DC MACHINES

WORKED EXAMPLES

Example : 1

A 8–pole dC generator running at 1,200 rpm, and with a flux of 25 m Wb per pole generates 440 V. Calculate the number of conductors, if the armature is : (i) lap-wound, (ii) wave-wound.

Solution :

Here P = 8; N = 1,200 ; f = 25 mWB = 0.025 Wb ; E = 440 V; Z = ?

(i) Lap winding : A = P = 8.

$$\therefore \quad E_g = 440 = \frac{\phi ZN}{60} \text{ or } Z = \frac{440\times 60}{0.025\times 1{,}200} = 880 \text{ V}$$

(ii) Wave winding : A = 2.

$$E_g = \frac{\phi ZNP}{60A} \text{ or } Z = \frac{440\times 60\times A}{\phi NP} = \frac{440\times 60\times 2}{0.025\times 1{,}200\times 8} = 220 \text{ V}$$

Example : 2

A 4–pole shunt generator with a lap-wound armature has an armature resistance of 0.1 ohm, and field circuit resistance of 50 ohms. The generator is supplying six 100 V, 40 W lamps. Find the current in each armature conductor, and the generated emf. The brush contact drop is IV per brush.

Solution :

Total load = 60×40 W = 2,400 W; R_a = 0.1Ω; R_{sh} = 50Ω; V = 100 V I_L = 2,400/100 = 24 A; I_{sh} = 100/50 = 2A;

(i) $I_a = I_L + I_{sh} = 24 + 2 = 26$ A.

(ii) $E_b = V + I_aR_a$ + Brush drop = 100 + 26×0.1 + 2 = 104.6 V.

Example : 3

The armature of a 8–pole d.c. generator has 960 conductors and runs at 400 rpm. The flux per pole is 40 mWb. (i) Calculate the induced emf, if the armature is lap-wound. (ii) At what speed should it be driven to generate 400 V, if the armature were wave-connected?

Solution :

Here P = 8, Z = 960, ϕ = 49 mWb = 4×10^{-2} Wb

(i) Lap winding : A = P = 8, N = 400 rpm

$$\therefore \quad E_g = \phi\frac{ZN}{60} = \frac{4\times 10^{-2}\times 960\times 400}{60} = 256 \text{ V}$$

(ii) Wave-winding : Eg = 400 V, P = 8, A = 2, Z = 960, speed = N′.

$$N' = \frac{400\times 60\times 2}{4\times 10^{-2}\times 960\times 8} = 156.25 \text{ rpm}$$

Example : 4

Calculate the flux per pole required for a 4–pole generator with 360 conductors generating 250 V at 1,000 rpm, when the armature is : (i) lap-wound, (ii) wave wound.

Solution :

Here P = 4, Z = 360, N = 1,000 rpm, E_g = 250 V.

(i) Lap-wound : A = P = 4.

$$\therefore \quad E_g = 250 = \frac{\phi ZN}{60} = \frac{\phi\times 360\times 1{,}000}{60}$$

which gives ϕ = 0.04167 Wb or 41.67 mWb.

(ii) Wave-wound : A = 2

$$\therefore \quad E_g = 250 = \frac{\phi' ZN}{60}\left(\frac{P}{A}\right) = \frac{\phi' \times 360 \times 1,000 \times 4}{60 \times 2}$$

which gives ϕ' = 0.02083 Wb or 20.83 mWb.

Example : 5

(a) A 4–pole generator, with wave wound armature, has 51 slots, each having 24 conductors. The flux per pole is 0.01 weber. At what speed must the armature rotate to give an induced emf of 220 V² ?

(b) What will be the voltage developed, if the winding is lap and the armature rotates it the same speed ?

Solution :

(a) Here P = 4; Z = 51 × 24 = 1,224; ϕ = 0.01 Wb ; N = ? A = 2 (for wave wound), and E_g = 220 V.

$$\therefore \quad E_g = 220 = \frac{\phi ZNP}{60A} = \frac{0.01 \times 1,224 \times N \times 4}{60 \times 2}$$

or speed $$N = \frac{220 \times 60 \times 2}{0.01 \times 1,224 \times 4} = 539.2 \text{ rpm}$$

(b) For lap wound, A = P = 4.

$$\therefore \quad E_g = \frac{\phi ZN}{60A} = \frac{\phi ZN}{60} = \frac{0.01 \times 1,224 \times 539.2}{60} = 110 \text{ V}$$

Example : 6

A shunt generator has an induced voltage of 254 V. When the machine is loaded, the terminal voltage drops down to 240 V. Neglecting armature reaction, determine the load current, if the armature resistance is 0.04 ohm, and the field circuit resistance is 24 ohms.

Solution

E_g = 254 V ; V = 240 V ; R_a = 0.04 Ω ; R_{sh} = 24 Ω.

$$\therefore \quad I_{sh} = 240\text{V} / 24\ \Omega = 10\text{A}$$

Now $E_g = V + I_a R_a$ or 254 = 240 + I_a × 0.04

$\therefore$ I_a = (254–240)/0.04 = 350 A.

Hence, load current,

$I_L = I_a = I_{sh}$ = 350 – 10 = 340 A.

Example : 7

A 500, V, 10–pole d.c. shunt generator runs at 750 rpm, supplyung a load at rated voltage. The armature has 600 conductors, and is lap-wound. If the armature current is 200 A, and the armature resistance is 0.15 ohm, find the flux per pole.

Solution :

V = 500 V; P = 10; A = P = 10 (for lap winding) ; N = 750 rpm ; Z = 600; I_a = 200 A; R_a = 0.15 Ω ; ϕ = ?

Now $E_g = V + I_a R_a$ = 500 + 200 × 0.15 = 530 V
Also

$$\therefore \quad E_g = 530 = \frac{\phi ZNP}{60A} = \frac{\phi ZN}{60} = \frac{\phi \times 600 \times 750}{60}$$

$\therefore$ Useful flux/pole, $\phi = \dfrac{530 \times 60}{600 \times 750}$ = 0.0707 Wb.

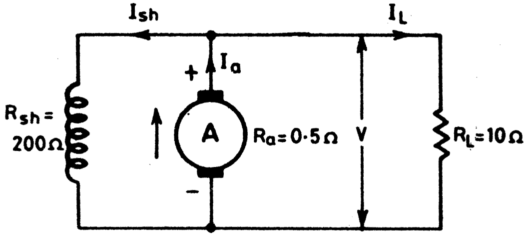
Example : 8

A four pole d.c. shunt generator with wave-connected armature has 41 slots, and 12 conductors per slot. Ra = 0.5 ohm, calculate the voltage across a 10 ohms load resistance connected across the armature terminals.

Solution :

Here P = 4, A = 2 (for wave winding) Z = 41 × 12 = 492 ;

R_a = 0.5 Ω; R_{sh} = 200 Ω ; φ = 125 × 10^{-3} Wb = 0.125 Wb; N = 1,000 rpm; R_L = 10 Ω.



$$\therefore \quad E_g = \frac{\phi ZNP}{60A} = \frac{0.125 \times 492 \times 1{,}000 \times 4}{60 \times 2} = 2{,}050 \text{ V}$$

$$V = E_g - (I_L + I_{sh})\, R_a = E_g - [(V/10) + (V/200)] \times 0.5$$

$$\therefore \quad V = E_g \times (400/421) = 2{,}050 \times (400/421) = 1{,}947.5 \text{ V.}$$

Example : 9

A 8–pole D.C. generator having a lap-wound armature is required to give the open-circuit voltage of 220 V. The total number of armature conductors is 2,000, and the total flux is 120 mWb. Calculate the speed at which the generator should be driven.

Solution :

Here φ = 120 mWb = 0.12 Wb ; Z = 2,000 ; P = 8 ; A = P = 8 (for lap winding); E_g = 220 V.

$$\therefore \quad E_g = 220 = \frac{\phi ZNP}{60A} = \frac{\phi ZN}{60} = \frac{0.12 \times 2{,}000 \times N}{60 \times 2}$$

$$\therefore \text{ Speed} \quad N = \frac{220 \times 60}{0.12 \times 2{,}000} = 55 \text{ rpm}$$

Example : 10

A four pole d.c. generator has a useful flux per pole of 0.07 Wb. The armature has 400 lap-wound conductors, each of resistance 0.002 Ω, and is rotating at a speed of 900 rpm.

If the armature current is 50 A, calculate the terminal voltage.

Solution :

Here P = 4 ; A = P = 4 (for lap winding) ; Z = 400 × 0.002 Ω = 0.8 Ω ; I_a = 50 A ; φ = 0.07 Wb ; N = 900 rpm.

$$\text{Now} \quad E_g = \frac{\phi ZNP}{60A} = \frac{\phi ZN}{60}$$

$$= \frac{0.07 \times 400 \times 900}{60} = 420 \text{ V}$$

∴ Terminal voltage,

$$V = E_g - IaRa = 420 - 50 \times 0.8 = 380 \text{ V.}$$

Example : 11

A 8-pole generator has a total of 500 conductors on its armature, and is designed to have 0.02 Wb of magnetic flux per pole, crossing its air-gap with normal excitation: (i) What voltage will be generated at a speed of 1,800 rpm, if the armature is : (a) wave-wound, (b) lap-wound? (ii) If allowable current is 5A per path, what will be the kW generated by machine in each case ?

Solution :

Here, P = 4; Z = 500 ; φ = 0.02 Wb ; N = 1,800 rpm.

(i) (a) Wave winding : A = 2

$$\therefore \quad E_g = \frac{\phi ZNP}{60A} = \frac{0.02 \times 500 \times 1{,}800 \times 4}{60 \times 2} = 600 \text{ V}$$

(b) Lap–winding : A = P = 4,

$$\therefore \quad E_g{}' = \frac{\phi ZN}{60} = \frac{0.02 \times 500 \times 1{,}800 \times 4}{60} = 300 \text{ V}$$

(ii) (a) Wave winding : Current in armature path,

$$Ia = A \times 5\ A = 2 \times 5A = 10\ A.$$

∴ Power generated = $E_g \times I_a$ = 600 V × 10A = 6,000 VA = 6kW.

(b) Lap winding : Current in armature path,

$$I_a' = A \times 5A = 4 \times 5A = 20\ A.$$

∴ Power generated = $E_g \times I_a$ = 300 V × 20A = 6,000 VA= 6 kW.

Example : 12

A 110 V d.c. shunt generator delivers a load current of 50 A. The armature resistance is 0.2 ohm, and the field circuit resistance is 55 ohms. The generator, rotating at a speed of 1,800 rpm, has 6 poles lap wound, and a total of 360 conductors. Calculate : (i) the no–load voltage at the armature, and (ii) the flux per pole.

Solution :

(a) V = 100 V; I_L = 50 A ; R_{sh} = 55 Ω ; R_a + 0.2 Ω ; N = 1,800 rpm ; P = 6 ; A = 6 (for lap wound) ; Z = 360.

∴ Shunt current, $I_{sh} = V/R_{sh}$ = 110/55 = 2A.

∴ Armature current, $I_a = I_r + I_{sh}$ = 50 + 2 = 52 V.

∴ No-load voltage in the armature,

$$E_g = V + I_aR_a = 110 + 52 \times 0.2 = 120.4\ V.$$

(ii) We know that :

$$E_g = \frac{\phi ZNP}{60A} = \frac{\phi ZN}{60} \qquad \text{(for lap wound)}$$

∴ Flux/pole = $E_g \times 60/ZN = 120.4 \times 60/360 \times 1{,}800 = 1.115 \times 10^{-3}$Wb.

Example : 13

A 8–pole dc generator has 500 armature conductors and a useful flux of 0.05 wb. What will be the e.m.f. generated, if it is lap–connected and runs at 1,200 r.p.m.? What must be

the which it is to be driven to produce the same e.m.f., if it is wave-wound?

Solution :

Here P = 8, Z = 500, ϕ = 0.05 Wb, N = 1,200 rpm.

(i) Lap–winding : A = P = 8

$$\therefore \quad E_g = \frac{\phi ZN}{60} = \frac{0.05 \times 500 \times 1{,}200}{60} = 500\ V$$

(ii) Wave-winding : A = 2, E_q = 500 V, N′ = ?

$$\therefore \quad E_g = \frac{\phi ZN'}{60}\left(\frac{P}{A}\right) \text{ or } 500 = \frac{0.05 \times 500 \times N'}{60}\left(\frac{8}{2}\right)$$

$$\text{or} \quad N' = \frac{500 \times 60 \times 2}{0.05 \times 500 \times 8} = 300 \text{ rpm}$$

Example : 14

The armature of a 4–pole, lap-wound shunt generator has 120 slots with 4 conductors per slot. The flux per pole is 0.05 Wb. The armature resistance is 0.05 Ω and shunt field resistance is 50 Ω. Find the speed of machine, when supplying 450 A at a terminal voltage of 250 V.

