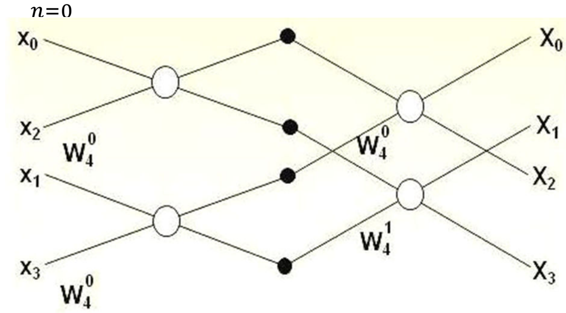
UNIT I- TRANSFORM

1. Compute the DFT of the sequence x(n)= {1,1,1,1}.
   DFT of x (n) is given by

$$X(k) = \sum_{n=0}^{N-1} \mathrm{x(n)}.e^{\frac{-j2\pi kn}{N}} \; ; \text{ Where K} = 0, 1, 2 \ldots\ldots \mathrm{N} - 1$$



W_4^0=1; W_4^1=e$^{-j2\pi/4}$ = -j

Input	S1	Output
1	1+1(1)=2	2+2(1)=4
1	1-1(1)=0	0+(-j)(0)=0
1	1+1(1)=2	2-2(1)=0
1	1-1(1)=0	0-(-j)(0)=0

X(K)={4,0,0,0}

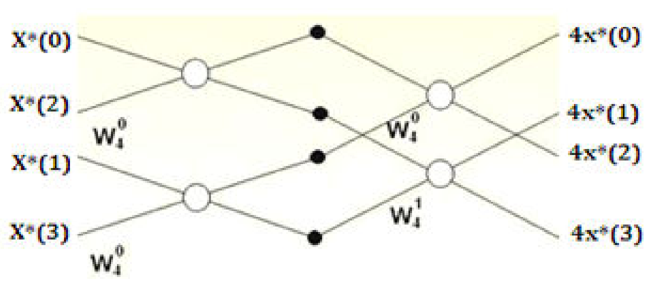
2. Perform circular convolution of two sequence x(n)={1,2,3} and h(n)={4,5,6}.

$$\begin{vmatrix} 4 & 6 & 5 \\ 5 & 4 & 6 \\ 6 & 5 & 4 \end{vmatrix} \begin{vmatrix} 1 \\ 2 \\ 3 \end{vmatrix} = \begin{vmatrix} 31 \\ 31 \\ 28 \end{vmatrix}$$

y(n)={31,31,28}

3. The first five DFT value for N=8 is as follows X(k)={28, -4+j9.656, -4+4j, -4+j1.656, -4, ...} compute rest of three DFT values.
   X(n)=X*(N-n)
   X(5)=X*(8-5)=X*(3)= -4 - j1.656
   X(6)=X*(8-6)=X*(2)= -4 - 4j
   X(7)=X*(8-7)=X*(1)= -4 - j9.656

4. Compute 4 point IDFT for X(k)={2, 3+j, -4, 3-j)



W_4^0=1; W_4^1=e$^{-j2\pi/4}$ = -j

Input	S1	Output
2	2+(-4)=-2	(-2)+(6)=4
-4	2-(-4)=6	6 + (-2j)(-j)=4
3-j	3-j +(3+j)=6	(-2)-(6)=-8
3+j	3-j -(3+j)=-2j	6 - (-2j)(-j)=8

x(n)={1,1,-2,2}

5. What is meant by radix-2FFT?

 The FFT algorithm is most efficient algorithm for calculating N-point DFT. If the number of output points N can be expressed as a power of 2, that is N=2^M, where M is an integer, then this algorithm is known as radix-2 FFT algorithm.

6. In direct computation of N-point DFT of a sequence, how many multiplications and additions are required? [Dec 2014] (Or) How is FFT faster? [May 2005] How many multiplications and additions are required to compute N point DFT using radix-2 FFT? [Dec 2009, May 2010]

 FFT is faster because it requires less number of complex multiplications and Complex additions compared to direct computation of DFT.

