Fixed Points or Invariant Points:

Under the transformation $w = f(z)$, if the image of z is z itself, then the point is called a fixed point or an invariant point of the transformation.

i.e., $w = f(z) = z$.

$\therefore$ For bilinear transformation

$$w = \frac{az+b}{cz+d} = z.$$

$$cz^2 + dz = az + b$$

$$cz^2 + (d-a)z - b = 0$$

which is quadratic in z,

$\therefore$ we get two fixed points.

1) Find the fixed points of the transformation

$$w = \frac{2z+4i}{iz+1}$$

Soln:

The invariant points of the transformation is given by

$$z = \frac{2z+4i}{iz+1}$$

$$iz^2 + z = 2z + 4i$$

$$iz^2 + z - 2z - 4i = 0$$

$$iz^2 - z - 4i = 0.$$

$$-z^2 - iz + 4 = 0$$

$$z^2 + iz - 4 = 0$$

$$z(z+i) = 4$$

$$z = 4 ;\ z = 4 - i$$

are the invariant or fixed points

2) Find the fixed pts of the transformation.

$$w = \frac{6z-9}{z}.$$

Soln:

$$z = \frac{6z-9}{z}$$

$$z^2 = 6z - 9$$

$$z^2 - 6z + 9 = 0.$$

$$(z-3)(z-3) = 0$$

$z = 3, 3$ are the fixed pts

2) Find the fixed pts of the transformation (i) $w = \frac{3z-4}{z-1}$ $z=2, z=2$ (ii) $w = \frac{4z-4}{z}$

3) Find the fixed pts of the transformation $w = \frac{2z-5}{z+4}$

Soln:

$$z = \frac{2z-5}{z+4}$$

$$z^2 + 4z = 2z - 5$$

$$z^2 + 4z - 2z + 5 = 0.$$

$$z^2 + 2z + 5 = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$z = \frac{-2 \pm \sqrt{4 - 4(1)(5)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-20}}{2}$$

$$= \frac{-2 \pm \sqrt{-16}}{2}$$

$$= \frac{-2 \pm 4i}{2} = -2 \pm 2i$$

$$z = -1+2i ;\ -1-2i$$

are the fixed pts.