Angle b/w the two Surfaces.

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

1) Find the angle b/w the surfaces $x = y^2 - 1$ and $x^2 y = 2$ at the point $(1,1,1)$

Soln:

Given: $x = y^2 - 1$
and $x^2 y = 2$.

$\therefore \phi_1 = x - y^2 + 1$

$\phi_2 = x^2 y - 2$

Angle b/w the curves

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

$$\nabla\phi_1 = \vec{i}\frac{\partial\phi_1}{\partial x} + \vec{j}\frac{\partial\phi_2}{\partial y} + \vec{k}\frac{\partial\phi_3}{\partial z}$$

$$\frac{\partial\phi_1}{\partial x} = 1 ; \frac{\partial\phi_1}{\partial y} = -2y ; \frac{\partial\phi_1}{\partial z} = 0$$

$$\nabla\phi_1 = \vec{i}(1) + \vec{j}(-2y) + \vec{k}(0)$$

$$= \vec{i} - 2y\vec{j} + 0\vec{k}$$ →at the pt

$$|\nabla\phi_1| = \sqrt{1^2 + (-2)^2 + 0^2}$$

$$= \sqrt{1+4} = \sqrt{5}.$$

$$(\nabla\phi_1)_{(1,1,1)} = \vec{i} - 2\vec{j}$$

$$\nabla\phi_2 = \vec{i}\frac{\partial\phi_2}{\partial x} + \vec{j}\frac{\partial\phi_2}{\partial y} + \vec{k}\frac{\partial\phi_2}{\partial z}$$

$$\frac{\partial\phi_2}{\partial x} = 2xy ; \frac{\partial\phi_2}{\partial y} = x^2 ; \frac{\partial\phi_2}{\partial z}$$

$$\nabla\phi_2 = 2xy\vec{i} + x^2\vec{j} + 0\vec{k}.$$

~~$|\nabla\phi_2| = \sqrt{}$~~

$$(\nabla\phi_2)_{(1,1,1)} = 2\vec{i} + \vec{j}.$$

$$|\nabla\phi_2| = \sqrt{2^2 + 1^2} = \sqrt{5}.$$

$$\therefore \cos\theta = \frac{(\vec{i} - 2\vec{j})(2\vec{i} + \vec{j})}{\sqrt{5}\sqrt{5}}$$

$$= \frac{2-2}{\sqrt{5 \times 5}} = \frac{0}{5}$$

$$\cos\theta = 0$$

$$\theta = \cos^{-1}(0)$$

$$= \cos^{-1}\left(\cos\frac{\pi}{2}\right)$$

$$\therefore \boxed{\theta = \frac{\pi}{2}}$$

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2) Find the angle b/w the surfaces, $x^2+y^2+z^2=5$ and $x^2+y^2+z^2-2x=5$ at $(0,1,2)$

Soln:

Given: $\phi_1 = x^2+y^2+z^2-5$

$\phi_2 = x^2+y^2+z^2-2x-5$

$\nabla\phi_1 = \vec{i}\frac{\partial\phi_1}{\partial x}+\vec{j}\frac{\partial\phi_1}{\partial y}+\vec{k}\frac{\partial\phi_1}{\partial z}$

$\frac{\partial\phi_1}{\partial x} = 2x;\ \frac{\partial\phi_1}{\partial y} = 2y;\ \frac{\partial\phi_1}{\partial z} = 2z$

$\nabla\phi_1 = \vec{i}(2x)+\vec{j}(2y)+\vec{k}(2z)$

$= 2x\vec{i}+2y\vec{j}+2z\vec{k}$

$(\nabla\phi_1)_{(0,1,2)}$

$= 2(0)\vec{i}+2(1)\vec{j}+2(2)\vec{k}$

$= 2\vec{j}+4\vec{k}$

$|\nabla\phi_1| = \sqrt{2^2+4^2} = \sqrt{20}$

$= \sqrt{4\times5} = 2\sqrt{5}.$

$\nabla\phi_2 = \vec{i}\frac{\partial\phi_2}{\partial x}+\vec{j}\frac{\partial\phi_2}{\partial y}+\vec{k}\frac{\partial\phi_2}{\partial z}$

