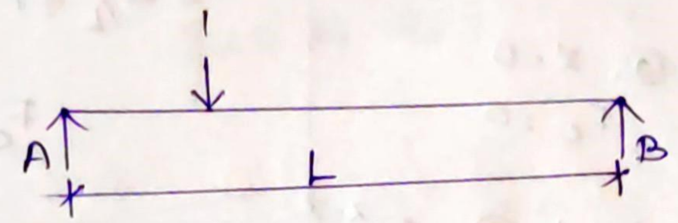
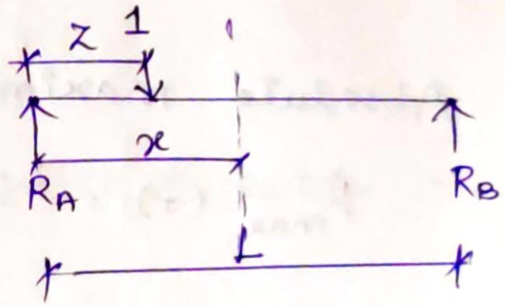
Unit - I : Influence line for Determinate beams

Influence line diagram for simply Supported beam:

Consider a S.S beam of Span L and moving unit load moves from left to right on the beam.



Let us consider a section C at a distance 'x' from A.



ILD for reaction @ A:

$\Sigma V = 0$

$R_A + R_B = 1$

$\Sigma M_B = 0$

$R_A \times L - 1(L - z) = 0$

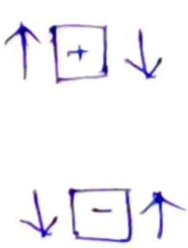
$R_A \times L = L - z$

$R_A = \frac{L - z}{L}$

When $z = 0$, $R_A = 1$ $\left(\frac{L-0}{L}\right) = L/L = 1$

$z = L$, $R_A = 0$ $\left(\frac{L-L}{L}\right) = 0$

Force



ILD for Reaction @ B:

$\Sigma M_A = 0$

$-R_B \times L + 1 \times z = 0$

$R_B = z/L$

when, $z = 0$, $R_B = 0$

$z = 1$, $R_B = 1$

ILD for Shear force @ c :

Consider the load will be in left half of the span

When $z < x$

$R_A - 1 = F_c$

$\frac{L-z}{L} - 1 = F_c$

$\frac{L - z - L}{L} = F_c$

$F_c = -z/L$

@ $z = 0$

$F_c = 0$

load @ c $z = x$

$F_c = -x/L$

load in right half of the span.

When $z > x$

$-R_B + 1 = F_c$

$-z/L + 1 = F_c$

$F_c = \frac{-z + L}{L}$

$F_c = \frac{L - z}{L}$

load @ c $F_c = \frac{L-x}{L}$

@ $z = L$, $F_c = 0$

Absolute maximum shear (F_{max})

$F_{max} (+) = \frac{L-z}{L} = 1$ (when $z = 0$)

$F_{max} (-) = -z/L = -1$ (When $z = 1$)

Maximum Bending moment :

B.M @ c From left side and load @ c.