Solution :

Here terminal voltage, V = 400 V, P = 4, Z = 120 × 4 = 480, A = 4 (lap-wound), f = 0.05 W, R_{sh} = 250/50 = 5A

$$\therefore \quad I_a = I_L + I_{sh} = 450 + 5 = 455\ A$$

Now $\quad E_g = V + I_aR_a = 250 + 455 \times 0.05 = 272.75\ V.$

$$\text{But} \quad E_g = \frac{\phi Z}{60}\left(\frac{P}{A}\right) = \frac{0.05 \times 480 \times N}{60}\left(\frac{4}{4}\right) = 272.75$$

$$\text{Hence,} \quad N = \frac{272.75 \times 60}{0.05 \times 480} = 681.87 \text{ rpm}$$

Example : 15

A 4–pole shunt generator with a lap wound armature has an armature resistance of 0.1 Ω and field circuit resistance of 50 Ω. The generator supplies sixty 100 V, 40 W lamps. Find the current in each armature conductor and the generator emf. The brush contact drop is 1V per brush.

Solution :

Here P = 4, R_a = 0.1 Ω, R_{sh} = 50 Ω ; power supplied to load = 60×40 W; terminal voltage, V = 100 V.

∴ Load current, $I_L = \frac{Power}{Voltage} = \frac{60 \times 40}{100} = 24A$

Field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{100}{50} = 2A$

(i) Total armature current, $I_a = I_L + I_{sh}$ = 24 + 2 = 26 A

Since generator is lap-wound, so number of parallel paths (A) = number of poles (P) = 4

∴ Current/conductor = Current/parallel path + I_a/A = 26/4 = 6.5 A.

(ii) Induced emf, E_g = v + Sum of all internal voltage drops

= V + I_aR_a + Brush drop

= 100 + 26×0.1 + 2 = 104.6 V.

Example : 16

A d.c. shunt generator has an induced voltage on open-circuit of 127 V. When the machine is on load, the voltage is 120 V. Find the load current, if the field circuit resistance is 15 ohms, and armature resistance is 0.02 ohms. Neglect arnature reactance.

Solution

V_0 = 127 V ; V_L = 120 V ; R_{sh} = 15 Ω ; R_a = 0.02 Ω ; R_{g0}

= 127 + I_aR_a = 127 V ; $E_{gL} = V_0 + I_aR_a = E_{b0} + I_aR_a$ = 120 + I_aR_a.

∴ I_aR_a = 7 V or I_a = 7/0.02 = 350 A, and I_{sh} = 120/15 = 8A.

∴ $I_L = I_a - I_{sh}$ = 350 – 8 = 342 A.

Example : 17

A 100 kw, 230 V shunt generator has R_a = 0.05 ohm, and R_{sh} = 57.5 ohms. If the generator operates at rated voltage, calculate voltage at : (i) full–load, and (ii) half full–load. Neglect brush contact drop.

Solution :

I_L = 100,000 W/230 V = 434.8 A ; I_{sh} = 230 V/57.5 W = 4A, and $I_a = I_L + I_{sh}$ = 434.8 + 4 = 438.8 A.

(i) Full–load emf, $E_g = V + I_aR_a$ = 230 + 434.8×0.05 = 252 V.

(ii) At half full–load, $I_L' = \frac{1}{2} \times 434.8A = 217.4$ A,

and I_a' = 217.4 + 4 = 221.4 A.

∴ Emf induced at half full–load,

$E_g' = V + I_a'R_a$ = 230 + 221.4×0.05 = 241 V.

Example : 18

Estimate the reduction in the speed of a dynamo working with constant excition on 500 V bus-bars to decrease its load from kW to 250 kW. The resistance between terminals is 0.015 ohm. Neglect armature reaction.

Solution :

(i) I_L = 500 + 1,000×0.015 = 515 Ω.

∴ $E_g = V + I_LR_a$ = 500 + 1,000×0.015 = 515 V.

(ii) $I_L' = 250{,}000$ W/500 V = 500 A.

$\therefore$ $E_g' = 500 + 500 \times 0.015 = 507.5$ V.

Now $$\frac{E_g}{E_g'} = \frac{N}{N'} = \frac{515}{507.5} = 1.0147$$

or $$\frac{E_g - E_g'}{E_g'} = \frac{N - N'}{N'} = \frac{515 - 507.5}{515} = 0.01456$$

Hence, percentage reduction in speed

$$= \frac{N - N'}{N} \times 100 = 1.456\% .$$

Example : 19

(a) Find the flux per pole of a 50 kW d.c. generator having 4 poles, and a lap-wound armature with 380 conductors. The machine is run at a speed of 800 rpm, and generates 460 V. Resistance of the armature, and shunt field are 0.5 ohm, and 300 ohms respectively.

(b) Find the current flowing in the armature at full-load, and the terminal voltage.

Solution :

(a) Load = 50,000 W ; P = 4 ; A = 4 (lap-wound) ; Z = 380 ; N = 800 E_g = 460 V; R_a = 0.5 Ω ; R_{sh} = 300 Ω.

Now $$E_g = 460 = \frac{\phi ZNP}{60A} = \frac{380 \times 800 \times 4 \times \phi}{60 \times 4}$$

$$\therefore \text{Flux / pole}, \phi = \frac{460 \times 60 \times 4}{380 \times 800 \times 4} = 0.0908 \text{ Wb}$$

(b) Let the terminal voltage be 'V' then, I_L = 50,000/V ; I_{sh} = V/300.

$$\therefore \quad I_a = \left(\frac{50{,}000}{V} + \frac{V}{300}\right)$$

But $E_g = V + I_a R_a$

$$\therefore 460 = V + \left(\frac{50{,}000}{V} + \frac{V}{300}\right) \times 0.5 = V + \frac{25{,}000}{V} + \frac{V}{600}$$

or 460 = V2(1 + 1/600) + 25,000

or $1.00167\ V^2 - 460\ V + 25{,}000 = 0$

or $$V = \frac{460 \pm \sqrt{460^2 - 4 \times 1.00167 \times 25{,}00}}{2 \times 1.00167}$$

$$= \frac{460 \pm 333.82}{2.00334} = 396.2 \text{ V}$$

Hence $$I_a = I_L + I_{sh} = \frac{50{,}000}{396.5} + \frac{396.5}{300} = 127.43\text{A}$$

Example : 20

A 6–pole, 500 V dc generator has a flux / pole of 50 mWb, prouced a field current of 10A. Each pole is wound with 600 turns. The resistance of entire field is 50 Ω. If the field is broken in 0.02 s, calculate : (i) the inductance of the field coils ; (ii) the induced emf, and (iii) the value of discharge resistance so that the induced emf should not exceed 1,000 V.

Solution

N = 600 : I = 10 A; $\phi = 50 \times 10^{-3} = 0.05$ Wb ; t = 0.02 s.

(i) Inductance for 6–pole (L) = Flux × linkage/Ampere = 6φ N/1

= 0.05 × 600/10 = 3.6 H.

(ii) Induced emf (E_g) $= \dfrac{\text{Change in flux linkage}}{\text{Time in second}} = \dfrac{\phi N}{6}$

$$= \frac{0.05 \times 600}{0.02} = 1{,}500 \text{ V}$$

(iii) If the induced emf is 1,000 V, then total resistance

$= 1{,}000 \text{ V}/10\text{A} = 100\ \Omega$

Hence, the value of discharge resistance to keep the induced emf not exceeding 1,000 V

$= (100 - 50)\ \Omega = 50\ \Omega.$

Example : 21

A 4–pole d.c. generator has 564 conductors on its armature and is driven at 800 rpm. The flux per pole being 20 mWb and current in each conductor is 60 A. Calculate : (a) the total current, (b) emf, and (c) power generated in the armature, if the armature is (i) wave-wound, (ii) lap-wound.

Solution :

Here P = 4, Z = 564, N 800 rpm, ϕ = 20 mWb = 2×10^{-2} Wb, current/armature conductor. I_Z = 60 A

(i) Wave-wound : A = 2 $\therefore$ Current/parallel path, I_Z = 60 A.

$\therefore$ Total armature current, $I_a = 60 \times 2 = 120$ A

Induced emf, $E_g = \dfrac{\phi ZN}{60}\left(\dfrac{P}{A}\right) = \dfrac{2 \times 10^{-2} \times 564 \times 800}{60} \times \dfrac{4}{2} = 3{,}008 \text{ V}$

$\therefore$ Armature power,

$P_a = E_g \times I_a = 3{,}008 \times 120 = 360,\ 960$ W pr 360.96 kW

(ii) Lap-wound : A = P = 4 ; I_Z = 60A

Total armature current, $I'_a = 60 \times 4 = 240\text{A}$

Induced emf, $E'_s = \dfrac{\phi ZN}{60} = \dfrac{2 \times 10^{-2} \times 564 \times 800}{60} = 1{,}504 \text{ V}$

$\therefore$ Armature posser, $P'_a = E'_a \times I'_a = 1{,}504 \times 240 = 360,\ 960$ W or 360.96 kW.

Note :

Power developed in both cases in same, because in lap-winding the number of parallel paths is doubled, but the armature current is halved.

Example : 22

A 3–phase, 50 Hz, 16–pole generator with connected winding has 144 slots with 10 conductors star slot. The flux pole of 24.8 mWb is sinusodally distributed. The coil is full-pitched. Find : (i) speed, (ii) the line emf.

Solution :

(i) Speed, $N = \dfrac{120f}{P} = \dfrac{120 \times 50}{16} = 375 \text{ rpm}$

(ii) No. of slots/phase = 144/3 = 48 ; No. of conductors/slot = 10

$\therefore$ No of conductors/phase = $48 \times 10 + 480$

and No. of turns/phase, $T = \dfrac{\text{No. of conductors / phase}}{2}$

$$= \frac{480}{2} = 240.$$

Now pitch factor, k_c = 1 (since the coils is full-pitched)

$\therefore$ No. of slots/pole = 144/16 = 9.

Angular displacement, $\beta = 180^{\circ}/9 = 20^{\circ}$

No. of slots/pole/phase, m = 9/3 = 3

$\therefore$ Winding (or ditribution) factors,

$$k_d = \frac{\sin m\beta/2}{m \sin\beta/2} = \frac{\sin(3\times 20^0/2)}{3\sin(20^0/2)} = \frac{\sin 30^0}{3\sin 10^0} = 0.96$$

$\therefore$ V_{ph} = 4.44 k_c k_d f ϕ T = $4.44\times 1\times 0.96\times 0.0248\times 240$ = 1,268.5 V

$\therefore$ $V_L = \sqrt{3}\times 1{,}268.5$ V = 2,197 V

Example : 23

A short-shunt cumulative compound d.c. generator supplies 7.5 kW at 230V. The shunt field, series field, and the armature resistances are 100, 0.3, and 0.4 ohm respectively. Calculate : (i) the induced emf, and (ii) the load resistance.

Solution :

I_L = 7,500/230 = 32.6 A = I_{se}.

$\therefore$ Voltage across generator/shunt field terminals,

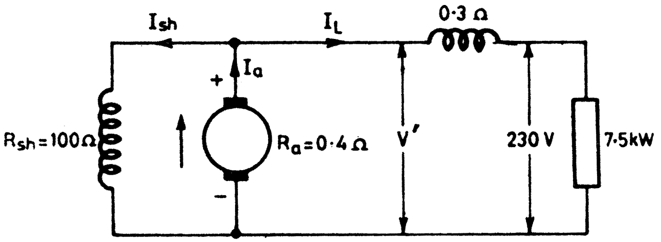
V' = 230 + Voltage drop in series winding

= 230 + 32.6×0.3 = 239.8 V

$\therefore$ $I_{sh} = V'/R_{sh}$ = 239.8/100 = 2.4 V

or $I_a = I_L + I_{sh}$ = 32.6 + 2.4 = 35 A

$\therefore$ $E_g = V' + I_aR_a$ = 239.8 + 35×0.4 V = 253.8 V



(ii) Load resistance,

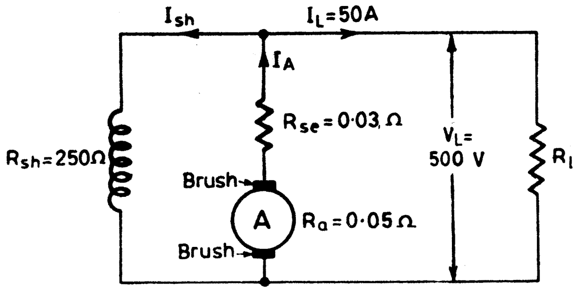
$R_L = V^2$/Poewr = 230^2/7,500 = 7.05 Ω.

Example : 24

A long shunt compund generator delivers a load current of 50 A at 500 V, and the resistances of armature, series and shunt fields are 0.05 ohm, 0.03 ohm, and 250 ohms respectively. Calculate the generated emf, and armature current. Allow 1.0 V per brush for contact drop.

Solution :

Here R_a = 0.05 Ω; R_{se} = 0.03 Ω ; R_{sh} = 250 Ω ; V_{brush}= IV/brush V_L = 500 V; I_L = 50 A.



$$\therefore \quad I_{sh} = \frac{V_L(=V_{sh})}{R_{sh}} = \frac{500}{250} = 2\text{ A}$$

and $I_a = I_L + I_{sh}$ = 50 + 2 = 52 A.

$\therefore$ $E_g = I_aR_a + 2\times V_{brush} + I_a R_{se} + V_L$

= 52×0.05 + 2×1 + 52×0.03 = 500 V = 506.16 V.