$\frac{\partial\phi_2}{\partial x} = 2x-2;\ \frac{\partial\phi_2}{\partial y} = 2y;\ \frac{\partial\phi_2}{\partial z} = 2z$

$\nabla\phi_2 = (2x-2)\vec{i}+2y\vec{j}+2z\vec{k}.$

$(\nabla\phi_2)_{(0,1,2)}$

$= -2\vec{i}+2\vec{j}+4\vec{k}.$

$|\nabla\phi_2| = \sqrt{(-2)^2+(2)^2+4^2}$

$= \sqrt{4+4+16} = \sqrt{24}$

$= \sqrt{4\times6} = 2\sqrt{6}.$

$\cos\theta = \frac{(2\vec{j}+4\vec{k})(-2\vec{i}+2\vec{j}+4\vec{k})}{2\sqrt{5}\ 2\sqrt{6}}$

$= \frac{4+16}{4\sqrt{30}} = \frac{20}{4\sqrt{30}}$

$= \frac{5}{\sqrt{5\times6}} = \sqrt{\frac{5}{6}}.$

$\cos\theta = \sqrt{\frac{5}{6}}$

$\theta = \cos^{-1}\left(\sqrt{\frac{5}{6}}\right).$

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Note:

If two surfaces cuts orthogonally $\nabla\phi_1 \cdot \nabla\phi_2 = 0$.

1) Find a and b such that the surface $ax^2 + byz = (a+2)x$ and $4x^2y + z^3 = 4$ cuts orthogonally at $(1,-1,2)$

Soln: Given: eqn of the Surfaces

$ax^2 + byz = (a+2)x$

$\therefore \phi_1 = ax^2 + byz - (a+2)x$ — (1)

$4x^2y + z^3 = 4$

$\therefore \phi_2 = 4x^2y + z^3 - 4$ — (2)

Using (1),

$$\nabla\phi_1 = \vec{i}\frac{\partial\phi_1}{\partial x} + \vec{j}\frac{\partial\phi_1}{\partial y} + \vec{k}\frac{\partial\phi_2}{\partial z}$$

$$= \vec{i}\left[2ax + 0 - (a+2)\right] + \vec{j}\left[0 + bz - 0\right] + \vec{k}\left[by\right]$$

$$= \vec{i}(2ax - (a+2)) + \vec{j}(bz) + \vec{k}\,by$$

$(\nabla\phi_1)_{(1,-1,2)}$

$$= \vec{i}(2a - (a+2)) + \vec{j}(b(2)) + \vec{k}(b(-1))$$

$$= (a-2)\vec{i} + 2b\vec{j} - b\vec{k}$$

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$\nabla\phi_1 = (a-2)\vec{i} + 2b\vec{j} - b\vec{k}$.

Using ②,

$$\nabla\phi_2 = \vec{i}\frac{\partial\phi_2}{\partial x} + \vec{j}\frac{\partial\phi_2}{\partial y} + \vec{k}\frac{\partial\phi_2}{\partial z}$$

$$= \vec{i}(8xy) + \vec{j}(4x^2) + \vec{k}(3z^2)$$

$$= 8xy\vec{i} + 4x^2\vec{j} + 3z^2\vec{k}$$

$$(\nabla\phi_2)_{(1,-1,2)} = -8\vec{i} + 4\vec{j} + 3(4)\vec{k}$$

$$= -8\vec{i} + 4\vec{j} + 12\vec{k}.$$

Two surfaces cut orthogonally

$$\nabla\phi_1 \cdot \nabla\phi_2 = 0$$

$$[(a-2)\vec{i} + 2b\vec{j} - b\vec{k}]$$

$$\cdot [-8\vec{i} + 4\vec{j} + 12\vec{k}] = 0$$

$$(a-2)(-8) + 8b - 12b = 0$$

$$-8a + 16 - 4b = 0$$

$$8a + 4b = 16$$

$$2a + b = 4 \quad \text{—— ①}$$

since $(1,-1,2)$ lies on

$$ax^2 + byz - (a+2)x = 0$$

$$a - 2b - (a+2) = 0$$

$$-2b = 2$$

$$\boxed{b = -1}$$ //

sub in ①, $2a - 1 = 4$

$$2a = 5$$

$$\boxed{a = 5/2}$$ //

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