Numerical integration.

Trapezoidal rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$

$$= \frac{h}{2}\left[(\text{sum of the first \& last ordinates}) + 2(\text{sum of remaining ordinates})\right]$$

1. Using Trapezoidal rule, evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals

$y(x) = \frac{1}{1+x^2}$, Length of the interval $= b-a = 1-(-1) = 2$.

divide 2 into 8 intervals, $h = \frac{2}{8} = 0.25$

x: -1	-0.75	-0.5	-0.25	0	0.25	0.5	0.75	1
y: 0.5	0.64	0.8	0.9412	1	0.9412	0.8	0.64	0.5

$$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \frac{0.25}{2}\left[(0.5+0.5) + 2(0.64+0.8+0.9412+1+0.9412+0.8+0.64)\right]$$

$$= \frac{0.25}{2}\left[1+2(5.7624)\right]$$

$$= \frac{0.25}{2}\left[12.5248\right]$$

$$= 1.5656$$

2. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ with $h = \frac{1}{6}$ by Trapezoidal rule.

$y(x) = \frac{1}{1+x^2}$, $h = \frac{1}{6}$

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
y	1	$\frac{36}{37}$	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{9}{13}$	$\frac{36}{61}$	$\frac{1}{2}$

$$\int_0^1 \frac{dx}{1+x^2} = \frac{1/6}{2}\left[\left(1+\frac{1}{2}\right) + 2\left(\frac{36}{37} + \frac{9}{10} + \frac{4}{5} + \frac{9}{13} + \frac{36}{61}\right)\right]$$

$$= \frac{1}{12}\left[\frac{3}{2} + 2(3.9554)\right]$$

$$= \frac{1}{2}\left[\frac{3}{2} + 7.9108\right]$$

$$= 0.7842$$