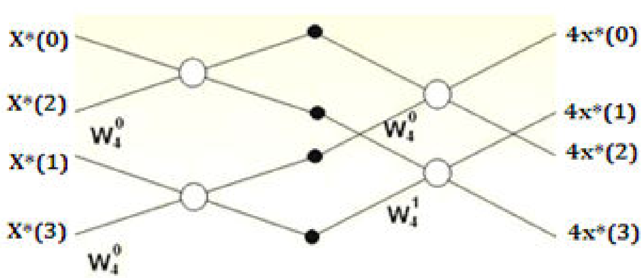
Operation	FFT	DFT
Complex multiplications	$\frac{N}{2} log_2 N$	N^2
Complex additions	$N log_2 N$	N (N - 1)

7. List any two properties of DFT.

 Let DFT{x (n)} =X (K), DFT{x1 (n)} =X1 (K), DFT{x2 (n)} =X 2(K)

 - Periodicity: X (K+N) =X (K) for all K.
 - Linearity: DFT[a1 x1 (n)+a2 x2(n)]=a1 X1 (K)+a2 X2 (K)
 - DFT of time reversed sequence: DFT[x(N-n)]=X(N-K)
 - Circular convolution :DFT[x1(n)*x2(n)]=X1(K) X2(K)

8. Compute the IDFT of Y(k)={1, 0, 1, 0}



W_4^0=1; W_4^1=$e^{-j2\pi/4}$ = -j

Input	S1	Output
1	1+1=2	(2)+(0)=2
1	1-1=0	0 + (0)(-j)=0
0	0+0=0	(2)-(0)=2
0	0-0=0	0 - (0)(-j)=0

x(n)={0.5, 0, 0.5, 0}

9. What is meant by radix-4 FFT?

The FFT algorithm is most efficient algorithm for calculating N-point DFT. If the number of output points N can be expressed as a power of 4, that is N = 4^M, where M is an integer, then this algorithm is known as radix-4 FFT algorithm.

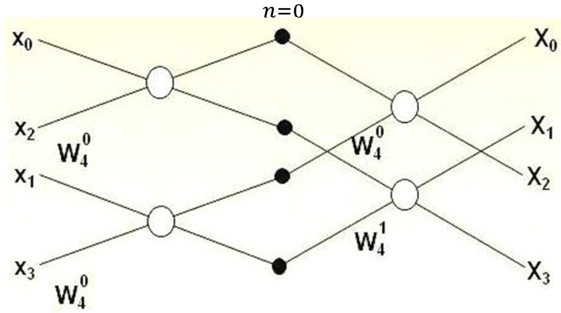
10. What is Twiddle factor?

In DFT computation, the term $W_N = e^{-j2\pi/n}$ is called as phase factor (or) twiddle factor. It is used to reduce the computational complexity and computational time.

11. Compute the DFT of the sequence x(n)={0, 1, 2, 3}.

DFT of x (n) is given by

$$X(k) = \sum_{n=0}^{N-1} \mathrm{x(n)}.e^{\frac{-j2\pi kn}{N}} \;;\; \text{Where K} = 0, 1, 2 \ldots\ldots \text{N} - 1$$



W_4^0=1; W_4^1=$e^{-j2\pi/4}$ = -j

Input	S1	Output
1	1+2(1)=3	3+4(1)=7
2	1-2(1)=-1	-1+2(-j)=-1-2j
1	1+3(1)=4	3-4(1)=-1
3	1-3(1)=2	-1-2(-j)=-1+2j

X(K)={7, -1-2j , -1, -1+2j}

12. Write down DFT pair of equation.

DFT of x (n) is given by

$$X(k) = \sum_{n=0}^{N-1} \mathrm{x(n)}.e^{\frac{-j2\pi kn}{N}} \;;\; \text{Where k} = 0, 1, 2 \ldots\ldots \text{N} - 1$$

IDFT of X (k) is given by

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} \mathrm{X(k)}.e^{\frac{j2\pi kn}{N}} \;;\; \text{Where n} = 0, 1, 2 \ldots\ldots \text{N} - 1$$

