2.) Given that $\frac{dy}{dx} = 1+y^2$, $y(0.6) = 0.6841$, $y(0.4) = 0.4228$, $y(0.2) = 0.2027$, $y(0) = 0$, find $y(-0.2)$ using Milne's method.

Given: $y' = 1+y^2$, $h = -0.2$.

$x_0 = 0.6$, $y_0 = 0.6841$.

$x_1 = 0.4$, $y_1 = 0.4228$

$x_2 = 0.2$, $y_2 = 0.2027$, $x_3 = 0$, $y_3 = 0$, $x_4 = -0.2$, $y_4 = ?$

By Milne's predictor formula,

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

Put $n = 3$

$$y_{4,p} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

Given $y' = 1+y^2$

$y_1' = 1+y_1^2 = 1+(0.4228)^2 = 1.1788$

$y_2' = 1+y_2^2 = 1+(0.2027)^2 = 1.0411$

$y_3' = 1+y_3^2 = 1+0 = 1$

$$y_{4,p} = 0.6841 + \frac{4(-0.2)}{3}\left[2(1.1788) - 1.0411 + 2(1)\right]$$

$= 0.6841 - 0.2667(3.3165)$

$= 0.6841 - 0.8844$

$= -0.2003.$

By Milne's corrector formula, we have

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left[y_{n-1}' + 4y_n' + y_{n+1}'\right]$$

Put $n=3.$

$$y_{4,c} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$y_4' = 1+y_4^2 = 1+(-0.2003)^2 = 1.0401.$

$$y_{4,c} = (0.2027) - \frac{0.2}{3}\left[1.0411 + 4(1) + 1.0401\right]$$

$= 0.2027 - 0.4054$

$= -0.2027.$