Example : 25

A 4–pole, 250 V d.c. long–shunt compound generator supplies a load of 10 kW at the rated voltage. The armature, series field, and the shunt field resistance are 0.1 ohm, 0.15 ohm, and 250 ohms respectively. The armature is lap wound with 50 slots, each slot containing 6 conductors. If the flux/pole is 50 mWb, calculate the speed of the generator.

Solution :

Here P = 4, A = 4 (for lap winding); Z = 50×6 = 300 ; ϕ = 50×10^{-3} = 0.05 Wb : V = 250 V ; load = 10,000 W ; R_a = 0.1 Ω R_{se} = 0.15 Ω; R_{sh} = 250 Ω.

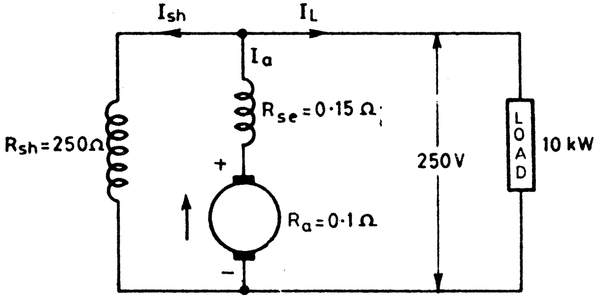
and $I_a = I_{se} = I_L = I_{sh}$ = 40 + 1 = 41 Ω.

∴ $E_g = V + I_a (R_a = R_{se})$ = 250 + 41 (0.1 + 0.15) = 260.25 V

Now

$$E_g = 260.25 = \frac{\phi ZN}{60}(\text{lap wound}) = \frac{0.05 \times 300 \times N}{60}$$

$$\therefore \text{ Speed } N = \frac{260.25 \times 60}{0.05 \times 300} = 1{,}041 \text{ rpm}$$



Example : 26

A 440 V d.c. compound generator has an armature, series field, and shunt field resistance of 0.5, 1.0, and 200 ohms respectively. Calculate the generated voltage while delivering 40 A to external circuit for both long shunt, and short–shunt connections.

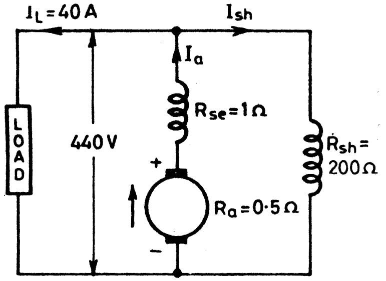
Solution :

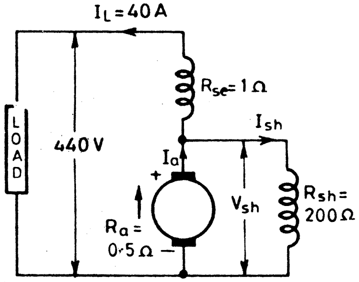
Here I_L = 40 A (for both cases)

(a) Long–shunt : I_{sh} = 440/200 = 2.2 A ; $I_a = I_L + I_{sh}$ = 40 + 2.2 = 42.2A = I_{se}.

∴ Emf induced, $E_g = 440 + I_a (R_a + R_{se})$

440 + 42.2 (0.5 + 1.0) = 503.3 V.





(b) short–shunt ; $I_{se} = I_2$ = 40 A.

∴ $V_{sh} = 440 + I_{se} \times R_{se}$ = 440 + 40×1 = 480 V.

$I_{sh} = V_{sh}/R_{sh}$ = 480/200 = 2.4 A.

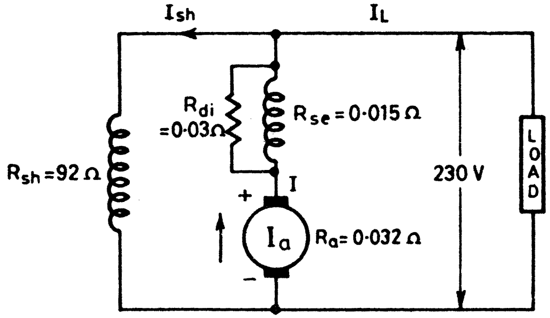
and $I_a = I_L + I_{sh}$ = 40 + 2.4 = 42.4 A

∴ Emf generated, $E'_g = V_{sh} + I_a R_a$ = 480 + 42.4×0.5 = 501.2 V.

Example : 27

In a long–shunt compund generator, the terminal voltage is 230 V, when it delivers 150 A. Determine : (i) induced emf, (ii) total power generated armature. The shunt field series field, divider, and armature resistances are 92, 0.015, 0.03, and 0.032 ohm respectively.

Solution :



I_{sh} = 230/92 = 2.5 A ; $I_a = I_L + I_{sh}$ = 150 + 2.5 = 152.5 Ω.

Combined resistance of R_{se} and R_{sh} $= \dfrac{0.03 \times 0.015}{0.03 \times 0.015} = 0.01\ \Omega$

∴ R_a = 0.032 + 0.01 = 0.042 Ω.

(i) emf induced, $E_g = V + I_aR_a$ = 230 + 152.5×0.042 = 236.4 V

(ii) Total power, E_gI_a = 236.4×152.5 = 36,051 W or 31.051 kW.

Example : 28

A shunt generator gives full–load output of 30 kW at a terminal voltage of 200 V. The armature, and shunt field resistances are 0.05, and 50 ohms respectively. The iron, and friction losses are 1,000 W. Calculate : (i) emf generated ; (ii) copper losses ; (iii) efficiency.

Solution :

I_L = 30,000/200 = 150 A ; I_{sh} 200/50 = 4A; $I_a = I_L + I_{sh}$ = 150 + 4 = 154 A ; R_a = 0.05 Ω.

(i) $E_g = V + I_aR_a$ = 200 + 154×0.05 = 206.16 V.

(ii) Cu loss = $I^2_{sh}\ R_{sh} + I^2_{sh}\ R_a = 4^2 \times 50 + (154)^2 \times 0.05$ = 1,985.5 W.

(iii) $\eta = \dfrac{\text{Output} \times 100}{\text{Input}} = \dfrac{\text{Output} \times 100}{\text{Output} + (\text{Cu} + \text{iron, and frictional losses})}$

$$= \frac{30{,}000\text{W} \times 100}{30{,}000 + 1{,}985.5 + 1{,}000} = 90.95\%.$$

Example : 29

The output of a d.c. shunt generator is 24 kW at a terminal voltage of 200 V. Armature resistance, and field resistance are 0.05 Ω, and 40 Ω respectively. If the iron, and friction losses

equal the Cu losses at this load, find : (i) output of the engine driving the generator ; (ii) efficiency of the generator.

Solution :

(i) I_L = 24,000/200 = 120A ; I_{sh} = 200/40= 5A ; $I_a = I_L + I_{sh}$ = 120 + 5 = 125 A.

∴ Eg = V + IaRa = 200 + 125×0.05 = 206.25 V.

Cu losses $= I_{sh}^2R_{sh} + I_a^2R_a = 5^2 \times 40 + 125^2 \times 0.05 = 1{,}781\text{W}$

(i) Output Input + Losses = 24,000 + 1,781 + 1,781 = 27, 562 W = 27.562 kW = Input of generator

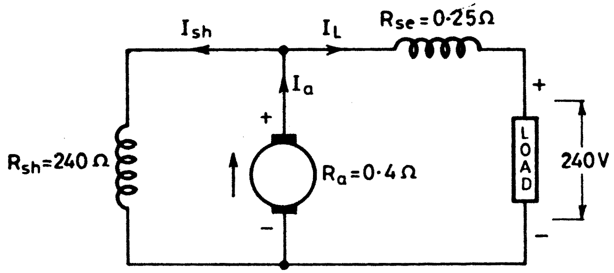
(ii) efficiency $= \dfrac{\text{Output} \times 100}{\text{Input}} = \dfrac{24{,}000 \times 100}{27{,}562} = 87.08\%$

Example : 30

A 20 kW, 440 V short–shunt compound d.c. generator has a full–load efficiency of 87%. If the resistance of the armature, series, and shunt–field are 0.4, 0.25. and 240 ohms respectively, calculate the combined bearing, windages, and core loss the machine.

Solution :

IL = Ise = 20,000/440 = 45.45A.



∴ Voltage at armature terminal

= $V + I_{se} \times R_{se}$ = 440 + 45.45×0.25 = 451.36 V.

∴ I_{sh} = 451.36/240 = 1.88 A,

and $\quad I_a = I_{sh} + I_L = 1.88 + 45.45 = 47.33$ A.

$\therefore$ Cu loss $= I_a^2 R_a + I_{sh}^2 R_{sh} = (47.3)^2 \times 0.4 + (1.88)^2 \times 240 = 1{,}744W$

Input $=$ Output/Efficiency $= 20{,}000$ W/0.87 $= 22{,}989$ W

$\therefore$ Total loss $= 22{,}989 - 20{,}000$ W $= 2989$ W

$\therefore$ Constant losses $=$ (Total–Cu) losses $= 2{,}989 - 1{,}744$
$= 1{,}245$ W.

Example : 31

A short–shunt compound generator supplies a current of 100 A at a voltage of 220 V. The resistance of shunt field, series, and armature are 50,0.025, and 0.05 respectively, total brush drop is 2V, and the iron friction loss amounts to 1 kW. Find : (i) the generated emf, (ii) the copper losses, (iii) the output of the prime mover driving the generator, (iv) generator efficiency.

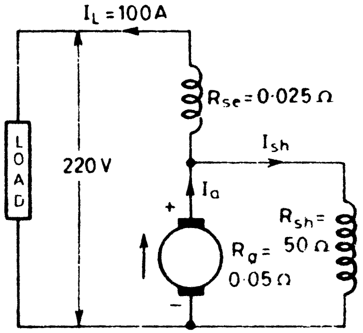
Solution

$I_L = 100A$; $V = 220$ V ; $R_{se} = 0.025\ \Omega$; $R_{sh} = 50\ \Omega$ brush drop $= 2$ V ; iron friction loss $= 1{,}000$ W.

$\therefore$ Voltage drop,

$V_{se} = R_{se} \times I_{se} = R_{se} \times I_L$

$= 0.025 \times 100 = 2.5$ V.



$\therefore \quad V_{sh} = 220 + 2.5 = 225.5$ V.

and $\quad I_{sh} = V_{sh}/I_{sh} = 225.5/50$

$= 4.51$ A.

$\therefore \quad I_a = I_L + I_{sh} = 100 + 4.51 = 104.51$ A.

(i) $E_g = V +$ Drop across (series + shunt + brush)

$= V_{sh} + I_a R_a +$ Brush drop

$= 224.45 + 104.51 \times 0.05 + 2V = 231.67$ V.

(ii) Cu losses $= I_a^2 R_a + I_{sh}^2 R_{sh} = (104.51)^2 \times 0.05 + (4.51)^2 \times 50$ W $= 1{,}563.1$ W.

(iii) Output $=$ Input $+$ Losses $= 220 \times 100$ W $+ (1{,}563.1 + 1{,}000)$ W

$= 24{,}563$ W or 24.563 kW $=$ Input of generator.

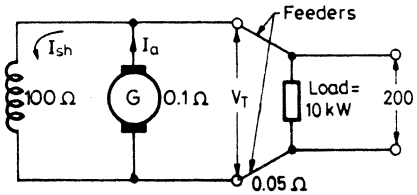
(iii) $\quad \text{Efficiency} = \dfrac{\text{Output} \times 100}{\text{Input}} = \dfrac{22{,}000W \times 100}{24{,}563W} = 89.56\%$.

Example : 32

A shunt generator supplies a load of 10 kW at 200 V through a pair of feeders of total resistance of 0.05 Ω. Armature resistance is 0.1 Ω. Shunt resistance is 100 Ω. Find the terminal voltage and generated emf of the generator.

Solution :

Circuits is :



Here $R_a = 0.1\ \Omega$; $R_{sh} = 100\ \Omega$; feeder resistance $= 0.05\ \Omega$; line voltage $V_L = 200$ V, load delivered $= 10$ kW $= 10^4$ W

Now $\quad P = V_L I_L$ or $I_L = \dfrac{P}{V_L} = \dfrac{10^4}{200} = 50A$

Voltage drop across the feeders $= I_L \times$ feeder resistance $= 50 \times 0.05 = 2.5$ V

$\therefore$ Terminal voltage $(V_t) = (200 + 2.5)$ V $= 202.5$ V

Field current, $I_{sh} = \dfrac{V_t}{R_{sh}} = \dfrac{202.5}{100} = 2.025A$

Amature current, $I_a = I_L + I_{sh}$ = (50 + 2.025) A = 52.025 A

∴ Emf generated, $E = V_t + I_a R_a$ = 202.5+52.025×0.1 207.703 A.

Example : 33

A 120 V, d.c. shunt motor has an armature resistance of 0.2 Ω and field resistance of 60 Ω. It runs at 1800 rpm, when it is taking a full–load current of 40 A. Find the speed of the motor, when it is operating with half full–load.

Solution :

I_{sh} = 120/60 = 2 A.

∴ I_{aq} (full–load)

= I_{a1} I_{sh} = 40–2 = 38 A.

∴ E_{b1} (full–load)

= $V–I_{ai} R_a$

= 120–38×0.2 = 11.24 V

I_{L2} (half–load) = 20 A,

so I_{a2} = 20 –2 = 18 A.

E_{b2}(half–load) = 120–18×0.2 = 116.4 V.