$M_c = R_A \times x$

$= \left(\frac{L-z}{L}\right) x$

$= \frac{x}{L} \times (L - z)$

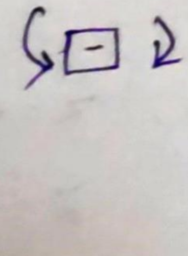
Put $z = x$

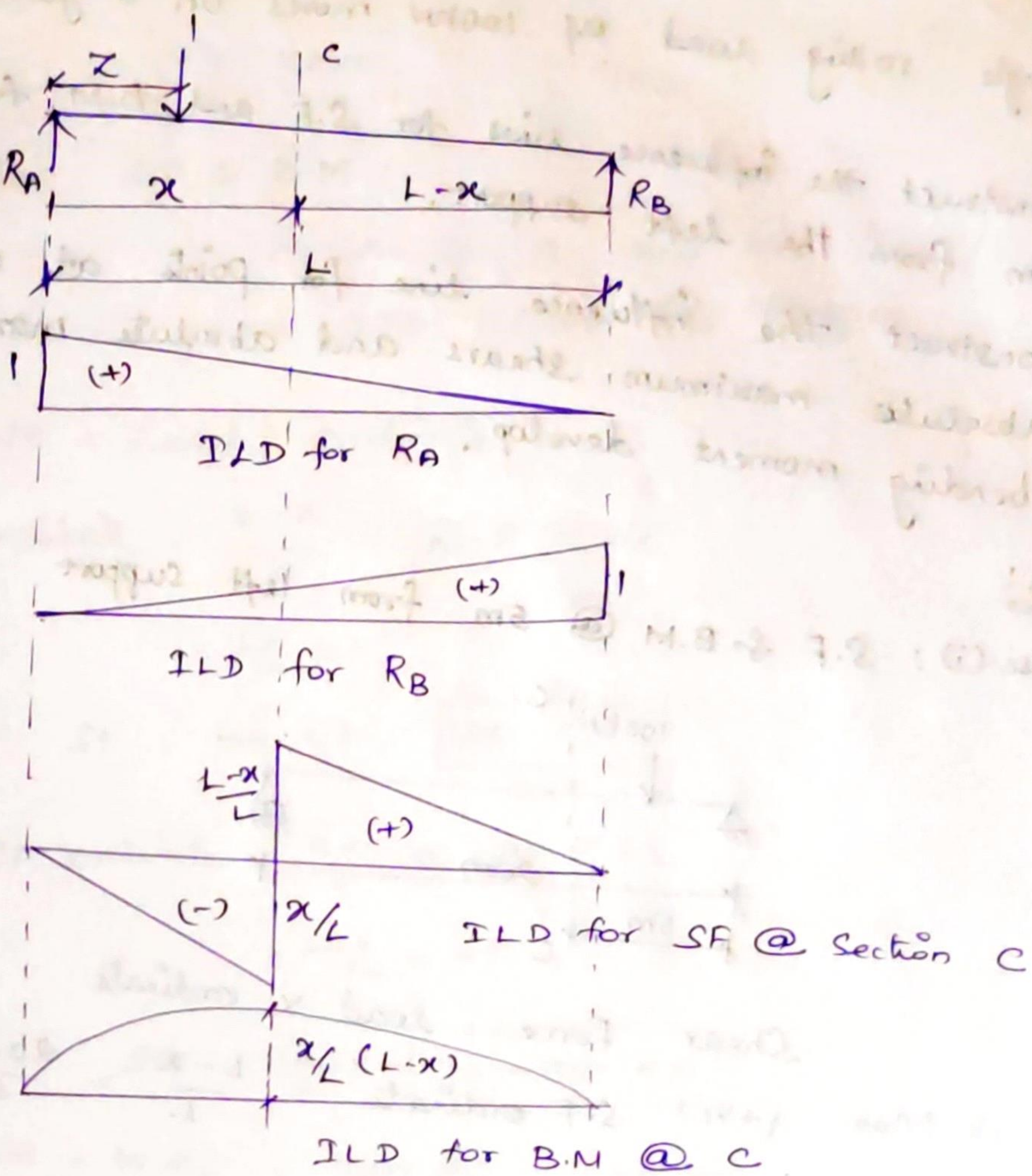
$M_c = \frac{x}{L} (L - x)$

B.M @ c from right side

$M_c = R_B (L - x)$

$= \frac{x}{L} (L - x)$





Absolute Maximum BM : B.M = load × ordinate

max moment occurs @ $x = L/2$

$$M_c = \left[\frac{x}{L}(L-x)\right] \times W$$

$$= \left[\frac{L/2}{L}\left(L - L/2\right)\right] \times W$$

$$= \left[\frac{L}{2L}\left(\frac{2L-L}{2}\right)\right] \times W$$

$$= \left[\frac{L}{2L}\left(L/2\right)\right] \times W$$

$$= L/4 \times W$$

$$M_c = \frac{WL}{4}$$

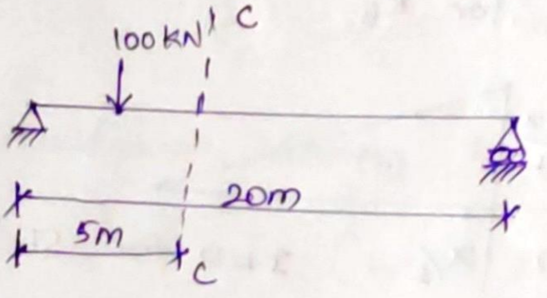
A single rolling load of 100kN moves on a girder of span 20m.

(i) Construct the influence lines for S.F and B.M for a section 5m from the left support.

(ii) Construct the influence line for points at which the absolute maximum shears and absolute maximum bending moment develop.

Soln:

Case (i): S.F & B.M @ 5m from left support



Shear Force = load × ordinate

(i) Max. (+ve) S.F ordinate $= \frac{L-x}{L} = \frac{20-5}{20} = \frac{15}{20}$ m/m

$= 0.75$ (no unit)

Max. (+ve) S.F = 100 × 0.75 (kN)

$= 75$ kN

(ii) Max (-ve) S.F ordinate $= \frac{x}{L} = \frac{5}{20} = 0.25$

Max. (-ve) S.F = 100 × 0.25

$= 25$ kN

(iii) Max. B.M = load × Ordinate

B.M ordinate $= \frac{x(L-x)}{L}$ m × m/m

$= 5\left(\frac{20-5}{20}\right)$

$= 3.75$ m (no unit)

Max. B.M = 100 × 3.75 (kN × m)

= **375 kN.m**

Case (2): Max S.F & B.M :

Max (+ve) SF occurs @ A, and max (-ve) SF occurs @ B and max B.M occurs @ midspan.

(i) (+ve) SF = Load × ordinate

(+ve) SF ordinate = $\frac{L-x}{L}$ @ A x = 0

= $L/L = 1$

Max (+ve) SF = 100 × 1 = **100 kN**

(ii) (-ve) SF ordinate = $-x/L$ @ B x = L

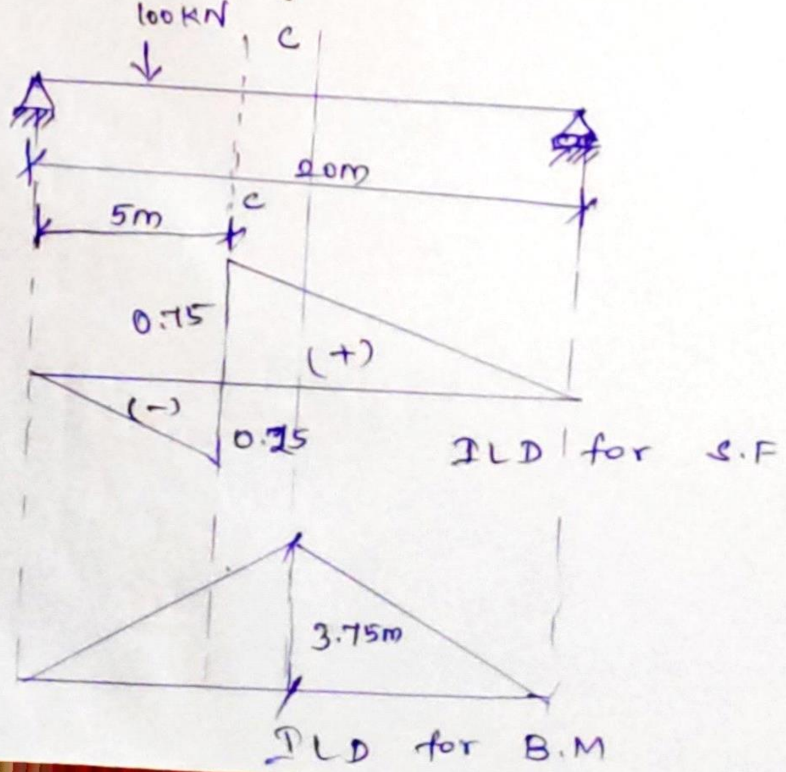
= $-L/L = -1$

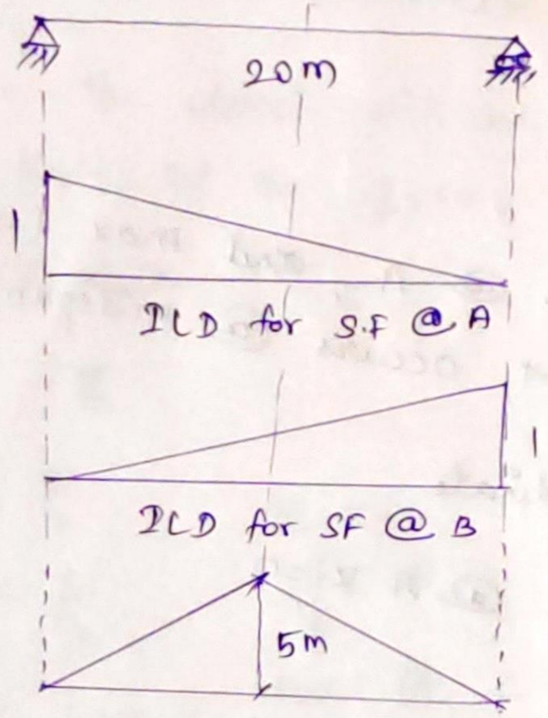
Max (-ve) SF = 100 × -1 = **-100 kN**

(iii) Max BM = $WL/4$ = $100 \times \frac{20}{4}$ (kN × m)

