Conformal Mapping

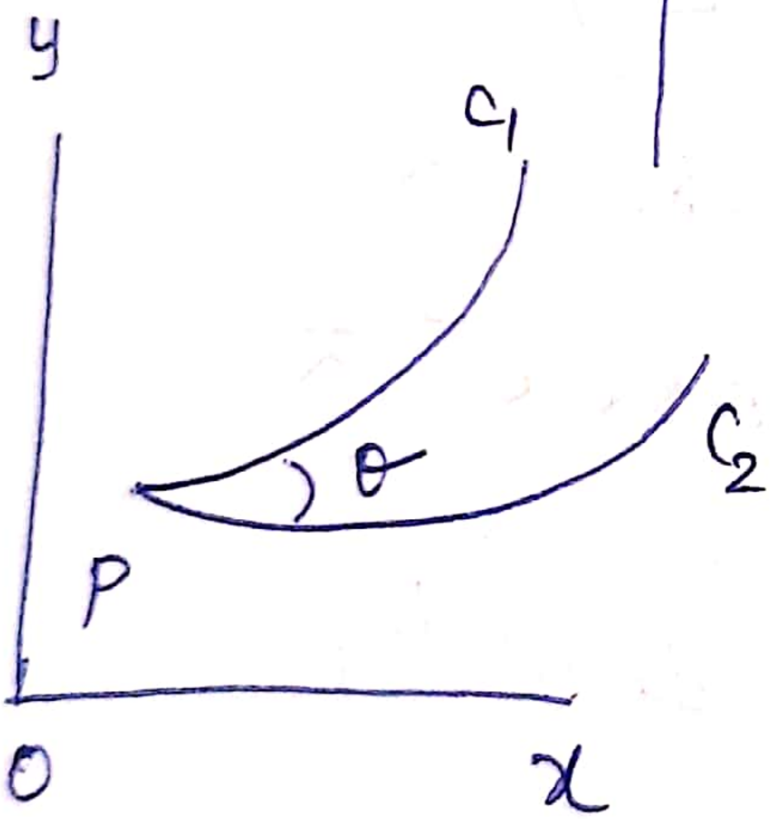
Defn:

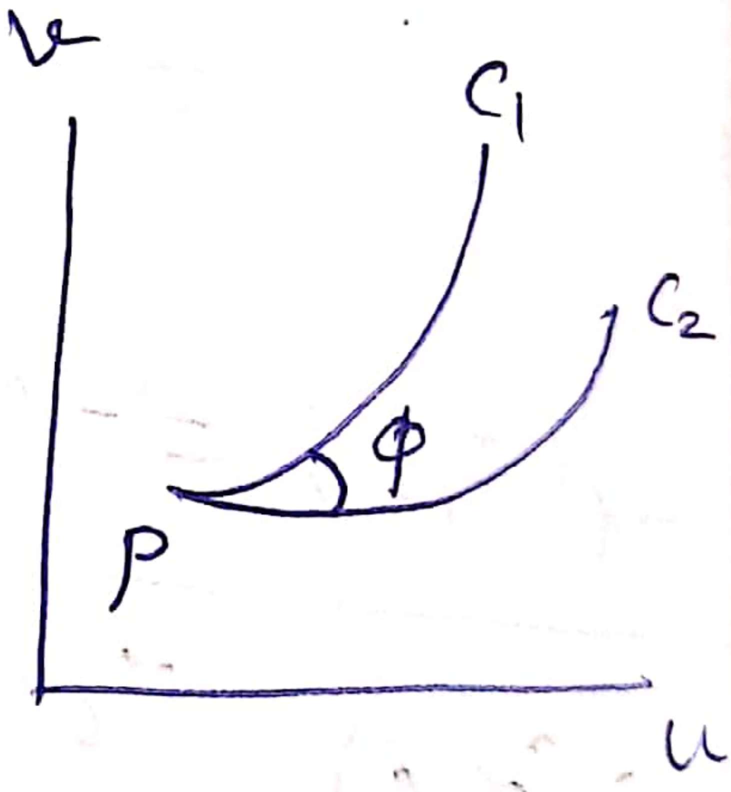
A mapping or transformation which preserves angles in magnitude and in sense between every pair of curves through a point is said to be conformal at that point.

(or)

A transformation or mapping that preserves angles between every pair of curves through a point both in magnitude and direction is said to be a conformal mapping at that point.

Complex	Complex
$z = x + iy$	$w = f(z) = u + iv$
z plane	w-plane
z-point $\longrightarrow$	w-point
curve C $\longrightarrow$	curve C'





Transformations:

(i) $w = c + z$ (Translation)

Problem:

What is the region of the w-plane into which the rectangular region in the z-plane bounded by the lines $x=0$; $y=0$; $x=1$; $y=2$ is mapped under the transformation $w = z + (2-i)$?

Soln: $w = z + (2-i)$

$u + iv = (x+iy) + (2-i)$

$= (x+2) + i(y-1)$

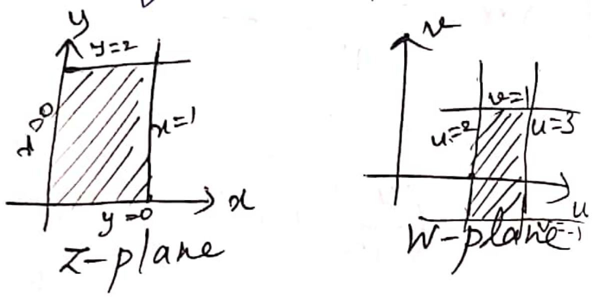
$u = x+2$; $v = y-1$.

when $x=0$ and $x=1$

$u = 2$ and $u = 3$.

when $y=0$ and $y=2$

$v=-1$ and $v=1$



Hence the given rec. region in the z-plane is translated into another rectangle with $u=2$; $u=3$
$v=-1$; $v=1$

as the boundaries

2) Find the image of the circle $|z| = 2$ by the transformation

$w = z + 3 + 2i$.

Soln:

W.K.T $z = x+iy$

$w = u+iv$.

$u+iv = x+iy+3+2i$

$= x+3+i(y+2)$

$\therefore u = x + 3 ; \quad v = y + 2$

$\Rightarrow x = u - 3 \qquad \Rightarrow y = v - 2$

Gn: $|z| = 2$

$$\sqrt{x^2 + y^2} = 2$$

$$(u-3)^2 + (v-2)^2 = 2^2$$

Hence the given circle $|z| = 2$ in z-plane is mapped to another circle with centre $(3, 2)$ and radius 2 in the w-plane