Now $$\frac{E_{b2}}{E_{b1}} = \frac{N_2(\text{half}-\text{load})}{N_1(\text{half}-\text{load})} = \frac{N_2(\text{half}-\text{load})}{1{,}800\text{ rpm}} = \frac{116.4}{112.4}$$

∴ Speed at half–load, N_2 (half–load) $= \dfrac{116.4 \times 1{,}800}{112.4} = 1{,}864$ rpm

Example : 34

A 250 V, shunt motor has a shunt field resistance of 250 ohms, and an armature resistance of 0.5 ohm. When running on no–load, it takes 5 A from the lines, and it speed is 1,500 rpm. Calculate its speed, when taking 50 A from the lines.

Solution :

No–load : V = 250 V ; I_{L1} = 5A ; R_{sh} = 250 Ω ; R_a = 0.5

Ω N_1 = 1,500 rpm; I_{sh} = 250/250 = I_{a1} = 5–1 = 4A.

∴ $E_{b1} = V - I_{a1}R_a$ = 250–4 × 0.5 = 248 A.

On load : I_{L2} = 50 A; I_{a2} = 50 – 1 = 49 A.

∴ $E_{b2} = V - I_{a2} R_a$ = 250 – 49×0.5 = 225.5 V.

Now $N_2/N_1 = E_{b2}/E_{b1}$ = 225.5/248 = N_2/1,500

N_2 = 22.5×1,500/248 = **1,364 rpm**.

Example : 35

A 220 V d.c. shunt motor runs at 1,000 rpm, when the armature current is 35 A. The resistance of the armature circuit is 0.3 ohm. Calculate the additional resistance required in the armature circuit to reduce the speed of the motor to 750 rpm, assuming that armature current then is 25 A.

Solution :

Let R be the resistance added in series with the field circuit.

$E_{b1} = V–I_a R_{a1}$ = 250 – 35×0.3 = 248.95 V (n_1 = 1,000 rpm)
$E_{b2} = V - I_{a2} R_a$ = 250 – 2.5×(0.3 + R) = 242.5 –25 R(N_2 + 750 rpm)

$$\therefore \quad \frac{N_2}{N_1} = \frac{750}{1{,}000} = \frac{E_{b2}}{E_{b1}} = \frac{(242.5-25R)}{248.95} = 0.75$$

∴ 248.95×0.75 = 242.5 – 25 R

∴ R = (242.5 – 186.1)/25 = 2.256 Ω.

Example : 36

A 220 V shunt motor, running at 700 rpm, has an armature resistance of 0.45 Ω, and takes armature current of 22.A. What resitance should be placed in series with the armature to reduce the speed to 450 rpm ?

Solution

$E_{b1} = V –I_a R_a$ = 220 – 22×0.45 = 210.1 V (N_1 = 700 rpm)

Now $$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1} = \frac{450}{700} = \frac{9}{14} = \frac{E_{b2}}{210.1\ V}$$

$\therefore$ E_{b2} = 210.1 × (9/14) V = 135.06 V.

Let R be the resistance placed in series with the armature. Then :

$$E_{b2} = 220 - 22 \times (0.45 + R) = 220 - 9.9 - 22\ R$$

$\therefore$ R = (220 − 9.9 − 135.06)/22 = 3.411 Ω.

Example : 37

A 4–pole, 500 V d.c. shunt motor has 720 wave–connected conductors on its armature. The full–load armature current is 60 A and the flux per pole 0.03 Wb. The armature resistacne is 1.2 Ω and the contact drop is 1 V per brush. Calculate the full–load speed of the motor.

Solution

Here P = 4, A = 2 (for wave connection), V = 500 V, Z = 720, ϕ = 0.03 Wb, R_a = 0.2 Ω, brush drop for 2 brushes = 2 × 1 = 2 V, $I_{(fl)}$ = 60 A

Now $E_b = V - I_{(fl)} \times R_a$ – Brush drop

= 500 − 60 × 0.2 − 2 = 500 − 14 = 486 V

Now $$E_b = \frac{\phi ZN}{60}\left(\frac{P}{A}\right) = \frac{0.03 \times 720 \times N}{60}\left(\frac{4}{2}\right) = 486$$

Hence, $$N_{(fl)} = \frac{486 \times 2 \times 60}{0.03 \times 720 \times 4} = 675 \text{ rpm}$$

Example : 38

A 230 V d.c. mtor has an armature circuit resistance of 0.2 W and field resistance of 0.1 Ω. At rated voltage, the motor draws a line current of 40 A and runs at a speed of 1,000 rpm. Find speed of the motor for a line current of 20 A at 230 V. Assume that the flux at 20 A line current is 60% of the flux at 40 A line current.

Solution :

Here R_a = 0.1 Ω, R_{se} = 0.2 Ω ; I_{L1} = 40 A; I_{L2} = 20 A; N_1 = 1,000 rpm, N_2 = ? ; $\phi_2 = 0.6\ \phi_1$

Now back emf, $E_{b1} = V - I_{L1}(R_a + R_{se})$ = 230 − 40 (0.3) = 218 V

and back emf, E_{b2} V − L_{L2} $(R_a + R_{se})$ = 230 − 2(0.3) − 40 (0.3) = 224V

Also $E_{b1} = K\phi_1 N_1$ and $E_{b2} = K\phi_2 N_2 = K \times 0.6\ \phi_1 N_2$

$$\frac{E_{b2}}{E_{b1}} = \frac{224}{218} = \frac{K \times 0.6\phi_1 N_2}{K\phi_1 N_1} = \frac{0.6N_2}{N_1} = \frac{0.6N_2}{1,000}$$

Hence, $$N_2 = \frac{224 \times 1,000}{218 \times 0.6} = 1,713 \text{ rpm}$$

Example : 39

Calculate the ohmic value of starting resistance for the following d.c. shunt motor : Output = 14,920 W; supply = 240 V; armature resistance = 0.25 ohm, and efficiency at full–load is 86%. The starting current is to be limited to 1.5 times full–load current. Ignore current in shunt windings.

Solution :

Input = 240 × I_L = Output/Efficiency = 14,920/0.86

$\therefore$ Full–load, current, I_L = 14,920/0.86 × 240 = 72.3 A.

$\therefore$ Starting current, I' = 72.3 × 1.5 = 108.45 A.

$\therefore$ Total resistance = 240/108.45 = 2.213 Ω

Hence, starting resistance = 2.213 − 0.25 = 1.963 Ω

Example : 40

A 500 V shunt motor takes 4 A on no–load. The armature resistance (including that of the brushes) is 0.2 ohm, and field current is 1A. Estimate the efficiency, when the input current is 100 A.

Solution :

N0–load : V = 500 V; I_{L1} = 4A ; R_a = 0.2 Ω; I_{sh} = 1A I_{a1} I_{L1} $-I_{sh}$ 41 = 3A.

∴ $E_{b1} = V - I_{a1} R_a = 500 - 3 \times 0.2 = 499.4$ V.

Loaded condition : I_{L2} = 100 A I_{a2} = 100 – 1 = 99 A.

$E_{b2} = V - I_{a2} R_a = 500 - 99 \times 0.2 = 480.2$ V.

Hence, efficiency at 100 A input current,

$$\eta = \frac{(\text{Input} - \text{Losses}) \times 100}{\text{Input}} = \left[\frac{500 \times 100 - (2{,}000 + 1{,}960)}{500 \times 100}\right] \times 100 = 92.08\%$$

Example : 41

A 230 V d.c. shunt d.c. shunt motor runs at 1,000 rpm on full–load, drawing a current of 10 A. The shunt–field resistance is 230 ohm, and the armature resistance is 0.5 ohm. Calculate the resistance to be inserted in series with the armature so that the speed at full load is 950 rpm.

Solution :

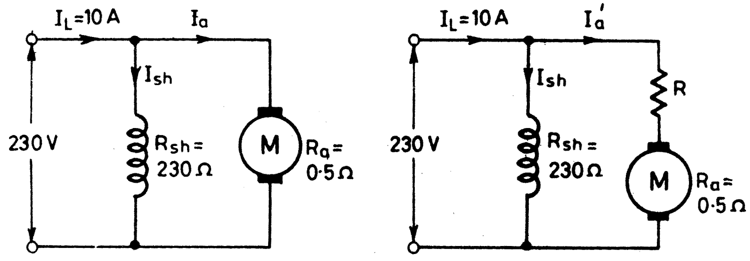
I_{sh} = 230/230 = 1 A ; I_{a1} = 10 – 1 + 9 A.

∴ $E_{b1} = 230 - 9 \times 0.5 = 225.5$ V.

Let 'R' be the required resistance in series with the armature, that total armature resistance, R_{az} = (0.5 + R) Ω.

∴ $E_{b2} = 230 - (0.5 + R) \times 9 = (225.5 - 9R)$ V.

Now $$\frac{E_{b2}}{E_{b1}} = \frac{225.5 - 9R}{225.5} = \frac{N_2}{N_1} = \frac{950}{1000} = 0.95$$



or $225.5 \times 0.95 = 225.5 - 9R$

or $9R = 225.5 \times 0.05 = 11.275$

or R = 1.253 Ω.

Example : 42

A series motor takes 20 A at 400 V, and run at 250 rpm. The armature, and field resistances are 0.6 ohm and 0.4 ohm respectively. Find the applied voltage, and current to run the motor at 350 rpm, if the torque required varies as the square of the speed.

Solution :

Torque $\alpha\, \phi\, I_a \alpha\, I_a^2\, \alpha\, N^2$

or I_a α N

(i) New value of current of 350 rpm = 350×20/250 = 28 A.

(ii) E_{b1} at 250 rpm = 400 – 20×(0.6 + 0.4) = 380 V.

E_{b2} at 350 rpm = V – 28×(0.6 + 0.4) = V – 28.

∴ $$\frac{E_{b1}}{E_{b2}} = \frac{380}{(V - 28)} = \frac{I_{a1} N_1}{I_{a2} N_2} = \frac{20 \times 250}{28 \times 350}$$

Whence, V = [380×28×350/20×250] + 28 = 744.8 + 28 = 772.8 V.

Example : 43

A 220 V d.c. shunt motor has an armature resistance of 0.5 ohm, and runs at 850 rpm., When taking a full–load current of 32 A, the shunt field resistance is 110 ohms. Calculate the speed at which the motor runs, if (i) 1.5 ohm resister were introduced in series with the armature, (iii) 300 ohms resister were connected in series with the field winding, and load torque remaining the same throughout. Assume that field is proportional to field current.

Solution :

I_{sh} = 320/100 = 2 A; I_a = 32 –2 = 30 A.

$\therefore$ $E_b = V - I_aR_a = 220 - 30\times 0.5 = 205$ V(N = 850 rpm)

(i) $R_{a1} = 0.5 + 1.5 = 2.0\ \Omega$.(Ish remaining the same)

$\therefore$ $E_{b1} = V - I_aR_{a1} = 220 - 30\times 2\ V = 160\ V$

$$\therefore \quad \frac{E_{b1}}{E_{b2}} = \frac{160}{205} = \frac{N_1}{N} = \frac{N_1}{850}$$

or $N_1 = 160\times 850/205 = 663$ rpm.

(ii) $R_{sh} = 110 = 30 = 140\ \Omega$; $I_{sh2} = 220/140 = 1.57$ A.

But torque, $T \propto \phi I_q \propto I_{sh}\times I_a$

$\therefore$ $T = k\times 2\times 30$(Normal case)

and $T_2 = T\ K\times 1.57\times I_{a2}$ (With field resistance)

or $1.57\ I_{a2} = 2\times 30$ or $I_{a2} = 2\times 30/1.57 = 38.2$ A.

$\therefore$ $E_{b2} = V - I_{a2}\ R_a = 220 - 38.2\times 0.5\ V = 200.9\ V$

$$\text{Now}\quad \frac{E_b}{E_{b2}} = \frac{N\phi}{N_2\phi_2} = \frac{NI_{sh}}{N_2 I_{sh2}} = \frac{850\times 20}{N_2\times 1.57}\ (\text{similarly } \phi \propto I_{sh})$$

$$\therefore \quad N_2 = \frac{850\times 2.0\times E_{b2}}{1.57\times E_b} = \frac{850\times 2.0\times 200.9}{1.57\times 205} = 1{,}061 \text{ rpm}$$

Example : 44

A 20 HP, 230 v, 1,150 RPM, 4–pole, Dc shunt motor Has a total of 620 conductors arranged in 2 Parallel paths, and yielding an armature circuit resistance of 0.2 ohm. When it delivers rated power at rated speed, it draws a line current of 74.8 A, and field current of 3.0 A. Calculate : (i) flux per pole; (ii) torque developed ; (iii) rotational looser ; (iv) total losses, expresseing as a percentage of power.

Solution :

20 HP = 20×735.5 W = 14,710 W; VL = 230 V; N = 1,150 rpm; P = 4; Z = 620; $R_a = 0.02\ \Omega$; I_L = 74.8 A. I_{sh} = 3 A; A = 2.

