1. The population of a certain town is given below. Find the rate of growth of population in 1931, 1941, 1961, 1971.

Year	1931	1941	1951	1961	1971
Population in thousands	40.62	60.80	79.95	103.56	132.65

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1931	40.62				
		20.18			
1941	60.80		-1.03		
		19.15		5.49	
1951	79.95		4.46		-4.47
		23.61		1.02	
1961	103.56		5.48		
		29.09			
1971	132.65				

i) To get $f'(1931)$ & $f'(1941)$ we use forward formula.

$$u = \frac{x-x_0}{h} = \frac{1931-1931}{10} = 0.$$

$$\left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \dots\right]$$

$$= \frac{1}{10}\left[20.18 - \frac{(-1.03)}{2} + \frac{(5.49)}{3} - \frac{(-4.47)}{4}\right]$$

$$= \frac{1}{10}\left[20.18 + 0.515 + 1.83 + 1.1175\right]$$

$$= 2.3643.$$

ii) To find $y'(1941)$

$$u = \frac{x-x_0}{h} = \frac{1941-1931}{10} = 1$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2}\Delta^2 y_0 + \frac{3u^2-6u+2}{6}\Delta^3 y_0 + \frac{4u^3-18u^2+22u-6}{24}\Delta^4 y_0\right]$$

$$= \frac{1}{10}\left[20.18 + \frac{1}{2}(-1.03) - \frac{1}{6}(5.49) + \frac{1}{12}(-4.471)\right]$$

$$= 1.83775$$

To find $f'(1961)$ & $f'(1971)$ we use backward formula.

$$v = \frac{x-x_n}{h} = \frac{1961-1971}{10} = -1$$

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2v+1}{2}\nabla^2 y_n + \frac{3v^2+6v+2}{6}\nabla^3 y_n + \frac{4v^3+18v^2+22v+6}{24}\nabla^4 y_n + \dots\right]$$

$$= \frac{1}{10}\left[29.09 - \frac{1}{2}(5.48) - \frac{1}{6}(1.02) - \frac{1}{12}(-4.47)\right]$$

$$= \frac{1}{10}\left[29.09 - 2.74 - 0.17 + 0.3725\right]$$

$$= 2.653$$

To find $y'(1971)$

$$v = \frac{x-x_n}{h} = \frac{1971-1971}{10} = 0$$

$$\left(\frac{dy}{dx}\right)_{v=0} = \frac{1}{h}\left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \dots\right]$$

$$= \frac{1}{10}\left[29.09 + 2.74 + 0.34 - 1.1175\right]$$

$$= 3.10525.$$

2. A jet fighter's position on an air craft carrier runway was timed during landing.

t (sec)	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y (m)	7.989	8.403	8.781	9.129	9.451	9.750	10.031

where y is the distance from the end of the carrier. Estimate velocity $\left(\frac{dy}{dt}\right)$ and acceleration $\left(\frac{d^2y}{dt^2}\right)$ at i) t = 1.1 & ii) t = 1.6 using numerical differentiation.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.0	7.989						
		0.414					
1.1	8.403		-0.036				
		0.378		0.006			
1.2	8.781		-0.03		-0.002		
		0.348		0.004		0.001	
1.3	9.129		-0.026		-0.001		0.002
		0.322		0.003		0.003	
1.4	9.451		-0.023		0.002		
		0.299		0.005			
1.5	9.750		-0.018				
		0.281					
1.6	10.031						

i) To find t = 1.1:

$h = 0.1$, $u = \frac{x - x_0}{h} = \frac{1.1 - 1.0}{0.1} = \frac{0.1}{0.1} = 1$

$$\left(\frac{dy}{dx}\right)_{1.1} = \left(\frac{dy}{dx}\right)_{u=1} = \frac{1}{h}\left[\Delta y_0 + \left(\frac{2u-1}{2}\right)\Delta^2 y_0 + \left(\frac{3u^2 - 6u + 2}{6}\right)\Delta^3 y_0 + \frac{4u^3 - 18u^2 + 22u - 6}{24}\Delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{0.1}\left[0.414 + \frac{1}{2}(-0.036) + \left(\frac{-1}{3}\right)(0.006) + \frac{1}{12}(-0.002) + \cdots\right]$$

$$= 3.9483$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left(\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{6u^2 - 18u + 11}{12}\Delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.036 + 0 + \left(-\frac{1}{12}\right)(-0.002) + \cdots\right]$$

$$= -3.5833$$

iii) to find $t = 1.6$.

$$v = \frac{t - t_n}{h} = \frac{1.6 - 1.6}{0.1} = 0.$$

$$\left(\frac{dy}{dx}\right)_{t=1.6} = \left(\frac{dy}{dx}\right)_{v=0} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \dots\right]$$

$$= \frac{1}{0.1}\left[0.281 + \frac{1}{2}(-0.018) + \frac{1}{3}(0.005) + \frac{1}{4}(0.002) + \frac{1}{5}(0.003) + \frac{1}{6}(0.002)\right)$$

$$= 2.751.$$

$$\left(\frac{d^2y}{dx^2}\right)_{v=0} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \dots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.018 + 0.005 + \frac{11}{12}(0.002) + \dots\right]$$

$$= -1.1167.$$