Inverse of a matrix - Gauss - Jordan method

Let us find the inverse of a non-singular square matrix A of order 3. If X is the inverse of A, then $AX = I$ where I is the unit matrix of order 3. Now we've to find the elements of X.

Let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ & $X = \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix}$

$\therefore AX = I$ reduces to.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

which is equivalent to

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{11} \\ x_{21} \\ x_{31} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{12} \\ x_{22} \\ x_{32} \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_{13} \\ x_{23} \\ x_{33} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

We can solve these eqs by Gauss Jordan method.

1. Find the inverse of the matrix $\begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$ by Gauss Jordan method.

Gauss Jordan method.

Let $A = \begin{pmatrix} 5 & -2 \\ 3 & 4 \end{pmatrix}$ & $X = \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$ be the inverse of A.

so that $AX = I$

The augmented matrix is $[A, I] \sim \left[\begin{array}{cc|cc} 5 & -2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right]$

$$\sim \left[\begin{array}{cc|cc} 1 & -2/5 & 1/5 & 0 \\ 3 & 4 & 0 & 1 \end{array}\right] \quad R_1 \to \frac{R_1}{5}$$

$$\sim \left[\begin{array}{cc|cc} 1 & -2/5 & 1/5 & 0 \\ 0 & 26/5 & -3/5 & 1 \end{array}\right] \quad R_2 \to R_2 - 3R_1$$

$$\sim \left[\begin{array}{cc|cc} 1 & -2/5 & 1/5 & 0 \\ 0 & 1 & -3/26 & 5/26 \end{array}\right] \quad R_2 \to R_2 \times \frac{5}{26}$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 2/13 & 1/13 \\ 0 & 1 & -3/26 & 5/26 \end{array}\right] \quad R_1 \to R_1 + \frac{2}{5} R_2$$

Hence the inverse of the given matrix is

$$\begin{bmatrix} 2/13 & 1/13 \\ -3/26 & 5/26 \end{bmatrix} = \frac{1}{26}\begin{bmatrix} 4 & 2 \\ -3 & 5 \end{bmatrix}$$

2. Find the inverse of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{pmatrix}$ using Gauss-Jordan

The Augmented matrix is

$$[A, I] \sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & 2 & 2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right] \quad R_3 \to R_3 + R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 4 & 0 & 1 & 1 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right] \quad R_1 \to 2R_1 + R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 4 & 0 & 1 & 1 & 0 \\ 0 & -8 & 0 & 10 & 2 & 6 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right] \quad R_2 \to -4R_2 + 6R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 4 & 0 & 0 & 12 & 4 & 6 \\ 0 & -8 & 0 & 10 & 2 & 6 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right] \quad R_1 \to 2R_1 + R_2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1/4 \\ R_2 \to R_2/-8 \\ R_3 \to R_3/-4 \end{array}$$

Hence the inverse of the given matrix is $\begin{pmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{pmatrix}$