$\therefore$ $I_a = I_i - I_{sh} = 74.8 - 3.0 = 71.8$ A

$E_b = V_L - I_aR_a = 230 - 71.8\times 0.02 = 215.64$V.

$$\text{(i)}\quad E_b = 215.64 = \frac{\phi ZNP}{60A} = \frac{\phi\times 620\times 1{,}150\times 4}{60\times 2}$$

$\therefore$ $\phi = 9.073$ mWb.

$$\text{(ii)}\ T_a = \frac{0.159\ P\ \phi\ I_a Z}{A} = \frac{0.159\times 4\times 9.073\times 10^{-3}\times 74.8\times 620}{2} = 133.8\ \text{Nm}$$

$$\text{(iii) Shaft power} = \frac{2\pi NT_{sh}}{60} = 14{,}710\ \text{W}$$

$$\therefore \text{Shaft torque,}\quad T_{sh} = \frac{14{,}710\times 60}{2\pi\times 1{,}150} = 122.15\ \text{Nm}$$

$\therefore$ Torque lost (T1) = $T_a - T_{sh}$ = 1.33.8 – 122.15 = 11.65 Nm

$\therefore$ Rotational losses = (Iron + Mech.) losses = $2\pi NT_{sh}/60$

= $2\pi\times 1{,}150\times 11.65/60$ = 1,403 W

(iv) Armature copper losses

$= I_a^2 R_a = 71.82 \times 0.2 = 1{,}031$ W.

$\therefore$ Field copper losses $\left(I_{sh}^2 R_{sh}\right) = I_{sh} \times V_{sh} = 3 \times 230 = 690$ W

$\therefore$ Total lossles = 1,403 + 1,031 + 690 = 3,124 W.

Hence, % losses $= \dfrac{3{,}124 \times 100}{14{,}710} = 21.24\%$

Example : 45

A 250 V d.c. shunt motor has armature resistance of 0.5 ohm, and is excited to give constant main field. At full–load, the motor runs at 400 rpm, and takes an armature current of 30 A. If a resistance of 1 ohm is placed in series with the armature, find : (i) the speed at the full–load torque; (ii) the speed at double full-load torque, and (iii) the stalling torque.

Solution :

(i) $E_b = V - I_a R_a = 250 - 30 \times 0.5 = 235$ V ; N = 400 rpm.

When 1 ohm is inserted in series with armature, the resistance of armature, R_{a1} = 0.5 + 1.5 ohm ; N_1 = ?

$\therefore$ $E_{b1} = 250 - 30 \times 1.5 = 205$V.

But $\dfrac{E_{b1}}{E_b} = \dfrac{N_1}{N}$ or $\dfrac{205}{235} = \dfrac{N_1}{400}$ (similarly ϕ is constant)

$\therefore$ Speed, $N_1 = 205 \times 400/235 = 350$ rpm

(ii) $R_{a2} = 1.5\ \Omega$; $I_{L2} = 30 \times 2 = 60$ A ; N_2 = ?

$\therefore$ $E_{b2} = 250 - 60 \times 1.5 = 160$ V.

$\therefore$ Speed, $N_2 = \dfrac{E_{b2} \times N}{E_b} = \dfrac{160 \times 400}{235} = 272$ rpm

(iii) Stalling torque : Under stalling conditions, the speed is zero, i.e., back emf is zero. Let I_0 be the current taken by the motor under stalling condition, then :

$E_{b0} = 250 - I \times 1.5 = 0$ or $I_0 = 250/1.5 = 166.7$ A.

But full-load current, I_L = 30 A.

$\therefore$ Stalling current $(I_0) = 766.7L_1/30 = 5.555 \times I_L$

Since torque is proportional to current, so stalling torque.

= 5.555 times full-load torque.

Example : 46

A 4 pole, 250 V d.c. shunt motor has a lap–connected armature with 960 conductors. The flux per pole is 2×10^{-2} Wb. Calculate : (i) the torque developed by the armature in Nm, and (ii) the useful torque in Nm, when the current taken by the motor is 30 A. The armature, and field resistances are 0.12 ohm, and 125 ohms respectively. The rotational losses amount to 825 W.

Solution :

P = A = 4 (for lap connection); V = 250 V; Z = 960 ; $\phi = 2 \times 10^{-2}$ Wb;

$I_L = 30$ A; $R_a = 0.12\ \Omega$; $R_{sh} = 125\ \Omega$; $W_e = 825$ W.

$\therefore$ $I_{sh} = 250/125 = 2$ A, and $I_a = 30 - 2 = 28$ A.

(i) Torque developed,

$$T_a = \frac{\phi Z I_a}{2\pi}\left(\frac{P}{A}\right) \text{Nm} = \frac{2 \times 10^{-2} \times 960 \times 28 \times 4}{2\pi \times 4} \text{Nm} = 86.73 \text{ Nm}$$

(ii) Losses in motor

= (Armature Cu + shunt Cu) loss + Rotational loss

$I_a^2 R_a + I_{sh} V + 825$ W $= 28^2 \times 0.12 + 2 \times 250 + 825$ W = 1,419 W.

∴ $E_b = V - I_aR_a = 250 - 28\times0.12 = 246.64$ V.

and input = $VI_L = 250\times30 = 7,500$ W

∴ Output = 7,500 –1,419 = 6,081 W.

But $E_b = 246.14 = \phi\ ZN(P/A)$ (if N is in rps)

$= 2\times10^{-2}\times960\times N\times(4/4)$

∴ N + 12.846 rps.

Hence, useful torque,

$$T = \frac{\text{Output}}{2\pi N} = \frac{6,081}{2\pi\times12.846} = 75.34\ \text{Nm}$$

Example : 47

A 8kW, 230 V, 1,250 rpm d.c. shunt motor has R_a = 0.7 ohm. The field current is adjusted, on no–load with a supply of 250 v, the motor runs at 1,250 rpm, and draws armature current of 1.6 A. A load torque is then applied to the motor shaff which causes I_a to rise to 40A, and the speed falls to 1,150 rpm. Determine the reduction in the flux per pole due to the armature reaction.

Solution

N_1 = 1,250 rpm ; R_a = 0.7 Ω; I_a = 1.6 A; I_{a2} = 40; N_2 + 150.

∴ $E_{b1} = V - I_{a1}R_a = 250 - 1.6\times0.7 = 248.88$ V.

and $E_{b2} = V - I_{a2}R_a = 250 - 40\times0.7 = 222$ V.

But $$\frac{E_{b2}}{E_{b1}} = \frac{222.0}{248.88} = \frac{N_2\phi_2}{N_1\phi_1} = \frac{1,150}{1,250}\left(\frac{\phi_2}{\phi_1}\right)$$

∴ $\dfrac{\phi_2}{\phi_1} = 220.0\times1,250/248.88\times1,150 = 0.9695$

Hence, reduction in flux due to armature reaction

$$= \frac{\phi_1 - \phi_2}{\phi_2}\times100 = \left[1 - \frac{\phi_2}{\phi_1}\right]\times100 = (1 - 0.9695)\times100 = 3.05\%$$

Example : 48

A 200 V shunt motor has an armature resistance of 0.4 ohm, and field resistance of 200 ohms. The runs at 750 rpm, and takes an armature current of 25 A. Assuming that the load torque remains constant, find the reduction in field resistance necessary to reduce to speed to 500 rpm. Neglect saturation.

Solution :

I_{a1} = 25 A; R_a = 0.4 Ω; V = 200 V; n_1 = 750 rpm ; R_{sh} = 200 Ω; I_{sh} = 200/200 = 1 A.

∴ E_{b1} at 750 rpm = $V - I_aR_a = 200 - 25\times0.4 = 190$ V

Let R be the reduced shunt field resistance, then I_{sh2} = (200/R) A.

Now $I_1\ \alpha\ \phi_1 I_{a1}\ \alpha,\ I_{sh}I_{a1}$ and $I_2\ \alpha\ I_{sh2}\ I_{a2}$

But $T_1 = T_2$, so $I_{sh2}.\ I_{a2} = I_{sh1}\ I_{a1}$

∴ $I_{sh2}.\ I_{a2}\ 1\times25$ or $I_{a2} = 25/I_{sh2} = 25/(200/R) = 0.125$ R.

∴ E_{b2} at 500 rpm = 200 – 0.0125 R × 0.4 = (200– 0.05 R) V

Now $$\frac{E_{b1}}{E_{b2}} = \frac{190}{(200 - 0.05R)} = \frac{I_{sh1}\times N_1}{I_{sh2}\times N_2} = \frac{1\times750}{(200/R)\times500} = \frac{1.5R}{200}$$

∴ $190\times200 = 3000\ R - 0.075\ R^2$

or $0.075\ R^2 - 300\ R + 38,00 = 0$

∴ $$R = \frac{300 \pm \sqrt{90,000 - 4\times0.075\times38,00}}{2\times0.075}$$

$$= \frac{300 \pm \sqrt{90,000 - 11,400}}{0.15} = \frac{300 \pm 280.36}{0.15} = 130.9\ \Omega$$

Hence, reduction in shunt resistance

$= 200 - 130.9 = 69.1\ \Omega$

Example : 49

A d.c. series motor with series field, and armature resistance of 0.06 ohm and 0.04 ohm respectively is connected across 220 V mains. The armature takes 40 A, and its speed is 900 rpm. Determine its speed when the armature takes 75 A, and excitation is increased by 15% due to saturation.

Solution :

$R_m = R_a + R_{se} = 0.04 + 0.06 = 0.1\ \Omega$; $V = 220$ V ; $I_{a1} = 40$ A ; $N_1 = 900$ rpm;

Now $E_{b1} = V - I_{a1} R_m = 220 - 40 \times 0.1 = 216$ V.

and $Ebe = V - I_{a2} R_m = 220 - 75 \times 0.1 = 212.5$ V.

But $$\frac{E_{b2}}{E_{b1}} = \frac{\phi_2 N_2}{\phi_1 N_1} = \frac{1.15 \times N_2}{900} = \frac{212.5}{216}$$

$\therefore$ Speed, $N_2 = 212.5 \times 900/216 \times 1.15 = 770$ rpm.

Example : 50

A 200 v d.c. series motor runs at 1,000 rpm, and takes 20 A. Combined resistance of armature, and field is 0.4 ohm. Calculate the resistance to be inserted in series so as to reduce the speed to 800 rpm, assuming torque varies as square of the speed.

Solution :

$I_L = I_{a1} = 20$ A; $N_1 = 1{,}000$ rpm; $N_2 = 800$ rpm; $R_{m1} = 0.4\ \Omega$; $R_{m2} = (0.4 + R)\ \Omega$.

Now $$\frac{T_2}{T_1} = \frac{I_{a2}\phi_2}{I_{a1}\phi_1} = \left(\frac{I_{a2}}{I_{a1}}\right)^2 = \left(\frac{N_2}{N_1}\right)^2 = \left(\frac{800}{1{,}000}\right)^2 \text{(similarly) } \phi \propto I_a)$$

$\therefore$ $I_{a2} = 0.8\ I_{a1} = 0.8 \times 20 = 16$A.

Now $E_{b1} = V - I_{a1} R_{m1} = 200 - 20 \times 0.4 = 192$ V.

and $E_{b2} = V - I_{a2} R_{m2} = 200 - 16 \times (0.4 + R) = (193.6 - 16 R)$ V.

Also $$\frac{E_{b2}}{E_{b1}} = \frac{N_2\phi_2}{N_1\phi_1} = \frac{N_2 I_{a2}}{N_1 I_{a1}} = \frac{800 \times 16}{1{,}000 \times 20} = 0.64$$

$\therefore$ $E_{b2} = 0.64 \times E_{b1} = 0.64 \times 192\ V = 122.88\ V = 193.6 - 16\ R$

Whence, $R = (193.6 - 122.88)/16 = 4.42\ \Omega$.

Example : 51

A 4-pole, 240-V, lap-wound, series motor has armature and series field resistance of 0.3 Ω, and 0.025 Ω respectively. There are 672 armature conductors. If the flux per pole is 0.25 Wb, and the total torque developed in the armature is 348 Nm, find the current taken by the motor, and its speed.

Solution :

(i) $T_a = 348$ Nm; $P = A = 4$ (lap wound), $V = 250$ V ; $Z = 672$; $\phi = 0.25$ Wb; $R_a = 0.03\ \Omega$; $R_{se} = 0.25\ \Omega$; $R_m = R_a + R_{se} = 0.3 + 0.25 = 0.55\ \Omega$.

Now $$T_a = \frac{\phi Z I_a}{2\pi}\left(\frac{P}{A}\right) \text{Nm}$$

$\therefore$ $$348 = \frac{0.25 \times 672 \times I_a}{2\pi}\left(\frac{4}{4}\right) \text{ or } I_a = 13.015\text{A}$$

(ii) Now $E_b = 250 - 0.55 \times 13.15 = 242.84\ V = \phi Z N \left(\frac{P}{A}\right)$

$\therefore \quad 242.84 = 0.25 \times 672 \times N \times (4/4) = 168\ N$

or speed, $N = 242.84/168 = 1.4455$ rps $= 1.4455 \times 60 =$ 86.73 rpm.

Example : 52

A 4–pole, 250 V series motor has a wave–connected armature with 1,254 conductors. The flux per pole is 22 mWb, when the motor is taking 50 A. Armature resistance is 0.2 ohm, and series field resistance is 0.2 Ω. Calculate the speed.