= **500 kNm**

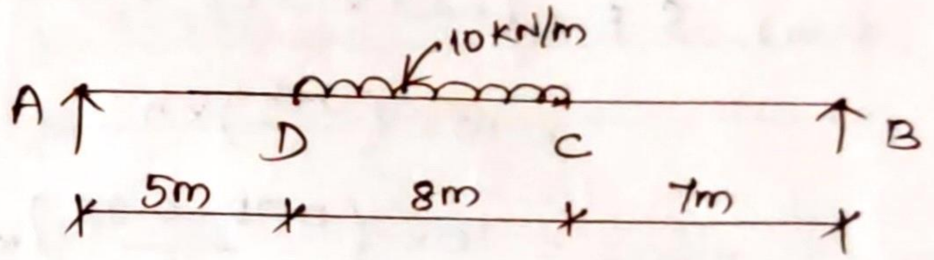
Influence Line Diagram :





ILD for B.M @ mid span

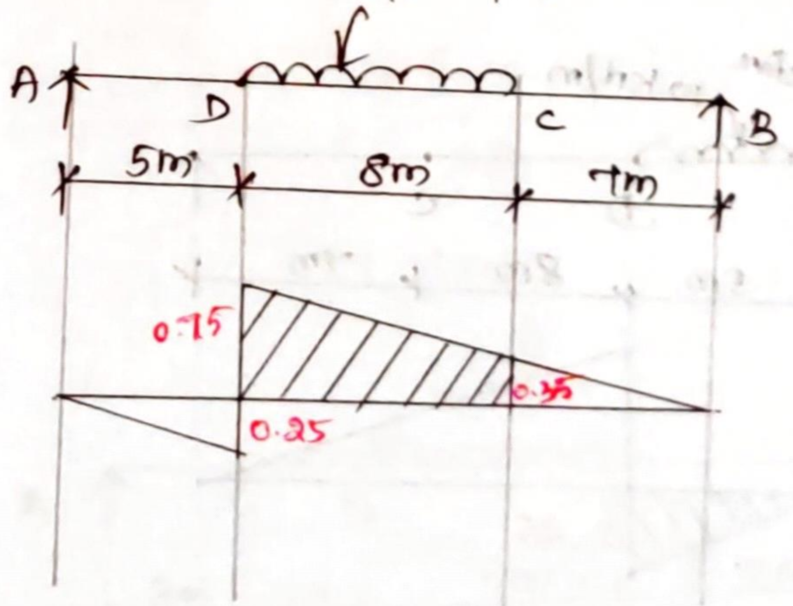
2. Draw the ILD for shear force and bending moment for a section at 5m from the left hand support of a SSB 20m long. Hence calculate the maximum B.M and S.F at the section, due to an uniformly distributed rolling load of length 8m and intensity 10 kN/m run.



Soln:

1. Maximum Positive Shear Force:

maximum (+ve) shear force occurs when the tail of the UDL is at D as it moves from left to right.



Max. (+ve) Shear force ordinate:

$$= \frac{L-x}{L} = \frac{20-5}{20} = 0.75$$

Max. (-ve) Shear force ordinate:

$$\frac{x}{L} = \frac{5}{20} = 0.25$$

Ordinate under C :

$$\frac{0.75}{15} = \frac{y}{7}$$

$$\therefore y = \frac{7 \times 0.75}{15}$$

$$y = 0.35$$

Max. (+ve) S.F = Load × ILD Area under load

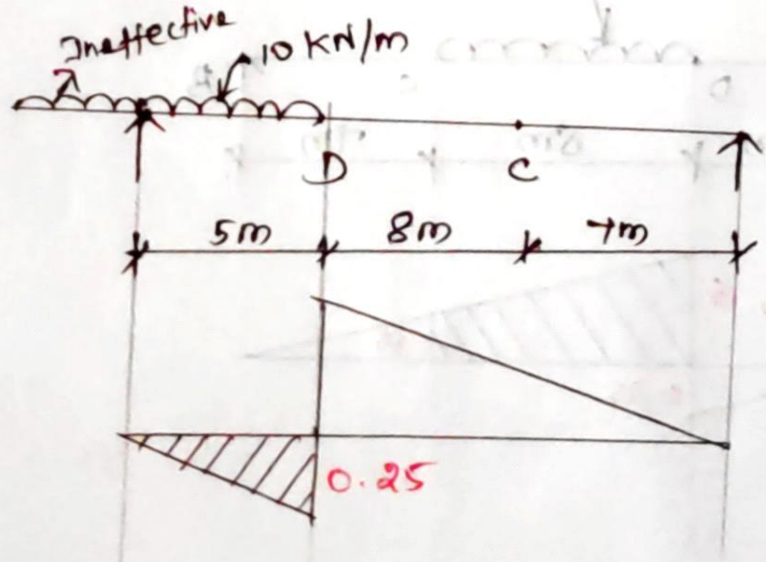
$$= 10 \times \left(\frac{a+b}{2}\right) \times h$$

$$= 10 \times \left(\frac{0.75 + 0.35}{2}\right) \times 8$$

$$= 44 \text{ KN}$$

2. Max. Negative Shear Force :

Max. negative shear force occurs when the head of the load is @ D as moves from left to right.



