Numerical Methods

Solutions of Equations.

Newton method (or) Newton Raphson Method.

Newton Rapson method is extensively used for analysis of flow in water distribution networks. It is used to find the root of non-linear equations.

Formula:

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}, \text{ provided } F'(x_n) \neq 0.$$

order = 2

Newton Raphson Condition:

$$|F(x)F''(x)| \leq |F'(x)|^2$$

Problems:

1. Find the smallest +ve root of the equation $x^3 - 2x + 0.5 = 0$

Let $f(x) = x^3 - 2x + 0.5$

$f'(x) = 3x^2 - 2.$

$F(0) = 0.5$

$F(1) = -0.5$

$\therefore$ The root lies b/w 0 & 1

$\because |F(0)| = |F(1)|$

Let us assume $x_0 = 0$

Newton Raphson formula: $x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$

Put $n=0$, $x_1 = x_0 - \frac{F(x_0)}{F'(x_0)} = 0 - \frac{0.5}{-2} = 0.25$

Put $n=1$, $x_2 = x_1 - \frac{F(x_1)}{F'(x_1)} = 0.25 - \frac{F(0.25)}{F'(0.25)} = 0.2586$

Put $n=2$, $x_3 = x_2 - \frac{F(x_2)}{F'(x_2)} = 0.2586.$

$\because$ x_2 & x_3 are equal roots, the smallest +ve root is 0.2586.

2. Compute the real root of $x \log x = 1.2$, correct to 3 decimal places using Newton Raphson method.

Let $F(x) = x \log x - 1.2$

$$F'(x) = x\left(\frac{1}{x}\right) + \log x (1) - 0 = 1 + \log x.$$

Now $F(0) = -ve$, $F(1) = -1.2$, $F(2) = -0.5979$, $F(3) = 0.2314$

$\therefore$ The root lies b/w 2 & 3

$\because |F(2)| > |F(3)|$, $\therefore$ let us assume $x_0 = 3$

Newton - Raphson formula is

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$$

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)} = 2.8434$$

$$x_2 = 2.8434 - \frac{F(2.8434)}{F'(2.8434)} = 2.7822$$

$$x_3 = 2.7822 - \frac{F(2.7822)}{F'(2.7822)} = 2.7576.$$

|||ly, $x_4 = 2.7476$, $x_8 = 2.7408$

$x_5 = 2.7435$, $x_9 = 2.7407$

$x_6 = 2.7418$, $x_{10} = 2.7407$.

$x_7 = 2.7411$

$\therefore$ The required root is 2.7407.

3. Find the $-ve$ root of $x^3 - \sin x + 1 = 0$.

Let $F(x) = x^3 - \sin x + 1.$

$F'(x) = 3x^2 - \cos x.$

$F(0) = 1$, $F(-1) = 0.8415$, $F(-2) = -6.0907.$

$\therefore$ The root lies b/w -1 & -2.

$\because |F(-1)| < |F(-2)|$. Let us assume $x_0 = -1$

$$x_1 = x_0 - \frac{F(x_0)}{F'(x_0)} = -1.3421$$

$$x_2 = -1.3421 - \frac{F(-1.3421)}{F'(-1.3421)} = -1.2564$$

$x_3 = -1.2491$, $x_4 = -1.2491$