Solution :

$E_b = V - I_a (R_a + R_{se}) = 250 - 6\backslash 50\ (0.2 + 0.2)$ = 230 V.

Now $E_b = \frac{\phi ZN}{60} \frac{P}{A} = \frac{22 \times 10^{-3} \times 1{,}254 \times N \times 4}{60 \times 2}$ (A = 2 for wave–winding)

$\therefore$ Speed, $N = \frac{230 \times 60 \times 2}{22 \times 10^{-3} \times 1{,}254 \times 4} = 250$ rpm

Example : 53

A 240 V series d.c. motor takes 40A, when giving its rated output at 1,500 rpm. Its resistance is 0.3 ohm. Calculate the value of resistance that must be added to obtain rated torque : (i) at starting, and (ii) at 1,000 rpm.

Solution :

Rated voltage, V = 240 V; rated current, I = 40 A; N = 1,500 rpm; $R_m = 0.3\ \Omega$.

$\therefore \quad E_b = V - I\ R_m = 240 - 40 \times 0.3 = 228$ V.

(i) At starting, back emf is zero, and to obtain rated torque when current drawn is rated one, (i.e., $I_1 = I = 40$ A), an additional resistance is needed to be connected in series.

$\therefore \quad E_{b1} = V - I_1 (R_m + R_1) = 240–40\ (0.3 + R_1) = 0$

or $\quad R_1 = (240 - 12)/40 = 5.7\Omega$

(ii) At 1,000 rpm, the rated torque can be obtained when I_a = I = 40 A; at N_2 = 1,000 rpm. Let R_2 be the required resistaance connected in series with circuit to give the rated torque.

$\therefore \quad E_{b2} = 240 - 40\ (0.3 + R_2) = 228 - 40\ R_2$ ....(i)

Now $\frac{N_2}{N} = \frac{E_{b2} \times I}{E_{b2} \times I_2} = \frac{E_{b2}}{E_b} = \frac{1{,}000}{1{,}500} = \frac{2}{3}$ (similarly $I = I_2$)

$\therefore\ E_{b2} = E_b \times (2/3) = 228 \times (2/3)$ V = 152 V ...(ii)

From Eqs. (i), and (ii), we get :

$152 = 228 - 40\ R_2$ or $R_2 = (228 - 152)/40 = 1.9\ \Omega$.

Example : 54

A d.c. shunt machine, connected to 250 V mains, has an armature resistance (including brushes) of 0.12 ohm, and the resistance of the field circuit is 100 ohms. Find the ratio of the speed of generator to the speed as motor, if the line current in each case is 80 A.

Solution :

V = 250 V; I_L = 80 A; $R_a = 0.12\ \Omega$; R_{sh} = 100 Ω; $I_{sh} = V/R_{sh} = 250/100 = 2.5$ A.

As a genrator : $I_a = I_L + I_{sh} = 80 + 2.5 = 82.5$ A; N_g?

$E_g = V + I_a R_a = 250 + 82.5 \times 0.12 = 260$ V....(i)

As a motor : $I'_a = I_L - I_{sh} = 80 - 2.5 = 77.5$ A,

$\therefore\ E_m = V - I'_a Ra = 250 - 77.5 \times 0.12 = 240.7$ V ....(ii)

But $\frac{N_g}{N_m} = \frac{E_g \times \phi_m}{E_m \times \phi_g} = \frac{260}{140} = 1.08$ (similarly I_L is same)

Example : 55

A 100 kW, belt driven shunt genrator running at 320 r.p.m on 220 V supply system continues to run as a motor when the belt breaks, then taking a power of 10 kW. Find it speed if armature resistance is 0.03 Ω, and field ciruit resistance Is 55 W. Take contanct drop under each brush = IV.

Solution :

As a genertor : I_L + W/V = 100,000/220 = 454.5 A ; I_{sh} = 220/55 = 4A; I_a = 454.5 + 4 = 458.5A; N_g = 320 rpm.

∴ $E_g = V + I_aR_a$ + Brush drop

$= 220 + 458.5 \times 0.03 + 2$ V = 235.8 V ...(i)

As a motor : I_L = 454.5 A ; I_{sh} = 229/55 = 4 A; $I_a = I_L - I_{sh}$ = 454.5 – 4 = 450.5 A ; N_m = ?

∴ $E_g = V - I_aR_a$ – Brush drop

$= 220 + 458.5 \times 0.03 - 2V$ = 204.5 V ....(ii)

Now $\frac{E_m}{E_g} = \frac{N_m}{N_g}$ or $N_m = \frac{E_m \times N_g}{E_g} = \frac{204.5 \times 320}{235.8} = 277.5$ rpm.

Example : 56

A 50 kW, 250 V d.c. shunt generator runs at 1,200 rpm. If this machine is run as a motor taking 30 kW at 250 V, what will be its speed ? The armature, and shunt field resistance are 0.1 ohm, and 125 ohms respectively. Brush drop is 2V.

Solution :

As a generator : I_{Lg} = 50,000 W/250 V = 200 A; I_{sh} = 250 V/ 125 W = 2A; I_{ag} = 200 + 2 = 202 A; N_g = 1,200 rpm.

∴ $E_g = V + I_aR_a$ + Brush drop

$= 250 + 202 \times 0.1 + 2$ V = 282.2 V

As a motor : I_{Lm} = 30,000 W/250 V = 120 A; I_{sh} = 2 A; I_a = 120 – 2A = 118 A ; N_m = ?

∴ $E_g = V - I_{am}R_a$ – Brush drop

$= 250 - 118 \times 0.1 - 2$ V = 236.2 V ...(ii)

Now $\frac{E_g}{E_m} = \frac{I_{ag}N_g}{I_{am}N_g} = \frac{202 \times 1{,}200}{118 \times N_m} = \frac{272.2}{236.2}$ [similarly $I_{Lg} \neq I_{Lm}$]

∴ $N_m + 202 \times 1{,}200 \times 236.2/118 \times 282.2$ = 1,720 rpm.

Example : 57

Magnetising curve for a d.c. generator running at 800 rpm is given by following:

Induced voltage (V)	***4***	***66***	***128***	***180***	***213***	***240***	***257***
Field current (A)	***0***	***0.2***	***0.4***	***0.6***	***0.8***	***1.0***	***1.4***

The machine is used as a "shunt motor", and is run at 1,000 rpm. Find the field circuit resistance required to generate 275 V on no–load.

Solution :

The reduced voltage for the same field current for the machine used as shunt–motor is given by:

$$E_2 = \frac{N_2}{N_1} \times E_1 = \frac{1{,}000}{800} \times E_1 = 1.25\ E_1$$

∴ New data for machine as well as shunt generator is :

Field current (A)	0	0.2	0.4	0.6	.08	1.0	1.4
Induced voltage at 800 rpm	4	66	128	180	213	240	257
Induced voltage at 1,000 rpm	5	82.5	160	225	266.25	300	321.25

∴ Field current for 275V ≈ 0.85 A.

(Assuming straight line relation between 0.8 and 1.0 A)

Neglecting the voltage drops, so the voltage across shunt field = 275 V

∴ Shunt field resistance, $R_{sh} = \frac{V}{I_{sh}} = \frac{275V}{0.85} = 323.5\ \Omega$.

TRANSFORMERS

WORKED EXAMPLES

Example 1:

A 50 kVA, single-phase transformer has 600 turns on primary, and 40 turns on secondary. The primary winding is connected to 2.2kV, 50 Hz supply. Determine (i) secondary voltage at no-load; (ii) primary, and secondary currents at full-load.

Solution:

(i) $E_2/E_1/N_1$

$\therefore \quad E_2 = N_2E_1/N_1 = 40\times2,200/600 = \mathbf{146.67\ V.}$

(ii) $$\text{kVA} = \frac{E_1I_1}{1,000} = \frac{E_2I_2}{1,000}$$

$$\therefore \quad I_1 = \frac{\text{kVA}\times1,000}{E_1} = \frac{50\times1000}{2,200} = \mathbf{22.72\ A}$$

and $$I_2 = \frac{\text{kVA}\times1,000}{E_2} = \frac{50\times1000}{146.67} = \mathbf{390.9\ A}$$

Example 2:

A single-phase 4 kVA transformer has 400 primary turns, and 1,000 secondary turn. The net cross-sectional area of the core is 60 cm². When the primary winding is connected to 500V, 50 Hz supply, calculate :

(i) the maximum value of flux density in the core;

(ii) the voltage induced in the seconday winding, and

(iii) the secondary full-load current.

Solution:

(i) Primary voltage, E_1=500V = 4.44 B_mAfN_1

$\therefore$ Max. value of flux density, Bm = $500/4.44\times60\times10^{-4}\times50\times400$

= **0.938 Wb/m²**.

(ii) Secondary voltage, $E_2 = N_2E_1/N_1 = 1,000\times500\times400$=**1,250 V**

(iii) Secondary current, $I_2 = VA/E_2 = 1,000\times4/1,250$=**3.2 A**

Example 3:

The design requirement of a 11,000/415 V, 50 Hz, single-phase core-type transformer are approximate emf/turn 15V, maximum flux density 1.5 T. Find a suitable number of primary, and secondary turns, and the net cross-sectional area of the core.

Solution :

(i) E_1 = 11,000 V; E_2 = 415 V; emf/turn = 15 V

$\therefore$ No. of primary turns, $N_1=E_1/15=11,000/15=$**734**.

and number of secondary turns, $N_2=E_2/15=$**28**.

(ii) Now $E_1=4.44\ B_m\ AfN_1=11,000=4.44\times1.5\times A\times50\times734$

or cross-sectional area of the core,

A=11,000/4.44×1.5×50×734=**0.045 m² or 450 cm²**.

Example 4:

The required no-load ratio in a single-phase, 50 Hz, core-type transformer is 6,000/250 V. Find the turns per limb on the high and low voltage sides, if the flux is to be about 0.06 Wb.

Solution :

Here E_1/E_2=6,000/250=24, f = 50 Hz and flux = 0.06 Wb.

Since it is specially mentioned that the flux is about 0.06 W, so it is the average value of flux.

But $\phi_{average} = \frac{\phi_m}{\pi/2} = 0.6366\phi_m \quad \because \ \phi_m = \frac{\phi_{average}}{0.6366} = \frac{0.06}{0.6366} = 0.09425$ Wb

Now $E_1=4.44f\phi_m T_1=6,000$

$$\therefore \quad T_1 = \frac{6,000}{4.44\times50\times0.09425} = 288$$

Also $T_2/T_1=E_1/E_2$ ∴ $T_2/288=1/24$

∴ $T_2=288/24=$**12**.

Hence, number of turns in : (i) H.V. (primary) winding = 288, and (ii) L.V. (secondary) winding=**12**.

Example 5:

A 80 kVA, 3200/400 volts transformer has 111 truns on secondary. Calculate : (i) number of turns on primary winding; (ii) secondary current; (iii) the cross-sectional area of the core, if the maximum flux density is 1.2 Telsa.

Solulion :

(i) No. of turns of primary winding,

$N_1=N_2E_1/E_2=111\times3{,}200/400=800$

(ii) Secondary current, $I_2=VA/E_2=80{,}000$ VA/400 V=**200 A**.

(iii) $E_2=4.44\times1.2\times A\times50\times111$ [Assuming 50 Hz frequency]

or cross-sectional area, A=**0.135 m² or 135 cm²**.

Example 6:

A 200/27.5 V, 400 Hz, step-down transformer is to be operated at 60 Hz. Find : (i) the highest safe input voltage, and (ii) transformation ratio in both frequency situations.

Solution:

(i) f = 400 Hz; f= 60 Hz; E_1 = 120 E_2=27.5 V.

Now $E_1=4.44f\phi_m N_1=120$ V

∴ $4.44\phi_m N_1=120/f=120/400=0.3$

Let E_1' be the highest safe input at 60 Hz, then:

$E_1'=4.44f'\phi_m N_1=0.3\times f'=0.3\times60=$**18 V**.

(ii) Output voltage

E_2' at E_1'=18 V is given by:

$E_2'=27.5(f'/f)=27.5\times(60/400)=4.125$ V

∴ Transformation ratio

E_1/E_2 = 120/27.5 = **4.36≈5**

and E_1'/E_2' = 18/4.125 = **4.36≈5**

i.e., transfromation remains the same or is independent of frequency.

Example 7:

A 4,600/230 V, 60 Hz, step-down transformer has core dimensions of 76.2 mm by 111.8 mm. A maximum flux density of 0.930 Wb/m² is to be used. Calculate the following, assuming 9% loss of area due to staking factor of laminations: (i) primary turns required; (ii) turns per volt; (iii) secondary turns required, and (iv) transmission ratio.

Solution :

Effective cross-sectional area of core,

$A=76.2\times111.8\times10^{-6}\times0.9m^2=7.752\times10^{-3}m^2$; max flux density, $B_m=0.93$ Wb/m²; frequency; f=60 Hz.

(i) Primary turns required,

$$N_1=\frac{E_1}{4.44B_mAf}=\frac{4{,}600}{4.44\times0.93\times7.752\times10^{-3}\times60}=\mathbf{2{,}395}.$$

(ii) Turns/volt=N_1/E_1=2,395/4,600=**0.52**.

(iii) Secondary turns, $N_2=E_2N_1/E_1=230\times2{,}395/4{,}600\approx$ **120**.

