Test of significance for single proportion:

Test statistic: $z = \dfrac{p - P}{\sqrt{\dfrac{PQ}{n}}}$, $p = \dfrac{x}{n}$

1. A manufacturer claimed that atleast 95% of the equipments which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at 5% level of significance.

Given: $n = 200$

Number of pieces conforming to specification $= 200 - 18 = 182$

p = proportion of pieces conforming to specifications $= \dfrac{182}{200} = 0.91$

P = population proportion $= \dfrac{95}{100} = 0.95$

$Q = 1 - P = 1 - 0.95 = 0.05$

Null hypothesis H_0: The proportion of pieces confirming to specifications (ie) $P = 95\%$

Alternative hypothesis: $H_1: P < 0.95$ (Left tailed test)

Level of significance: $\alpha = 5\%$ [fixed]

Test statistic: $z = \dfrac{p - P}{\sqrt{\dfrac{PQ}{n}}} = \dfrac{0.91 - 0.95}{\sqrt{\dfrac{0.95 \times 0.05}{200}}} = \dfrac{-0.04}{0.0154}$

$z = -2.59$

Critical value: Z_α at 5% [left tail] is -1.645

Conclusion: $|Z| = 2.59 > 1.645 = |Z_\alpha|$.

$\therefore$ Null hypothesis is rejected at 5% L.of S

Hence the manufacturer claim is rejected.

2. In a sample of 1000 people in karnataka, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat eaters are equally popular in this state at 1% level of significance?

Given: $n = 1000$

p = sample proportion of rice eaters = $\frac{540}{1000} = 0.54$

P = Population proportion of rice eaters = $\frac{1}{2} = 0.5$

$\therefore Q = 1 - P = 1 - 0.5 = 0.5$

Null hypothesis: Both rice and wheat are equally popular in the state. $H_0: P = 0.5$

Alternative hypothesis: $H_1: P \neq 0.5$ [two-tailed]

Level of significance: $\alpha = 1\% = 0.01$.

Test statistic: $Z = \dfrac{p - P}{\sqrt{\frac{PQ}{n}}} = \dfrac{0.54 - 0.50}{\sqrt{\frac{(0.5)(0.5)}{1000}}} = 2.532$

Critical value: Z_α at 1% [2-tailed] is 2.58.

Conclusion: $Z = 2.532 < 2.58 = Z_\alpha$.

$\therefore$ Null hypothesis is accepted.

(ie) Both rice and wheat eaters are equally popular in the state.

3) In a big city 325 men out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in the city are smokers?

Given : $n = 600$, Number of smokers $= 325$

p = sample proportion $= \frac{325}{600} = 0.5417$

P = population proportion $= \frac{1}{2} = 0.5$

$Q = 1 - P = 1 - 0.5 = 0.5$

Null Hypothesis: H_0 - The number of smokers and non-smokers are equal in the city (ie) $H_0 : P = 0.5$

Alternative hypothesis: $H_1 : P > 0.5$ [Right-tailed]

Level of significance: $\alpha = 5\%$. [fixed]

Test statistic: $z = \frac{p - P}{\sqrt{PQ/n}} = \frac{0.5417 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{600}}} = 2.04$

Critical value: z_α at 5%. [Right-tail] is 1.645

Conclusion: $z = 2.04 > 1.645 = z_\alpha$.

$\therefore$ Null hypothesis is rejected.

$\Rightarrow$ The majority of men in the city are smokers.

4) 40 people were attacked by a disease and only 36 survived. Will you reject the hypothesis that the survival rate (if attacked by the disease) is 85% in favour of the hypothesis that it is more at $\boxed{5\%}$ level of significance?

Given: $n = 40$, No. of people survived $= 36$.

p = sample proportion $= \frac{36}{40} = \frac{9}{10} = 0.9$.

P = population proportion $= 85\% = 0.85$

$Q = 1 - P = 1 - 0.85 = 0.15$

Null hypothesis: $H_0 : P = 0.85$ (ie) 85% of the people survived from the disease.

Alternative hypothesis: $H_1 : P > 0.85$ [1-tailed right]

Level of significance: $\alpha = 5\% = 0.05$

Test statistic: $z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.9 - 0.85}{\sqrt{\frac{(0.85)(0.15)}{40}}} = 0.886$

Critical value: Z_α at 5% [1-tail right] is 1.645

Conclusion : $|z| = 0.886 < 1.645 = |Z_\alpha|$

$\therefore$ Ho is accepted at 5%.

Hence 85% of the people survived from the attack

5) A coin is tossed 400 times and it turns up. head 216 times. Discuss whether the coin may be regarded as unbaised one.

Given: $n = 400$, $p = \frac{216}{400} = 0.5156$.

P = population proportion for hedd $= \frac{1}{2}$

$Q = 1 - P = 1 - \frac{1}{2} = \frac{1}{2}$

Null hypothesis: Ho : The coin is unbaised one (ie) $H_0: P = \frac{1}{2}$

Alternative hypothesis: The coin is baised one $H_1: P \neq \frac{1}{2}$

Level of significance: $\alpha = 5\%$ is fixed

Test statistic: $z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.5156 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{400}}} = \frac{0.0156}{0.025}$

$z = \frac{156}{250} = 0.624$

Critical value: Z_α at 5% [2-tailed] is 1.96.

Conclusion: $|z| = 0.624 < 1.96 = Z_\alpha$.

$\therefore$ Ho is accepted. (ie) The coin is unbaised.

HW Similar to 4) $n = 20$, $p = \frac{18}{20} = 0.9$, $P = 0.85$, $Q = 0.85$, $Z = 0.626$

Similar to 5) $n = 256$, $p = \frac{132}{256} = 0.5156$, $P = \frac{1}{2}$, $Q = \frac{1}{2}$, $Z = 0.4992$