Max. (-ve) S.F = load × Area

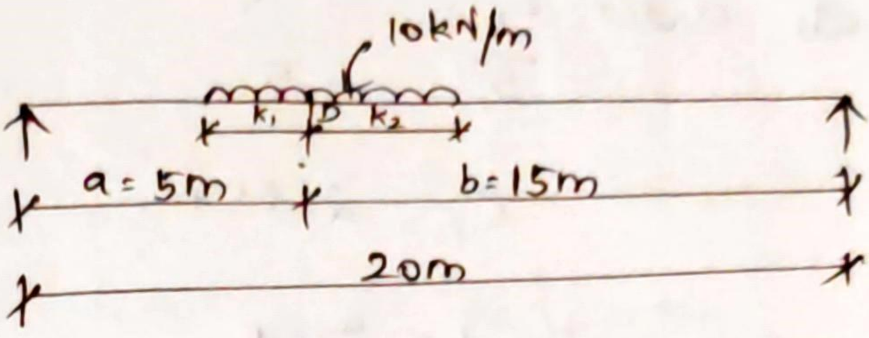
$$= 10 \times \tfrac{1}{2} \times bh$$

$$= 10 \times \tfrac{1}{2} \times 5 \times 0.25$$

$$= 6.25 \text{ KN}$$

3. Maximum Bending Moment :

Max. B.M at D due to UDL shorter than span occurs, when the section divides the load in the same ratio.



$$\frac{\text{Length of the section}}{\text{Whole length of the beam}} = \frac{\text{Length of the load within the section}}{\text{Whole length of the load}}$$

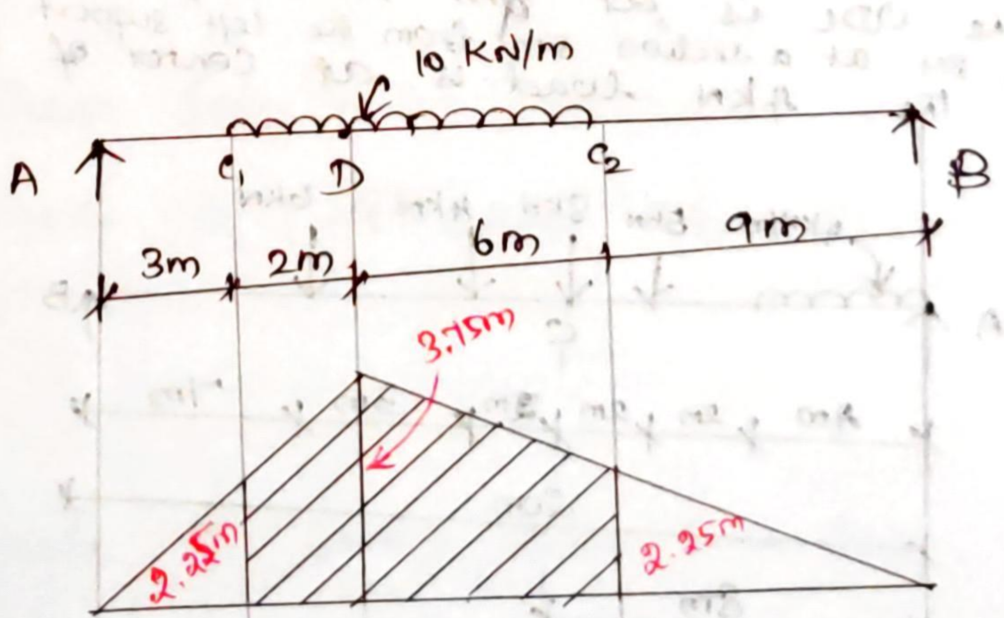
$$\frac{5}{20} = \frac{k_1}{8}$$

$$k_1 = 8 \times \frac{5}{20} = 2m$$

$$k_2 = 8 - 2 = 6m$$

ILD @ D,

$$\frac{x(L-x)}{L} = \frac{5(20-5)}{20} = 3.75m$$



Ordinate under, $\frac{C_2}{9} = \frac{3.75}{15}$

$C_2 = 2.25m$

Ordinate under, $\frac{C_1}{5} = \frac{3.75}{5}$

$C_1 = 2.25m$

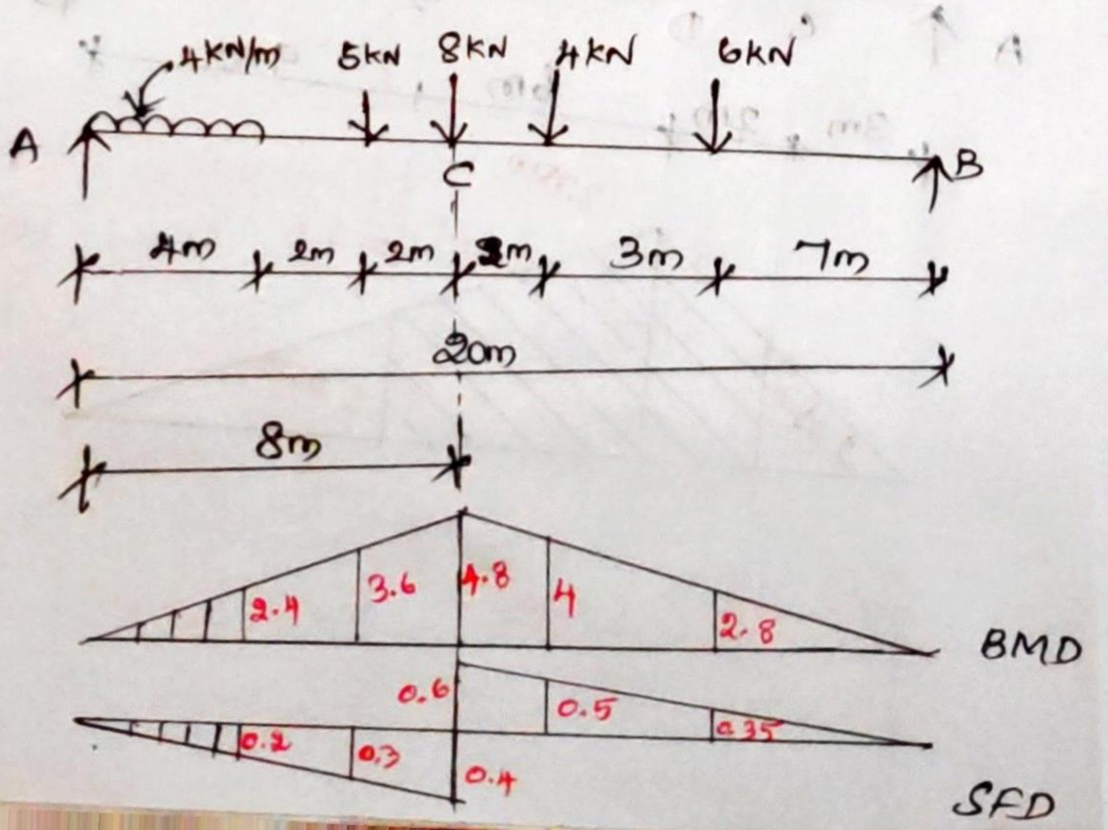
Max. B.M = Load x Area

$= 10 \times \left(\frac{a+b}{2} \times h\right)$

$= 10 \times \left(\frac{2.25 + 3.75}{2}\right) \times 8$

$= 240$ KNm.