(iv) Transmission ratio, $K=E_1/E_2=N_1/N_2=2{,}395/120\approx$ **20**.

Example 8:

The follow data apply to a single-phase transformer:

Out:100 kVa; Secondary voltage : 400 V; Primary turns: 200: Secondary turns: 40. Neglecting the losses, calcuate : (i) the primary applied voltage; (ii) the normal primary and secondary currents; (iii) the secondary current, when the load is 25 kW at 0.8 power factor.

Solution :

(i) Primary voltage,

$E_1=E_2N_1/N_2=400\times200/40=$**2,000 V**.

(ii) Normal primary current,

$I_1=$kVA×1,000/E_1=100×1,000/2,000=**50A**.

Normal secondary current,

$I_2=$kVA×1,000/E_2=100×1,000/400=**250A**.

(iii) Secondary current at 0.8 p.f.

kW×1,000×p.f./E_2=25×1,000/0.8×400=**78.125 A**.

Example 9:

A single-phase transformer has 1,000 turns on the primary, and 200 turns on the secondary. The no-load current is 3 A at p.f. of 0.2 lagging. Calculate the primary current, and p.f., when secondary current is 280 A at a p.f. of 0.8 lagging.

Solution :

(i) No-load current= $3\angle-\cos^{-1}0.2=3\angle-78.46^0$

$$=3\left(\cos78.46^{\circ}-\mathrm{j}\sin78.46^{\circ}\right)=(0.6-2.94)\mathrm{A}$$

Now $I_1=I_2(N_2/N_1)$=280(200/1,000) at p.f. 0.8 lag

$$=56\angle-\cos^{-1}0.8=(44.8-\mathrm{j}33.6)\mathrm{A}$$

∴ Total primary current $=(0.6-\mathrm{j}2.94)+(44.8-\mathrm{j}33.6)\mathrm{A}$

$$=(45.4-\mathrm{j}36.54)\mathrm{A}=\mathbf{58.28\angle-38.82^{o}\,A}$$

(ii) P.f. of primary current = cos(–38.82°)=**0.779(lag)**.

Example 10:

A 1,000/200 V transformer takes 0.3 at p.f. of 0.2 on open circuit. Find the magnetising, and iron loss component of no-load primary current.

Solution :

Here I_0=0.3 A; $\cos\phi_0$=0.2; E_1=1,000 V; E_2=200 V

(i) Iron loss component of primary current,

$$I_\omega=I_0\cos\phi_0=0.3\times0.2\mathrm{A}=\mathbf{0.06A}.$$

(ii) Magnetising component of primary current,

$$I_\mu=\sqrt{I_0^2-I_\omega^2}=\sqrt{0.3^2-0.06^2}=\mathbf{0.294A}.$$

Example 11:

A 230/110 V, 1ϕ transformer takes an input of 350 VA at no-load, and at rated voltage. The core loss is 110 W. Find : (i) the iron loss component of no-load current; (ii) magnetising component of no-load, and (iii) no-load p.f.

Solution :

V_1I_0=350 or I_0=350 VA/230 V=1.52 A.

(iii) No-load p.f., $\cos\phi_0$=110V/350V=**0.314**

(i) Iron component of no-load current,

$$I_\omega=I_0\cos\phi_0=1.52\times0.314=\mathbf{0.477A}$$

(ii) Magnetising component of no-load current,

$$I_\omega=\sqrt{I_0^2-I_\omega^2}=\sqrt{1.52^2-0.477^2}=\mathbf{1.44A}$$

Example 12:

A 400/200 V, single-phase transformer is supplying a load of 25 A at p.f. of 0.855 lagging. On no-load, the current, and p.f. are 2.0 A, and 0.208 lag respectively. Calculate the current taken from, the supply, and specify its phase.

Solution :

K=200 V/400 V=1/2; I2=25 A at 0.866 p.f. lag=25∠–30°A;

I0=2A at 0.208 p.f. lag=2∠–78° A.

$$I_1 = KI_2 = 1/2 \times 25\angle -30^\circ = 12.5\angle -30^\circ\,A$$

Now $I_1 = I_2 + I_0 = 12.5\angle -30^\circ + 2\angle -78^\circ\,A$

=(10.825–j6.250)+(0.416–j1.956) A=(2.241–j8.206) A

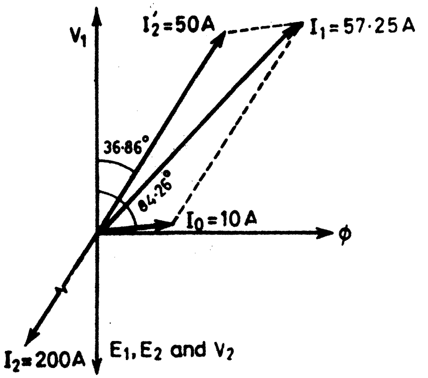
$= 14.737\angle -\mathbf{33.84^\circ\,A}$

Hence, phase angle, ϕ_1=**33.84° (lag)**.

Example 13:

A 1-phase transformer takes 10 A on no-load at a power factor of 0.1. The turns ratio is 4 : 1. If a load is supplied by the secondary at 200 A, and a power factor of 0.8, find the primary current, and the power factor. Neglect the internal voltage drops in a transformer.

Solution :



I_2=200 A; $\phi_2=\cos^{-1}0.8=36.86^\circ$; I_0=10 A; $\phi_0=\cos^{-1}0.1=84.26^\circ$;

$I_2' = I_1(N_1/N_2) = 200(1/4) = 50\ A$; angle between I_2 and $I_0 = 84.26^\circ - 36.86 = 47.4^\circ\,A$.

(i) $I_1 = \sqrt{\left(I_0^2 + \left(I_2'\right)^2\right)} + \sqrt{2I_0I_2'\cos 47.4^\circ\,A}$

$= \sqrt{10^2 + 50^2 + 2} \times \sqrt{10 \times 5 \times 0.6769}A$

=57.25A

(ii) P.f. = cosϕ

$$= \cos\left[\tan^{-1}\left(\frac{I_2'\sin\phi_2 + I_0\sin\phi_0}{I_2'\cos\phi_2 + I_0\cos\phi_0}\right)\right]$$

$$= \cos\left[\tan^{-1}\frac{50\times0.6+10\times0.99}{50\times0.8+10\times0.01}\right]$$

$$= \cos\left(\tan^{-1}0.995\right) = \cos 44.86^\circ = \mathbf{0.708}$$

Example 14:

Calculate the voltage regulation of a transformer in which the ohmic loss is 10% of the output, and the reactive loss is 5% of the voltage, when the p.f. is : (i) 0.8 lagging; (ii) 0.8 leading.

Solution :

(i) $\%V.R. = \frac{(I_2R_2\cos\phi + I_2X_2\sin\phi)100}{E_2} = 10\times0.8 + 5\times0.6 = \mathbf{11.0}\%$

(ii) $\%V.R. = \frac{(I_2R_2\cos\phi - I_2X_2\sin\phi)100}{E_2} = 10\times0.8 - 5\times0.6 = 5.\mathbf{0}\%$

Example 15:

A single-phase 100 kVA, 2,000/200 V, 50 Hz transformer has impedance drop of 10%, and resistance drop of 5%. Calculate : (i) the regulation at full-load 0.8 p.f. lagging; (ii) the value of p.f. at which regulation is zero.

Solution :

(i) cosϕ=0.8; sinϕ=0.6; I_2Z_2/E_2=10/100

or I_2Z_2=(10/100)×200=20 V and I_2R_2/E_2=5/100 or I_2R_2=(5/100)×200=10 V

$$\therefore\ I_2X_2 = \sqrt{(I_2Z_2)^2 - (I_2R_2)^2} = \sqrt{20^2 - 10^2} = 17.32\ V$$

(i) Regulation $= \frac{[I_2R_2\cos\phi + I_2X_2\sin\phi]\times 100}{E_2}$

$$= \frac{[20\times 0.8 + 10\times 0.6]\times 100}{200} = \mathbf{9.2\%}$$

(ii) For zero regulation, p.f. must be leading.

$$\therefore \quad 0 = \frac{[I_2R_2\cos\phi - I_2X_2\sin\phi]}{E_2}$$

or $\quad \tan\phi = I_2R_2 / I_2X_2 = 10/17.32 = 0.5774 = \tan 30^\circ$

$\therefore$ P.f., $\cos\phi = \cos 30^\circ = \mathbf{0.866(leading)}$.

Example 16:

A 50 kVA transformer has a efficiency of 98% at full-load, 0.8 p.f., and an efficiency of 96.9% at 1/4 full-load, unity p.f. Determine the iron loss and full-load copper loss.

Solution :

Efficiency at full-load 0.8 p.f.,

$$0.98 = \frac{50{,}000\times 0.8}{50{,}000\times 0.8 + W_i + W_C} = \frac{40{,}000}{40{,}000 + W_i + W_C}$$

$$\therefore \quad W_i + W_C = 40{,}000\left[\frac{1}{0.98} - 1\right] = 816.3 \text{ W}$$

Efficiency at 1/4 full-load,

$$0.98 = \frac{50{,}000\times(1/4)}{50{,}000\times(1/4) + W_i + (1/4)^2 W_C} = \frac{12{,}500}{12{,}500 + W_i + W_C/16} \quad \text{(i)}$$

or $\quad W_i + W_C/16 = 12{,}500\left[\frac{1}{0.969} - 1\right] = 399.9 \text{ W} \quad$ (ii)

Eq. (i) – Eq. (ii) gives :

(15/16) W_C=416.4 or $\mathbf{W_C=444.2\ W}$ (iii)

from : (i) and (ii), we get $\mathbf{W_i=372.1\ W}$ (iv)

Example 17:

A 600 kVA, single-phase transformer has an efficiency of 92% at both full-load and half-load at unity p.f. Determine its efficiency at 75% of full-load and 0.9 p.f. lag.

Solution :

We have from given data :

$$0.92 = \frac{600\times 1}{600\times 1 + W_i + W_C} = \frac{600\times(1/2)\times 1}{600\times(1/2)\times 1 + W_i + 0.25W_C}$$

$$\therefore \quad W_i + W_C = 600\left[\frac{1}{0.92} - 1\right] = 52.174 \text{ kW} \quad \text{(i)}$$

and $\quad W_i + 0.25W_C = 300\left[\frac{1}{0.92} - 1\right] = 26.086 \text{ kW} \quad$ (ii)

Solving (i) and (ii), we get : W_i=34.783 kW and W_C=17.391 kW.

Efficiency at 0.75 full-load, 0.9 p.f. : Output=600×0.75=450 kVA, copper loss =$(0.75)^2$×34.783=19.565 kW, iron loss=17.391 kW (constant)

$$\therefore \text{ Efficiency} = \frac{450\times 0.9}{450\times 0.9 + 17.391 + 19.565} = 0.9164 \text{ or } \mathbf{91.64\%}.$$

Example 18:

(i) A 220/440V, 10 kVA, 50Hz single-phase transformer has at full-load, a copper loss of 120 W. If it has an efficiency of 98% at full-load, and unity p.f., determine the iron losses; (ii) What would be the efficiency at half full-load, and 0.8 p.f. lagging?

Solution :

$$\text{Efficiency} = 0.98 = \frac{\text{kVA}\times 1{,}000\times \text{p.f.}}{\text{kVA}\times 1{,}000\times \text{p.f.} + W_i + W_C}$$

$$= \frac{10{,}000 \times 1}{10{,}000 \times 1 + W_i + 120}$$

$\therefore$ Iron losses, $W_i = 10{,}000\left[\frac{1}{0.98} - 1\right] - 120\,W = 204 - 120\,W = \mathbf{84\ W}$.

(ii) Efficiency at half full-load, and 0.8 p.f. lagging

$$\eta = \frac{0.5 \times 10{,}000 \times 0.8}{0.5 \times 10{,}000 \times 0.8 + 84 + (0.5)^2 \times 120} = \frac{4{,}000}{4{,}000 + 84 + 30} = \mathbf{97.23\%}$$

Example 19:

A 40 kA transformer has a core loss of 450 W, and full-load copper loss of 850 W. If the power factor of the load is 0.8, calculate: (i) the full-load efficiency; (ii) the maximum efficiency, and (iii) the load in kVA at which the maximum efficency occurs.

Solution :

(i) Full-load efficiency at 0.8 p.f.,

$$\eta = \frac{\text{kVA} \times \text{p.f.}}{\text{kVA} \times \text{p.f.+losses}} = \frac{40 \times 0.8\text{kW}}{40 \times 0.8\text{kW}(W_i + W_C)}$$

$$= \frac{32\text{kW}}{32\text{kW} + (0.45 + 0.850)\text{kW}} = \mathbf{96.1\%}$$

(ii) Let at x times full-load, the efficiency is maximum. Then total losses = $0.45+0.85x^2$ kW.

But at maximum efficiency iron losses equal copper losses.

$0.45=0.85x^2$ or $x=(0.45/0.85)^{1/2}=0.7276$

$\therefore$ Output at max. efficiency = 40×0.7276×0.8=23.28kW

and input at max. efficiency = 23.28+0.45+0.45=24.18kW

Hence, max. efficiency, $\eta_{max} = \frac{23.28}{24.18} \times 100 = \mathbf{96.3\%}$

(iii) Load in kVA at maximum efficiency

= 40×0.7276 kVA=**29.1kVA**.

