2) Using R.K method of 4^{th} order solve $y' = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$

Given $y' = \frac{y^2 - x^2}{y^2 + x^2}$, $f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$, $x_0 = 0, y_0 = 1,$

$x_1 = 0.2$, $h = 0.2 - 0 = 0.2$, $y_1 = ?$

$k_1 = hf(x_0, y_0) = (0.2) f(0, 1) = 0.2 \frac{1^2 - 0^2}{1^2 + 0^2}$

$= 0.2$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) = (0.2) f(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2})$

$= (0.2) f(0.1, 1.1)$

$= 0.2 \frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}$

$= (0.2) \frac{(1.2)}{1.22} = 0.1967.$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}) = (0.2) f(0 + \frac{0.2}{2}, 1 + \frac{0.1967}{2})$

$= (0.2) f(0.1, 1.0984)$

$= 0.2 \frac{(1.0984)^2 - (0.1)^2}{(1.0984)^2 + (0.1)^2}$

$= 0.2 (0.9836)$

$= 0.1967$

$k_4 = hf(x_0 + h, y_0 + k_3) = (0.2) f(0 + 0.2, 1 + 0.1967)$

$= (0.2) f(0.2, 1.1967)$

$= (0.2) \frac{(1.1967)^2 - (0.2)^2}{(1.1967)^2 + (0.2)^2} = (0.2)(0.9457)$

$= 0.1891$

$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= \frac{1}{6}[0.2 + 2(0.1967) + 2(0.1967) + 0.1891]$

$= 0.1960.$

$y_1 = y_0 + \Delta y = 1 + 0.1960$

$= 1.1960.$