Diagonally dominant:

If the numerical value of the leading diagonal element in each row is greater than or equal to the sum of the numerical values of the other elements in that row, then it is called diagonally dominant.

ii) Gauss Seidel Method:

Let the system of simultaneous eqn be

$a_1x + b_1y + c_1z = d_1$; $a_2x + b_2y + c_2z = d_2$; $a_3x + b_3y + c_3z = d_3$

Assume:

$|a_1| > |b_1| + |c_1|$

$|b_2| > |a_2| + |c_2|$

$|c_3| > |a_3| + |b_3|$

The diagonals elements should be dominant so that the iteration process can be applied. This system of eqs can also be written as:

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z);\ y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z);\ z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

I Iteration: Let $y^{(0)} = z^{(0)} = 0$.

$$x^{(1)} = \frac{1}{a_1}(d_1 - b_1 y^{(0)} - c_1 z^{(0)})$$

$$y^{(1)} = \frac{1}{b_2}(d_2 - a_2 x^{(1)} - c_2 z^{(0)})$$

$$z^{(1)} = \frac{1}{c_3}(d_3 - a_3 x^{(1)} - b_3 y^{(1)})$$

II Iteration:

$$x^{(2)} = \frac{1}{a_1}(d_1 - b_1 y^{(1)} - c_1 z^{(1)})$$

$$y^{(2)} = \frac{1}{b_2}(d_2 - a_2 x^{(2)} - c_2 z^{(1)})$$

$$z^{(2)} = \frac{1}{c_3}(d_3 - a_3 x^{(2)} - b_3 y^{(2)})$$

This process is repeated till the difference b/w 2 consecutive approximations are negligible

1. Solve by Gauss Seidel method:

$x + y + 54z = 110$; $27x + 6y - 5z = 85$; $6x + 15y + 2z = 72$.

Let us rearrange the equations:

$27x + 6y - 5z = 85$; $6x + 15y + 2z = 72$; $x + y + 54z = 110$.
→ ① → ② → ③

$|27| > |6| + |5|$; $|15| > |6| + |2|$; $|54| > |1| + |1|$

① ⇒ $x = \frac{1}{27}(85 - 6y + 5z)$; ② ⇒ $y = \frac{1}{15}(72 - 6x - 2z)$

③ ⇒ $z = \frac{1}{54}(110 - x - y)$.

Let $y_0 = z_0 = 0$.

Iteration	$x = \frac{1}{27}(85 - 6y + 5z)$	$y = \frac{1}{15}(72 - 6x - 2z)$	$z = \frac{1}{54}(110 - x - y)$
1	3.148	3.541	1.913
2	2.432	3.572	1.926
3	2.426	3.573	1.926
4	2.425	3.573	1.926
5	2.425	3.573	1.926.

$\therefore$ The solution is $x = 2.425$, $y = 3.573$, $z = 1.926$.

2) Solve the following system by Gauss Seidel method

$9x - y + 2z = 9$; $x + 10y - 2z = 15$; $2x - 2y - 13z = -17$

$\rightarrow$ ① $\rightarrow$ ② $\rightarrow$ ③.

Clearly the co-efficient matrix is diagonally dominant so we can apply Gauss Seidel method

① $\Rightarrow$ $x = \frac{1}{9}(9 + y - 2z)$

② $\Rightarrow$ $y = \frac{1}{10}(15 - x + 2z)$

③ $\Rightarrow$ $z = \frac{1}{13}(2x - 2y + 17)$

Iteration	$x = \frac{1}{9}(9 + y - 2z)$	$y = \frac{1}{10}(15 - x + 2z)$	$z = \frac{1}{13}(2x - 2y + 17)$
1	1	1.4	1.3626
2	0.8528	1.6872	1.1795
3	0.9254	1.6433	1.1972
4	0.9165	1.6478	1.1952
5	0.9175	1.6473	1.1954
6	0.9174	1.6473	1.1954.

$\therefore$ The solution is $x = 0.9174$

$y = 1.6473$

$z = 1.1954$