3. Four wheel loads of 6, 4, 8 and 5kN cross a girder of span 20m, from left to right followed by UDL of 4 kN/m and 4m long with the 6kN load leading. The spacing b/w the loads in the same order are 3m, 2m and 2m. The head of the UDL is at 8m from the last 5kN load left support. Calculate SF and BM at a section 8m from the left support when the 4kN load is at center of span.



(i) Bending Moment :

The Ordinate of I.L for B.M at C, $x = 8m$

$= \frac{x}{L}(L-x)$

$= \frac{8}{20}(20-8)$

$= \frac{8}{20} \times 12$

$= 4.8$

4KN load is at the center.

Ordinate under 6KN load $= \frac{4.8}{12} \times 7$

$= 2.8$

Ordinate under 4KN load $= \frac{4.8}{12} \times 10$

$= 4$

Ordinate under 5KN load $= \frac{4.8}{8} \times 6$

$= 3.6$

Ordinate under the head of UDL $= \frac{4.8}{8} \times 4$

$= 2.4$

$M_C = (6 \times 2.8) + (4 \times 4) + (8 \times 4.8) + (5 \times 3.6) + 4\left[\frac{1}{2} \times 4 \times 2.4\right]$

$= 108.4$ KN.m

(ii) Shear Force :

Ordinate of (-ve) shear force ILD $= -\frac{x}{L}$

$= -\frac{8}{20}$

$= -0.4$

Ordinate of (+ve) shear force ILD $= \frac{L-x}{L}$

$= \frac{20-8}{20}$

$= 0.6$

Ordinate under 6 kN load $= \frac{0.6}{12} \times 7 = 0.35$

Ordinate under 4 kN load $= \frac{0.6}{12} \times 10 = 0.5$

Ordinate under 5 kN load $= \frac{0.4}{8} \times 6 = 0.3$

Ordinate under head of UDL $= \frac{0.4}{8} \times 4 = 0.2$

Consider 8 kN load is just left of C.

Max (−ve) S.F $= -\left\{ (8 \times 0.4) + (5 \times 0.3) + (\tfrac{1}{2} \times 4 \times 0.2 \times 4) \right\}$

$+ \left\{ (4 \times 0.5) + (6 \times 0.35) \right\}$

$= -2.2$ kN

Max (+ve) S.F $= \left\{ (6 \times 0.35) + (4 \times 0.5) + (8 \times 0.6) \right\} -$

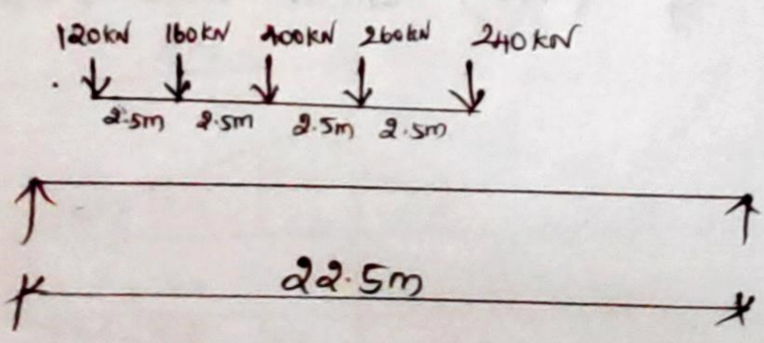
$\left\{ (0.5 \times 0.3) + (\tfrac{1}{2} \times 4 \times 0.2 \times 4) \right\}$

$= 5.8$ kN

Max S.F = Greater of (+ve) and (−ve)

$= 5.8$ kN

4. A train of 5 wheel loads crosses a simply supported beam of span 22.5 m. Using influence lines, calculate the maximum positive and negative shear forces at mid span and absolute maximum bending moment anywhere in the span.

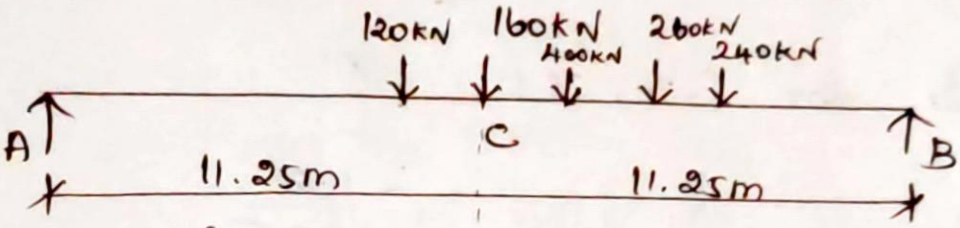


(i) Maximum (+ve) Shear Force :

To determine the load position to get the maximum positive shear force. Let us keep all the loads to the right of C. Then move W_1 load to the left of 'C' by 2.5m. If the sign of shear increment (S_i) is negative it will indicate that W_1 shall be retained at C.

$$S_i = \frac{W \times d}{l} - W_1$$

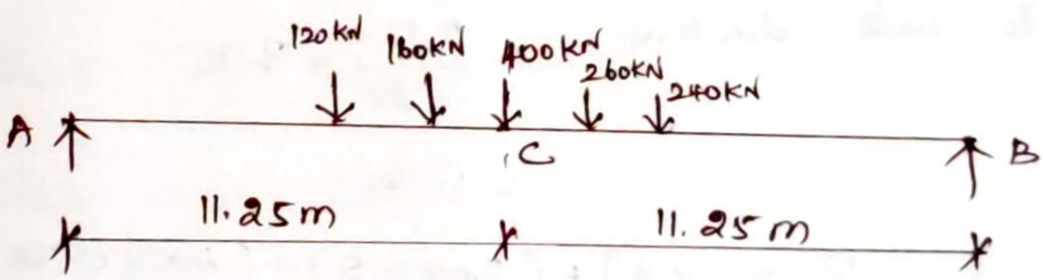
W = Total load
d - distance b/w the load train
l = length of span



$$S_i = \frac{(120+160+400+260+240) \times 2.5}{22.5} - 120$$

S_i = 11.11 KN (+ve)

