3. Find the inverse of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{pmatrix}$

The Augmented matrix is

$$[A, I] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 4 & 8 & -3 & 1 & 0 \\ 0 & -6 & -7 & 0 & 0 & 1 \end{array}\right] \quad R_2 \to R_2 - 3R_1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 4 & 8 & -3 & 1 & 0 \\ 0 & 1 & 7/6 & 0 & 0 & -1/6 \end{array}\right] \quad R_3 \to \frac{R_3}{-6}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 7/6 & 0 & 0 & -1/6 \\ 0 & 0 & 8 & -3 & 1 & 0 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 7/6 & 0 & 0 & -1/6 \\ 0 & 0 & 1 & -3/8 & 1/8 & 0 \end{array}\right] \quad R_3 \to \frac{R_3}{8}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 7/16 & -7/48 & -1/6 \\ 0 & 0 & 1 & -3/8 & 1/8 & 0 \end{array}\right] \quad R_2 \to R_2 - \frac{7}{6} R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5/8 & 1/8 & 0 \\ 0 & 1 & 0 & -1/16 & -7/48 & -1/6 \\ 0 & 0 & 1 & -3/8 & 1/8 & 0 \end{array}\right]$$

Hence the inverse of the given matrix is

$$\begin{bmatrix} 5/8 & 1/8 & 0 \\ -1/16 & -7/48 & -1/6 \\ -3/8 & 1/8 & 0 \end{bmatrix}$$

4. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$ using Gauss Jordan method

The Augmented matrix is

$$[A, I] \sim \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 1 & 4 & 9 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 4 & 9 & 0 & 0 & 1 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 & 0 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 4 & 9 & 0 & 0 & 1 \\ 0 & -10 & -24 & 0 & 1 & -3 \\ 0 & -7 & -17 & 1 & 0 & -2 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 4 & 9 & 0 & 0 & 1 \\ 0 & -10 & -24 & 0 & 1 & -3 \\ 0 & 0 & 2 & -10 & 7 & -1 \end{array}\right] \quad R_3 \to 7R_2 - 10R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 8 & 0 & 90 & -63 & 11 \\ 0 & -10 & 0 & -120 & 85 & -15 \\ 0 & 0 & 2 & -10 & 7 & -1 \end{array}\right] \quad \begin{array}{l} R_1 \to 2R_1 - 9R_3 \\ R_2 \to R_2 + 12R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 20 & 0 & 0 & -60 & 50 & -10 \\ 0 & -10 & 0 & -120 & 85 & -15 \\ 0 & 0 & 2 & -10 & 7 & -1 \end{array}\right] \quad R_1 \to 10R_1 + 8R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 5/2 & -1/2 \\ 0 & 1 & 0 & 12 & -17/2 & 3/2 \\ 0 & 0 & 1 & -5 & 7/2 & -1/2 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1/20 \\ R_2 \to R_2/-10 \\ R_3 \to R_3/2 \end{array}$$

Hence the inverse of the given matrix is

$$\begin{pmatrix} -3 & 5/2 & -1/2 \\ 12 & -17/2 & 3/2 \\ -5 & 7/2 & -1/2 \end{pmatrix}.$$