Euler's method.

$$y_{n+1} = y_n + h f(x_n, y_n) \quad , n = 0, 1, 2, \ldots$$

Modified Euler's method

$$y_{n+1} = y_n + h\left[f\left(x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)\right)\right]$$

1. Using Euler's method, find the solution of the initial value problem $\frac{dy}{dx} = \log(x+y)$, $y(0) = 2$ at $x = 0.2$ by assuming $h = 0.2$

$f(x,y) = \log(x+y)$

$x_0 = 0, y_0 = 2, x_1 = 0.2, \quad h = x_1 - x_0 = 0.2 - 0 = 0.2$

$$y_{n+1} = y_n + h f(x_n, y_n)$$

$$y(0.2) = y_1 = y_0 + h f(x_0, y_0) = 2 + (0.2) \log(0+2)$$

$$= 2 + 0.2 \log 2$$

$$= 2 + 0.2 (0.3010)$$

$$= 2.0602.$$

2. Using Euler's method, find $y(0.2)$ and $y(0.4)$ from $\frac{dy}{dx} = x + y$, $y(0) = 1$ with $h = 0.2$

$f(x,y) = x + y$, $x_0 = 0$, $y_0 = 1$, $x_1 = 0.2$, $x_2 = 0.4$, $h = 0.2 - 0 = 0.2$

$$y_{n+1} = y_n + h f(x_n, y_n)$$

$$y(0.2) = y_1 = y_0 + h f(x_0, y_0) = 1 + (0.2)(0+1)$$

$$= 1 + 0.2 = 1.2$$

$$y(0.4) = y_2 = y_1 + h f(x_1, y_1) = 1.2 + (0.2)(0.2 + 1.2)$$

$$= 1.2 + (0.2)(1.4)$$

$$= 1.2 + 0.28$$

$$= 1.48.$$

3. Using Euler's method, solve $y' = x + y + xy$, $y(0) = 1$

Compute y at $x = 0.1$ by taking $h = 0.05$.

$$f(x, y) = x + y + xy$$

$x_0 = 0$, $y_0 = 1$, $h = 0.05$

$\therefore x_1 = 0 + 0.05 = 0.05$

$x_2 = 0.05 + 0.05 = 0.1$

$$y_1 = y(0.05) = y_0 + hf(x_0, y_0)$$

$$= 1 + (0.05)(x_0 + y_0 + x_0 y_0)$$

$$= 1 + (0.05)(0 + 1 + 0)$$

$$= 1.05.$$

$$y(0.1) = y_2 = y_1 + hf(x_1, y_1) = 1.05 + (0.05)(0.05 + 1.05 + (0.05)(1.05$$

$$= 1.05 + (0.05)[1.1 + 0.0525]$$

$$= 1.05 + (0.05)(1.1525)$$

$$= 1.05 + 0.057625$$

$$= 1.107625$$

$$= 1.1076$$