Since S_i is positive the shear force increases due to the shifting of W to the left of C. Again let us move W_2 to the left of C by 2.5m to check whether the shear force further increases (or) not.



$$S_i = \frac{1180 \times 2.5}{22.5} - 160$$

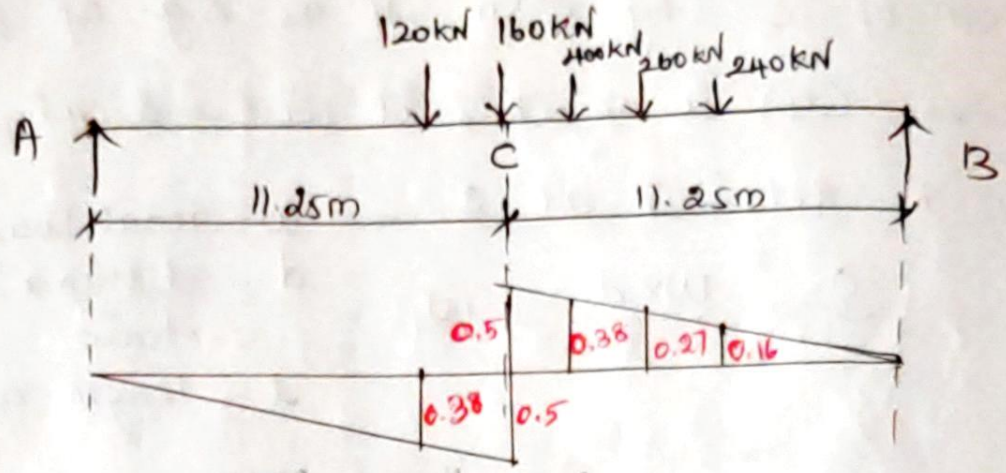
$= -28.89$ KN (-ve)

It indicates that to get maximum (+ve) shear force W_2 should stay just right of C.

Ordinate under 160 kN load $= \frac{L-x}{L}$

$= \frac{22.5-11.25}{22.5}$

$= 0.5$



Ordinate for (-ve) S.F $= -\frac{x}{L}$

$= -\frac{11.25}{22.5}$

$= -0.5$

Ordinate under 120 kN $= -\frac{0.5}{11.25} \times 8.75$

$= -0.38$

Ordinate under 400 kN $= \frac{0.5}{11.25} \times 8.75$

$= 0.38$

Ordinate under 260 kN $= \frac{0.5}{11.25} \times 6.25$

$= 0.27$

Ordinate under 240 kN $= \frac{0.5}{11.25} \times 3.75$

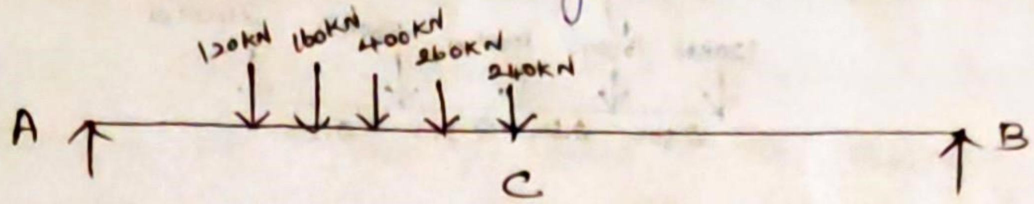
$= 0.16$

Max S.F $= (120 \times -0.38) + (160 \times 0.5) + (400 \times 0.38) + (260 \times 0.27) + (240 \times 0.16)$

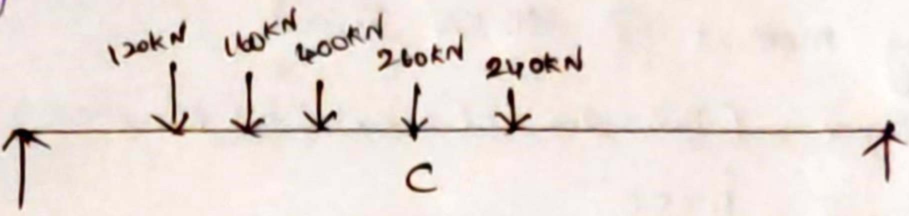
$= 295$ kN

(ii) Maximum Negative Shear Force:

To determine the position of loads to get the maximum negative S.F move the loads only one by one to the right of C and compute the value of S_i, if S_i become (-ve) it will be indicate a decrease on negative SF due to the load



First let us move the load W_5 to the right of 'C' by 2.5m

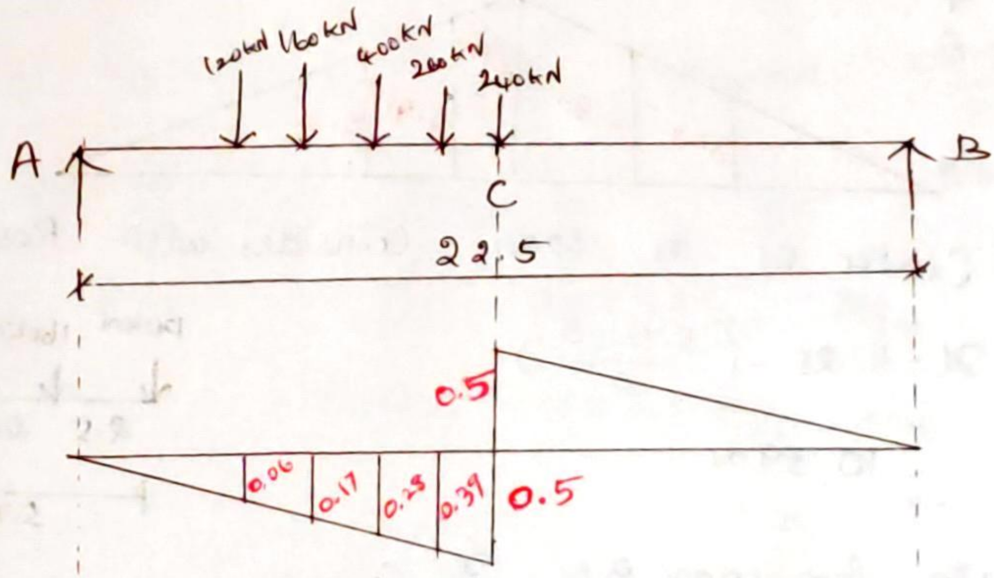


$$S_i = \frac{W \times d}{L} - W_5$$

$$= \frac{1180 \times 2.5}{22.5} - 240$$

$$S_i = -108.84 \text{ (-ve)}$$

Since it indicates that W_5 should stay just to the left of 'C'.