13. Calculate % saving in computing through radix 2 DFT algorithm of DFT coefficients. Assume N=512. [Dec 2010]

DFT

The number of complex multiplications required using direct computation is

N^2=512^2=2,62,144

The number of complex addition required using direct computation is

N(N-1)=512(512-1)=2,61,632

FFT

The number of complex multiplication required using FFT is

(N/2) $\log_2$ N=(512/2) $\log_2$ 512=2,304

The number of complex addition required in FFT isN $\log_2$ N=512 $\log_2$512=4608

% Saving in Multiplication is $\frac{2,62,144}{2,304} * 100 = 11377\%$

% Saving in Addition is $\frac{2,61,632}{4608} * 100 = 5677\%$

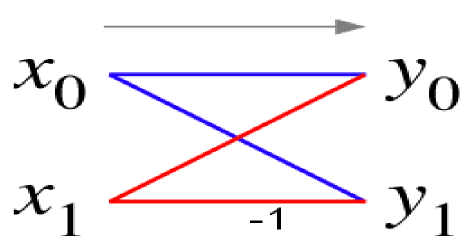
14. What are the advantages of FFT over DFTs?

The DFT computation time can be effectively reduced by incorporating the symmetry and periodicity properties of twiddle factor into the DFT computation. FFT is an algorithm which is used to compute DFT in a faster way.

It is based on the fundamental principle of decomposing the computation of DFT of a sequence of length N into successively smaller DFTs. It reduces the computational complexity and time complexity. There are two methods of FFT namely, Decimation-in-frequency (DIF) and Decimation-in-time (DIT).

15. What is meant by 'in place' in DIT and DIF algorithms? (or) What is inplace computation?

The basic butterfly diagram used in DIT is shown in fig. Here the two lines emerging from two nodes on the left side cross each other and connected to the two nodes on the right hand side. These nodes represent memory locations. At the input nodes, the inputs are stored. After the outputs are calculated, the same memory location is used to store the new values in place of the input values. The algorithm uses same location to store both the input and output sequences are called as 'in-place' algorithm.



16. How many stages are required in case of a 64 point radix-2 DIT FFT algorithm?[

We know that $2^m = N$,

Where 'N' is the no of sample points and 'm' is the no of stages involved in DIT FFT computation

Here, N=64 $2^m = 64$

$$2^m = 2^4\,; m = 4$$

Thus, no of stages involved in case of a 64 point radix-2 DIT FFT algorithm: m= 4

17. What is bit reversal?

When the binary representation of one number is the mirror image of the binary representation of the other, then both the numbers are said to be in bit reversal order. For example, in a three-bit system, binary equivalent of one and four are bit-reversed values of each other, since the three-bit binary representation of one, 001, is the mirror image of the three-bit binary representation of four, 100.

18. What are the differences and similarities between DIF and DIT?

DIT radix – 2 FFT	DIF radix – 2 FFT
The time domain sequence is decimated.	The frequency domain sequence is decimated.
When the input is in bit reversed order, the output will be in normal order and vice versa.	When the input is in bit normal order, the output will be in bit reversed order and vice versa.
In each stage of computations, the phase factors are multiplied before add and subtract operations.	In each stage of computations, the phase factors are multiplied after add and subtract operations.
The value of N should be expressed such that $N = 2^m$ and this algorithm consists of m stages of computations.	The value of N should be expressed such that $N = 2^m$ and this algorithm consists of m stages of computations.
Total number of arithmetic operations is NlogN complex additions and (N/2) logN complex multiplications.	Total number of arithmetic operations is NlogN complex additions and (N/2) logN complex multiplications.

19. What are the applications of FFT algorithms? [May 2009]
    1. Linear filtering
    2. Correlation analysis
    3. Power spectrum analysis
    4. Frequency analysis

20. Distinguish between linear convolution and Circular Convolution. [Dec 2005]

S. No	Linear Convolution	Circular Convolution
1.	If x(n) is a sequence of L number of samples and h(n) with m number of samples, after convolution y(n) will contain N = L + M – 1 samples.	If x(n) is a sequence of L number of samples and h(n) with m number of samples, after convolution y(n) will contain N = Max(L,M) samples
2.	Linear convolution can be used to find the response of a linear filter.	Circular convolution can be used to find the response of a linear filter
3.	Zero padding is not necessary to find the response of a linear filter.	Zero padding is necessary to find the response of a linear filter.