VISCOELASTICITY:

The fluidlike behavior of a material (such as water and oil) can be described in terms of stress and strain as in the elastic solids, but the proportionality constant, viscosity (1/), is derived from the following relationship:

$$\sigma = \eta \, d\epsilon / dt$$

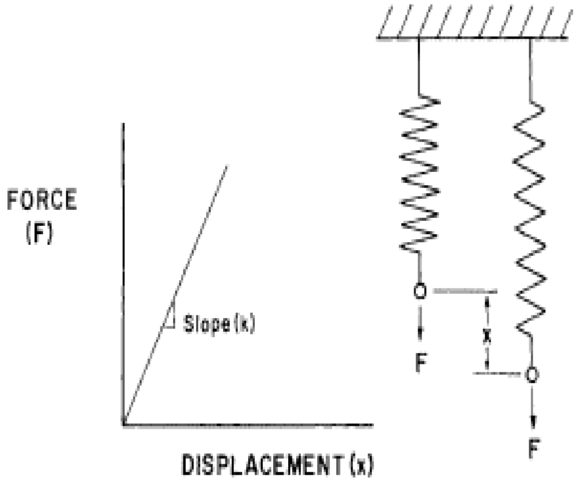

Figure 2-14. Force versus displacement of a spring.

It is noted that the stress and strain are shear rather than tensile or compressive although the same symbols are used to avoid complications. A mechanical analog (dashpot) can be used to simulate the viscous behavior of equation (2-14) as shown in Figure 2-15. An automobile shock-absorbing cylinder has a similar construction, oil being the damping fluid. By examining equation (2-14) one can see that the stress is time dependent, that is, if the deformation is accomplished in a very short time (dt ~ 0), then the stress becomes infinite. On the other hand, if the deformation is achieved slowly (dt ~ infinity), the stress approaches zero regardless of the viscosity value.

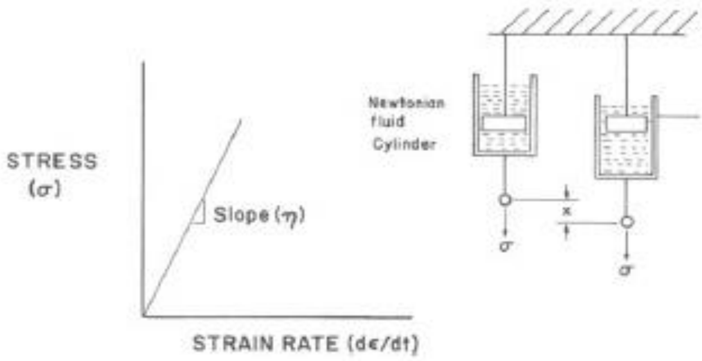

Figure 2-15. Stress versus strain rate of a dashpot. Note the stress and strain are in shear.

The stress-strain behaviour of the spring and dashpot can be represented as shown in Figure 2-16. If the spring and dashpot are arranged in series and parallel, they are called Maxwell and Voigt (or Kelvin) models, respectively. Remember that equation (2-6) does not involve time, implying the spring acts instantaneously when stressed. When the Maxwell model is stressed suddenly, the spring reacts instantaneously while the dashpot cannot react since the piston of the dashpot cannot move due to the infinite stress required by the surrounding fluid. However, if we hold the Maxwell model after instantaneous deformation, the dashpot will react due to the restriction of the spring and this will take time (dt = finite). The foregoing description can be expressed concisely by a simple mathematical formulation. In general, the response to stress by the Maxwell model will result in cumulative strain, that is, total strain (t: t) is a combination of the strain of spring (t:s) and dashpot (t:d):

$$\epsilon_t = \epsilon_s + \epsilon_d$$

Differentiating both sides,

$$d\epsilon_t/dt = d\epsilon_s/dt + d\epsilon_d/dt$$

$$d\sigma_s/dt = E\,d\epsilon_s/dt$$

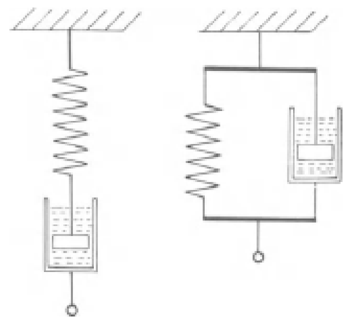

Figure 2-16. Two-element viscoelastic models.

and substituting equations (2-17) and (2-14) into (2-16),

$$d\epsilon_t/dt = (1/E)\,d\sigma_s/dt + \sigma_s/\eta \qquad (2\text{-}18)$$

Also, one can see that the total stress is the same for the spring and the dashpot since each member has the same applied load internally (or else it breaks!). Thus, equation (2-18) becomes

$$d\epsilon/dt = (1/E)\,d\sigma/dt + \sigma/\eta \qquad (2\text{-}19)$$

where ϵ is the total strain (ϵ_t). Equation (2-19) can be applied easily for a simple mechanical test condition such as *stress relaxation* in which the specimen is strained (or stressed) instantaneously and the relaxation of the load is monitored while the specimen is held at a constant length (strain). Thus, the strain rate becomes zero ($d\epsilon/dt = 0$) and equation (2-19) can be rewritten as

$$(1/E)\,d\sigma/dt + \sigma/\eta = 0 \qquad (2\text{-}20)$$

Therefore,

$$d\sigma/\sigma = -E\,dt/\eta \qquad (2\text{-}21)$$

and by integrating and knowing $\sigma = \sigma_0$ at $t = 0$,

$$\sigma/\sigma_0 = \exp(-Et/\eta) \qquad (2\text{-}22)$$

The constant η/E can be substituted with another constant τ called *relaxation time* and equation (2-22) will become

$$\sigma = \sigma_0 \exp(-t/\tau) = \sigma_0/\exp(t/\tau) \tag{2-23}$$

Examining equation (2-23) one can see that if the relaxation time is short, then the stress σ at a given time becomes small. On the other hand, if the relaxation time is long, then the stress σ is the same as the original stress, σ_0.

Similar analysis can be made with the *Voigt* model. In this case the strain of spring and dashpot represent the total strain, that is,

$$\epsilon_t = \epsilon_s = \epsilon_d = \epsilon \tag{2-24}$$

The total stress is a cumulative of spring and dashpot:

$$\sigma_t = \sigma_s + \sigma_d = 0 \tag{2-25}$$

Substituting equations (2-6) and (2-24) into (2-25),

$$\sigma = E\epsilon + \eta \, d\epsilon/dt \tag{2-26}$$

If a stress is applied and the stress is removed after a certain time, then

$$0 = E\epsilon + \eta(d\epsilon/dt) \tag{2-27}$$

which is similar to equation (2-20) and can be solved likewise; hence,

$$\epsilon_{\text{recovery}} = \epsilon_0 \exp(-Et/\eta) \tag{2-28}$$

where ϵ_0 is the strain at the time of stress removal. The constant η/E is termed *retardation time* λ for this *creep recovery* process. Since the strain is being removed from the original strain ϵ_0,

$$\epsilon(t) = \epsilon_0[1 - \exp(-t/\lambda)] \tag{2-29}$$