Ordinate under 240 KN $= -\frac{x}{L} = \frac{-11.25}{22.5} = -0.5$

Ordinate under 260 KN $= \frac{-0.5}{11.25} \times 8.75 = -0.39$

Ordinate under 400 KN $= \frac{-0.5}{11.25} \times 6.25 = -0.28$

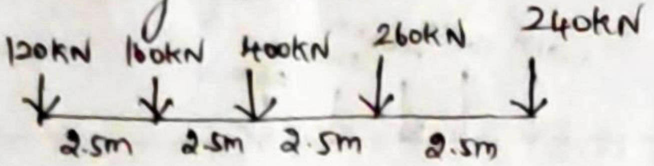
Ordinate under 160 KN $= \frac{-0.5}{11.25} \times 3.75 = -0.17$

Ordinate under 120 KN $= \frac{-0.5}{11.25} \times 1.25 = -0.06$

Max. S.F $= -[(240 \times 0.5) + (260 \times 0.39) + (400 \times 0.28) + (160 \times 0.17) + (120 \times 0.06)]$

$= -367.8$ KN

(iii) Absolute Bending moment :



$R = 120 + 160 + 400 + 260 + 240$

$= 1180$ KN

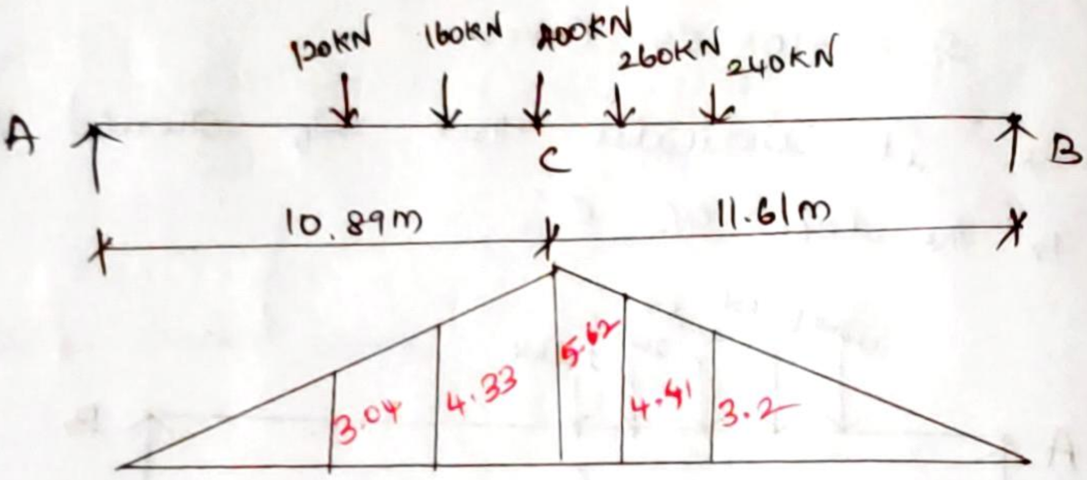
Taking moment @ 120 KN load,

$R \times \bar{x} = (160 \times 2.5) + (400 \times 5) + (260 \times 7.5) + (240 \times 10)$

$1180 \times \bar{x} = 6750$

$\bar{x} = 5.72$ m from 120KN load

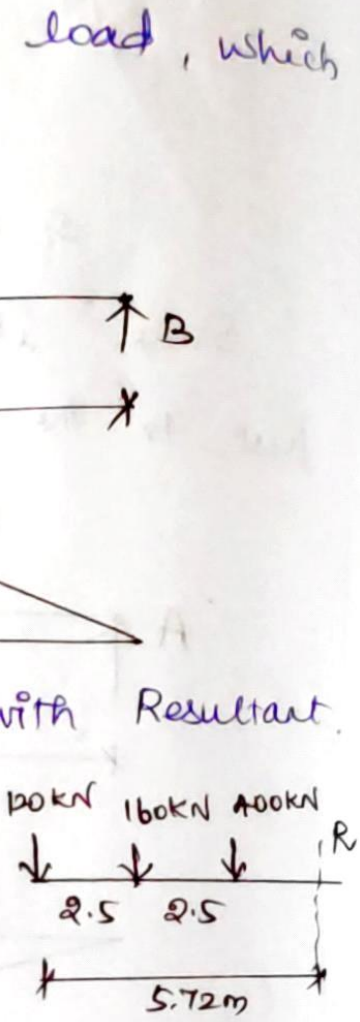
Absolute B.M occurs under the load, which is nearest to the resultant 'R'.



