Test of significance of difference of proportion.

$$z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} \quad \text{where} \quad p = \frac{n_1P_1 + n_2P_2}{n_1+n_2} \;\&\; q = 1-p$$

1. Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that % proportions of men & women in favour of the proposal are same at 5%.

Given: $n_1 = 400$, $n_2 = 600$

Proportion of men $= P_1 = \frac{200}{400} = 0.5$

Proportion of women $= P_2 = \frac{325}{600} = 0.541$

Null hypothesis H_0: Assume that there is no significant difference. The opinion of men and women are same

$H_0 : P_1 = P_2 = P$

Alternative hypothesis: H_1: $P_1 \neq P_2$ [two-tailed]

Level of significance: $\alpha = 5\%$. [fixed]

Test statistic: $z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$ where $p = \frac{n_1P_1+n_2P_2}{n_1+n_2}$

$$p = \frac{400(0.5)+600(0.541)}{400+600} = \frac{525}{1000} = 0.525$$

$$q = 1-p = 1-0.525 = 0.475$$

$$\therefore z = \frac{0.5 - 0.541}{\sqrt{(0.525)(0.475)\left(\frac{1}{400}+\frac{1}{600}\right)}} = \frac{-0.041}{0.032} = -1.34$$

Critical value: z_α at 5% [two-tailed] is 1.96

Conclusion: $|z| = 1.34 < 1.96 = |z_\alpha|$.

$\therefore$ Null hypothesis is accepted

Hence proportions of men and women in favour of the proposal are same.

2) Before an increase in excised duty on tea, 800 persons out of a sample of 1000 persons were found to be tea drinkers. After an increase in excise duty, 800 people were tea drinkers in a sample of 1200. Using standard error of proportion state whether there is significant decrease in the consumption of tea after the increase in excise duty.

Given: $n_1 = 1000$, $n_2 = 1200$.

$$p_1 = \frac{800}{100} = 0.8 , \quad p_2 = \frac{800}{1200} = 0.667 \qquad \Rightarrow p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$\therefore p = \frac{1000(0.8) + 1200(0.667)}{1000 + 1200} = \frac{1600}{2200} = 0.727$$

$$\Rightarrow q = 1 - p = 1 - 0.727 = 0.273$$

Null hypothesis: H_0: Assume that there is no significant difference in the consumption of tea before and after the increase in excise duty. (ie) $H_0: p_1 = p_2$

Alternative hypothesis: $H_1: p_1 > p_2$ [right tailed]

Level of significance: $\alpha = 5\%$. [fixed]

Test statistic:
$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.8 - 0.667}{\sqrt{(0.727)(0.273)\left(\frac{1}{1000} + \frac{1}{1200}\right)}}$$

$$= \frac{0.8 - 0.667}{0.019} = 7$$

Critical value: z_α at 5% [right tailed] is 1.645

Conclusion: $|z| = 7 > 1.645 = z_\alpha$

$\therefore$ Null hypothesis is rejected.

$\therefore$ There is a difference in the consumption of tea before and after the increase in excise duty.

3) In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant?

Given: $n_1 = 900$, $n_2 = 1600$

$$p_1 = \frac{20}{100} = 0.2, \quad p_2 = \frac{18.5}{100} = 0.185$$

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{900(0.2) + 1600(0.185)}{900 + 1600} = \frac{476}{2500} = 0.1904$$

$\Rightarrow q = 1 - p = 0.8096.$

Null hypothesis: There is no significant difference b/w the 2 proportions.

Alternative hypothesis: The difference b/w the 2 proportions are significant

Level of significance: $\alpha = 5\%$ [fixed]

Test statistic: $$Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{0.2 - 0.185}{\sqrt{(0.1904)(0.8096)\left(\frac{1}{900} + \frac{1}{1600}\right)}}$$

$$= \frac{0.015}{0.016} = 0.9375$$

Critical value: Z_α at 5% [2-tailed] is 1.96.

Conclusion: $Z = 0.9375 < 1.96 = Z_\alpha$.

∴ Null hypothesis is accepted. (ie) $H_0: P_1 = P_2$

the two proportions are

Hence the difference between

not significant.