

Iterative method: i) Gauss Jacobi method

Let the system of simultaneous equation be

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

The diagonal elements should be dominant, so that the iteration process can be applied. This system of eqs can also be written as,

$$x = \frac{1}{a_1} (d_1 - b_1y - c_1z); y = \frac{1}{b_2} (d_2 - a_2x - c_2z); z = \frac{1}{c_3} (d_3 - a_3x - b_3y)$$

Let us assume $x_0 = y_0 = z_0 = 0$.

1st iteration:

$$x^{(1)} = \frac{1}{a_1} (d_1 - b_1y^{(0)} - c_1z^{(0)})$$

$$y^{(1)} = \frac{1}{b_2} (d_2 - a_2x^{(1)} - c_2z^{(0)})$$

$$z^{(1)} = \frac{1}{c_3} (d_3 - a_3x^{(1)} - b_3y^{(1)})$$

2nd iteration:

$$x^{(2)} = \frac{1}{a_1} (d_1 - b_1 y^{(1)} - c_1 z^{(1)})$$

$$y^{(2)} = \frac{1}{b_2} (d_2 - a_2 x^{(1)} - c_2 z^{(1)})$$

$$z^{(2)} = \frac{1}{c_3} (d_3 - a_3 x^{(1)} - b_3 y^{(1)})$$

This process is repeated till the difference b/w two consecutive approximations are negligible.

1) Solve the following eqs by using Jacobi's iteration method

$$30x - 2y + 3z = 75; \quad x + 17y - 2z = 48; \quad x + y + 9z = 15$$

$$|30| > |-2| + |3|; \quad |17| > |1| + |-2|; \quad |9| > |1| + |1|$$

∴ The diagonal elements are dominant, the iteration process is applied here. The given system can be written as:

$$x = \frac{1}{30} (75 + 2y - 3z); \quad y = \frac{1}{17} (48 - x + 2z); \quad z = \frac{1}{9} (15 - x - y)$$

Let us assume $x_0 = y_0 = z_0 = 0$.

Iteration	$x = \frac{1}{30} (75 + 2y - 3z)$	$y = \frac{1}{17} (48 - x + 2z)$	$z = \frac{1}{9} (15 - x - y)$
1	2.5	2.8235	2.6667
2	2.5216	2.8726	1.0752
3	2.5840	2.8017	1.0673
4	2.5655	2.7971	1.0683
5	2.5796	2.7983	1.0708
6	2.5795	2.7978	1.0691
7	2.5796	2.7976	1.0692
8	2.5796	2.7976	1.0692

∴ The solution is $x = 2.5796, y = 2.7976, z = 1.0692$.

2. Solve the following system of eqs by Gauss Jacobi method

$$10x + 2y + z = 9; \quad x + 10y - z = -22; \quad -2x + 3y + 10z = 22$$

$$|10| > |2| + |1|; \quad |10| > |1| + |-1|; \quad |10| > |-2| + |3|$$

∴ The diagonal elements are dominant, the iteration process is applied here.

The given system can be written as.

$$x = \frac{1}{10} (9 - 2y - z); \quad y = \frac{1}{10} (-22 - x + z); \quad z = \frac{1}{10} (22 + 2x - 3y)$$

Let us assume $x_0 = y_0 = z_0 = 0$.

Iteration	$x = \frac{1}{10} (9 - 2y - z)$	$y = \frac{1}{10} (-22 - x + z)$	$z = \frac{1}{10} (22 + 2x - 3y)$
1	0.9	-2.2	2.2
2	1.12	-2.07	3.04
3	1	-2.008	3.045
4	0.9971	-1.9955	3.0024
5	0.9989	-1.9995	2.9981
6	1	-2.0001	2.9996
7	1	-2	3
8	1	-2	3

\therefore The solution is $x=1, y=-2, z=3$