Center of the span coinsides with Resultant

$x = 11.25 - \left(\frac{5.72 - 5}{2}\right)$

$= 10.89$ m



Ordinate for max B.M @ 400 KN $= \frac{x}{L}(L - x)$

$= \frac{10.89}{22.5}(22.5 - 10.89)$

$= 5.62$

Ordinate under 160 KN $= \frac{5.62}{10.89} \times 8.39 = 4.33$

Ordinate under 120 KN $= \frac{5.62}{10.89} \times 5.89 = 3.04$

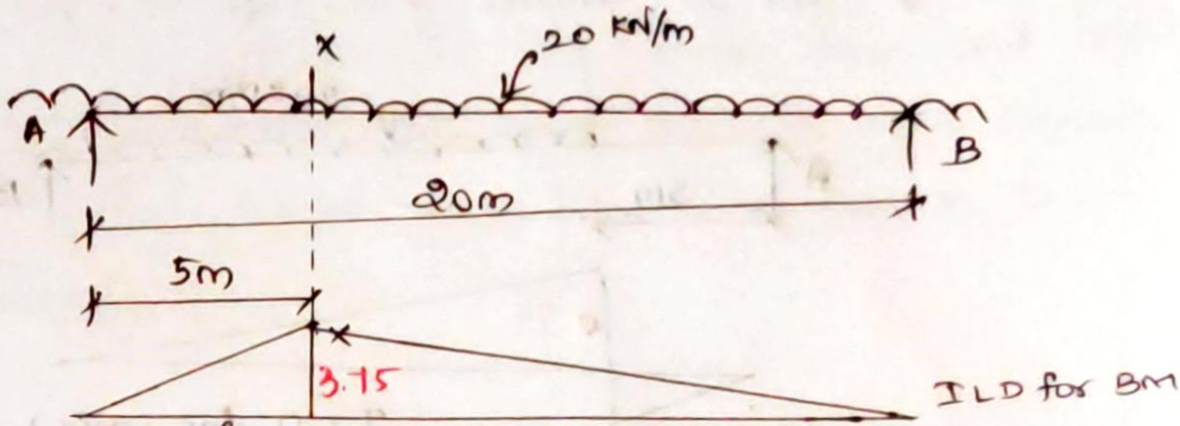
Ordinate under 260 KN $= \frac{5.62}{11.61} \times 9.11 = 4.41$

Ordinate under 240KN $= \frac{5.62}{11.61} \times 6.61 = 3.2$

Max. B.M $= (120 \times 3.04) + (160 \times 4.33) + (400 \times 5.62) + (260 \times 4.41) + (240 \times 3.2)$

$= 5220.2$ KNm

5. In a simply supported girder AB of span 20m, determine the maximum bending moment and maximum shear force at a section 5m from A, due to the passage of a uniformly Distributed load of intensity 20 kN/m longer than span.



Soln :

(i) Maximum Bending Moment :

Since the UDL is longer than the span, the criteria for maximum B.M at a section is that the whole span should be loaded as shown.

$R_A = R_B = \frac{wl}{2} = \frac{20 \times 20}{2} = 200 kN$

$\frac{x(L-x)}{L} = \frac{(20-5)5}{20} = 3.75$

B.M at $x = 5m$, $\Sigma M_x = 0$ (or)

$B.M = (3.75 \times \frac{1}{2} \times 5 \times 20) + (3.75 \times \frac{1}{2} \times 15 \times 20)$
$= 187.5 + 562.5$
$= 750 kNm$

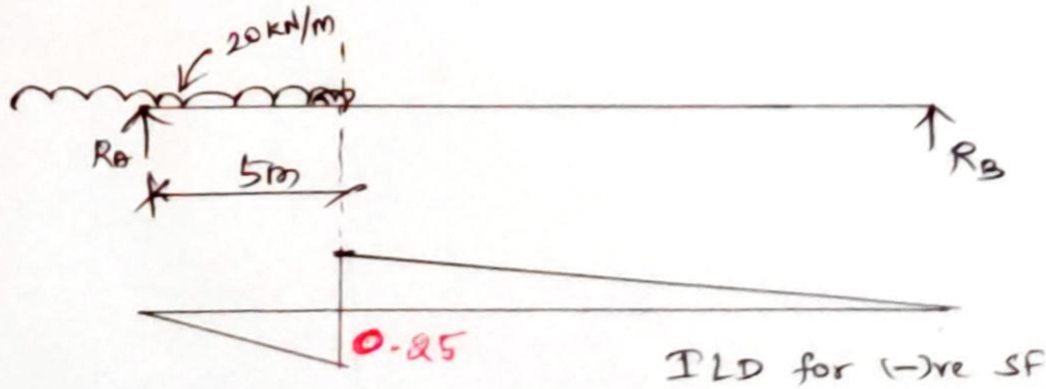
$R_A \times 5 - 20 \times 5 \times 5/2 = M_x$

$200 \times 5 - 20 \times 5 \times 5/2 = M_x$

$M_x = 750 kNm.$

(ii) Maximum Negative Shear Force :

It occurs when the head of the load reaches the section,



Ordinate of negative shear force $= -\frac{x}{L} = -\frac{5}{20}$

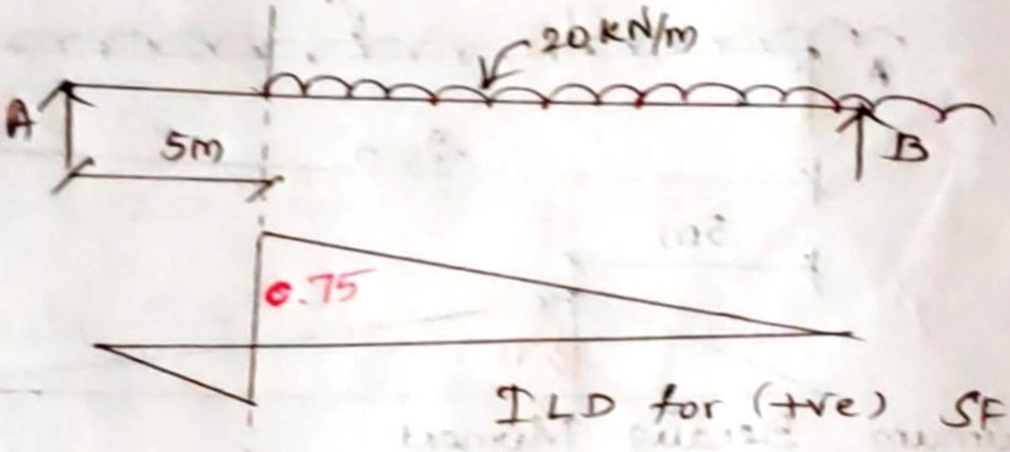
$= -0.25$

Max. Negative Shear force $= (\frac{1}{2} \times 5 \times 0.25) \times 20$

$= 12.5$ kN.

(ii) Max. Positive Shear Force:

It occurs @ when the tail of the lo is at x as it moves from left to right.



Ordinate for positive shear force $= \frac{L-x}{L} = \frac{20-5}{20}$

$= 0.75$

Max. (+ve) SF $= (\frac{1}{2} \times 15 \times 0.75) \times 20$

$= 112.5$ kN