Example 20:

An iron-cored transformer has 200 turns in the primary and 100 turns on the secondary. A supply of 400 V, 50 Hz is given to the primary and an impedance of (4+j3)Ω is connected across secondary. Assume ideal behaviour and calculate: (i) voltage and current through the load, (ii) the primary current, (iii) the power taken from the supply, (iv) the input impedance of the transformer.

Solution :

(i) $V_2=V_1(N_2/N_1)=400\times(100/200)=\mathbf{200\ V}$

$I_2=V_2/Z_{load}=200/\sqrt{4^2+3^2}\,\underline{|36.87^\circ} = \mathbf{40\underline{|-36.87^\circ}\,A}$

(ii) $I_1=I_2\times(N_2/N_1)=40\underline{|36.87^\circ} \times (100/200) = \mathbf{20\underline{|-36.87^\circ}\,A}$

(iii) $P=V_1I_1\cos\phi=400\times20\times0.8=$ **6,400 W or 6.3 kw**.

(iv) $Z_1=Z_2(N_1/N_2)^2=(4+j3)\times(200/100)^2=\mathbf{(16+j12)\Omega}$

Example 21:

A single-phase 2,200/220 V, 50 Hz transformer has core area of 3,600 mm² and maximum flux density 1.67 T. Determine the number of turns of primary and secondary windings.

Solution :

Here V_1=2,200 V, V_2=220 V, f=50 Hz, A = $3{,}600\times10^{-6}$ m² = 3.6×10^{-3} m², B=1.67 T

$\therefore$ Max. flux, ϕ_m=Area×flux density=$3.6\times10^{-3}\times1.67$ Wb

Now $V_1=4.44\,\phi_m fN_1$ or $N_1/(4.44\phi_m f)$

$$\therefore \quad N_1 = \frac{2{,}200}{4.44 \times 3.6 \times 10^{-3} \times 1.67 \times 50} = \mathbf{1{,}648}$$

and $N_2=N_1(V_2/V_1)$ = 1,648 (220/2,200) = 164.8 $\approx$ **165**

Example 22:

In a 25 kVA, 2000/200 V transfer, the iron and copper losses are 350 watts and 400 watts respectively. Calculate the effciency at full-load and 0.8 power factor lagging. Determine the maximum effeciency and the corresponding load.

Solution :

$$\text{Efficiency} = \frac{\text{kVA} \times 1{,}000 \times \text{p.f.}}{\text{kVA} \times 1{,}000 \times \text{p.f.} + W_i + W_C}$$

$$= \frac{25 \times 1{,}000 \times 0.8}{25 \times 1{,}000 \times 0.8 + 350 + 400} = \frac{20{,}000}{20{,}750}$$

= **0.9638 or 96.38%**.

Let at x times full-load, the efficiency is maximum. Then, total loss = $(350+400\ x^2)$ W

But at maximum efficiency iron losses equal copper losses.

$\therefore$ $350 = 400\ x^2$ or $x=(350/400)^{1/2} = 0.9354$

$\therefore$ Output at maximum efficiency = 25,000×0.9354×0.8=18,708 W

and input at maximum efficiency = 18,708+350+350=19,408 W

Hence, max. efficiency $\eta_{max} = \frac{18{,}708}{19{,}408} =$ **0.9639 or 96.39%**

Example 23:

Full-load efficiency of a 4,000/400 V, 40 kVA, single-phase transformer is 94%. Maximum efficiency occurs at 90%, of full-load. Find iron loss and full-load copper loss of the transformer. The load power factor being 0.8 lagging.

Solution :

Efficiency at full-load 0.8 p.f.,

$$0.94 = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{40 \times 0.8 \text{ kW}}{40 \times 0.8\text{kW} + (W_i + W_C)} = \frac{32\text{kW}}{32\text{kW} + (W_i + W_C)}$$

$$\therefore \quad (W_i + W_C) = 32\left[\frac{1}{0.94} - 1\right] = 2.043 \text{ kW} \qquad \text{(i)}$$

Efficiency at 90% full-load and 0.8 p.f.

$$= \frac{40 \times 0.9 \times 0.8\text{kW}}{40 \times 0.9 \times 0.8\text{kW} + W_1 + (0.9)^2 W_C} = \frac{28.8\text{kW}}{28.8\text{kW} + W_1 + 0.81 W_C}$$

But at maximum efficiency, iron loss equals copper loss, i.e., $W_1=0.81W_C$ (ii)

For Eqs. (i) and (ii), we get:

$0.81\ W_C+W_C=2.043$ kW or $W_C=2.043$ kW/1.81 = **1.129 kW**.

and $W_1=0.81\times1.125$ kW=**0.914 kW**.]

Example 24:

A 50 kVA trasformer has an efficiency of 98% at full-load, 0.8 p.f. and 97% at half full-load, 0.8 p.f. Determine the full-load copper loss and iron loss. Find the load at which maximum efficiency occurs and also maximum efficiency.

Solution :

(i) Efficiency at full-load, 0.8 p.f.,

$$0.98 = \frac{50 \times 0.8}{50 \times 0.8 + W_i + W_C} = \frac{40}{40 + W_i + W_C}$$

$$\therefore \quad W_i + W_C = 40\left[\frac{1}{0.98} - 1\right] = 0.8163\text{kW} = 816.3\text{W} \qquad \text{(i)}$$

Efficiency at hal full load, 0.8 p.f.,

$$0.97 = \frac{50 \times (1/2) \times 0.8}{50 \times (1/2) \times 0.8 + W_i + (1/2)^2 W_C} = \frac{20}{20 + W_i + W_C/4}$$

$$\therefore \ W_i + W_C/4 = 20\left[\frac{1}{0.97} - 1\right] = 0.6186\text{kW} = 618.6\text{W} \qquad \text{(ii)}$$

Eq. (i) – Eq. (ii) gives : $3W_C/4=197.7$ or W_C=**263.6 W** (iii)

From (i) and (ii), we get : W_i = **552.7 W**.

(ii) Now kVA output at maximum efficiency

$$= \text{Rated kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{Full-load copper loss}}} = 50 \times \sqrt{\frac{552.7}{263.6}} = 72.392 \text{ kVA}$$

∴ Maximum efficiency occurs when supplying **72.392 kVA**.

(iii) Output at maximum efficiency = 72.392 kVA, iron loss (W_i)=552.7 W

$$\text{Copper loss} = \text{Full-load copper loss} \times \left[\frac{\text{Load at max. efficiency}}{\text{Full load}}\right]^2$$

$$= 263.7\left[\frac{72.392}{50,000}\right]^2 = 552.7 \text{ W}$$

Hence, maximum efficiency (at unity p.f.)

$$= \frac{72,392 \times 1}{72,392 + 552.7 + 552.7} = 0.98496 \text{ or } \mathbf{98.496\%}$$

Note : At maximum efficiency, copper loss (W_C)=iron loss (W_i)

Example 25:

A 40 kVA, 2,000/250 V, 50 Hz, 1-ph transformer has : (i) efficiency of 97% of full-load and 0.8 p.f.; (ii) efficiency of 98% at half-load, and unity p.f. Determine: (i) iron, and copper losses at full-load; (ii) load at which copper loss is 400 W; (iii) efficiency at the load in (u), and 0.8 lag p.f.

Solution :

(i) $$0.97 = \frac{40,000 \times 0.8}{40,000 \times 0.8 + W_i + W_C} \quad \text{(At full-load)} \qquad \text{(i)}$$

and $$0.98 = \frac{0.5 \times 40,000 \times 1.0}{0.5 \times 40,000 \times 1 + (0.5)^2 W_C} \quad \text{(At half full-load)} \qquad \text{(ii)}$$

From Eqs. (i) and (ii), we get:

$W_i + W_C = 989.7\text{W}$ and $W_i + 0.25W_C = 408.16\text{W}$

∴ Copper loss, W_C=(989.75–408.16)/0.75=**775.45 W**.

and iron loss, W_i=**214.25 W**.

(ii) Load at which copper loss is 400 W be x 40 kVA, then:

775.45 W×x^2=400 or x=0.7182

or load =40×0.7182 kVA=**28.73 kVA**.

(iii) Efficiency, $$\eta = \frac{28.73 \times 1,000 \times 0.8}{28.74 \times 1,000 \times 0.8 + 214.25 + 400} = \mathbf{97.4\%}$$

Example 26:

The iron loss of a 80kVA, 1,000/250V, 1-ph, 50Hz transformer is 800 W. The copper loss, when primary carries 50A is 400 W. Estimate : (i) area of cross-section of limb, if working flux density is 1 tesla, and there are 1,000 turns on primary (h.t. winding); (ii) current ratio (primary to secondary); (iii) efficeincy at full-load, and 0.8 p.f. and (iv) efficiency for a load, when copper loss will equal iron loss, and p.f. remains 0.8 lag.

Solution :

(i) $E_1 = 4.44\phi_m f N_1 = 4.44 \ B_m A f N_1$

∴ Area of cross-section, $$A = \frac{E_1}{4.44 \times B_n f N_1} = \frac{1,000}{4.44 \times 1 \times 50 \times 1000}$$

$$= \mathbf{4.5 \times 10^{-3} \, m^2} \text{ or } \mathbf{45 \, cm^2}.$$

(ii) Current ratio, $I_1/I_2 = E_2/E_1$=250/1,000=**0.25**

(iii) Full-load current =80,000/1,000=**80 A**

∴ Copper loss at full-load =$(80/50)^2$×400=1,024 W

Efficiency at full-load $$= \frac{VA\cos\phi}{VA\cos\phi + (W_i + W_C)}$$

$$= \frac{80{,}000 \times 0.8}{80{,}000 \times 0.8 + 800 + 1{,}024}$$

= **0.9723 or 97.23%**

(iv) When W_i=W_C=800W, then corresponding current

= $(800/400)^{1/2}$ ×50A = 70.71 A.

$$\therefore \text{ Efficiency, } \eta_{max} = \frac{VI\cos\phi}{VI\cos\phi + 2W_i} = \frac{1{,}000 \times 70.71 \times 0.8}{1{,}000 \times 70.71 \times 0.8 + 2 \times 800}$$

= **0.9725 or 97.25%**.

Example 27:

A 25 kVA, 1,910/240 V, single-phase, 50 cps transformer is employed to setp-down the voltage, and the voltage on l.v. side is kept constant. Calculate : (i) the turn ratio; (ii) full-load current on l.v. side; (iii) load-current referred to h.v. side; (iv) load impedance on l.v. side for full-load; (v) the impedance referred to the h.v. side.

Solution :

(i) Turn ratio, $N_1/N_2=V_1/V_2$=1,910/240=**7.95**.

(ii) Full-load current on l.v. side,

I_2=25,000 VA/240 V = **104.16 A**.

(iii) Load current referred to h.v. side,

I_1=25,000 VA/1,910 V = **13.1 A**.

(iv) Load impedance on l.v. side for full-load,

$Z_2=V_2/I_2$=240/104=**2.3Ω**

(v) Impedance referred to h.v. side,

$Z_1=Z_2(N_1/N_2)2=2.3\times(7.95)^2$=**145.36Ω**

Example 28:

The open-circuit test readings on a 400/200 V, single-phase transformer conducted from L.V. side are : voltage = 200 V; current=0.7A; power=95 W. Calculate the no-load circuit parameters referred to H.V. side.

Solution :

$\cos\phi_0=W_{OC}/V_{OC}.I_{OC}$=95/200×0.7=0.6786; $\sin\phi_0$=0.7345; $R_{02}=V_{OC}/I_O\cos\phi_0$=200/0.7×0.6786=421 W; $X_{02}=V_{OC}/I_0\sin\phi_0$=200/0.7×0.7365=387.83 Ω.

Hence no-load circuit parameters referred to H.V. side are:

$R_{01}=R_{02}\times(N_1/N_2)^2=421\times(400/200)^2$=**1,684 Ω**

$X_{01}=X_{02}\times(N_1/N_2)^2=387.93\times(400/200)^2$=**1,551.7 Ω**

Example 29:

The S.C test on a single-phase transformer with primary winding short-circuited, and 30V applied to the secondary gave a wattmeter reading of 60W, and secondary current of 10A. If the normal voltage is 200 V, transformer ratio is 1:2, and full-load secondary current is 10A, calculate the secondary terminal potential difference at full-load current for : (a) unity p.f., and (b) p.f. 0.8 lagging.

Solution :

I_2=10A, $R_2=P_{SC}/I_{SC}^2=60/10^2$=0.6Ω; $Z_2=V_{2SC}/I_{2(f1)}$=30/10=3Ω;

$X_2=\sqrt{3^2-0.6^2}$=2.94Ω; E_1=200V; $E_2=E_1(N_2/N_1)$=200(2/1)=400V

(i) At unity p.f.: $(E_2–V_2)=I_2[R_2\cos\phi+X_2\sin\phi]$=10[0.6×1+0]=6V

∴ V_2 at load=E_2–Voltage drop=400–6=**394 V**.

(ii) At 0.8 p.f. lagging : cosϕ = 0.8; sinϕ = 0.6

∴ Voltage drop $(E_2–V_2)=I_2[R_2\cos\phi+X_2\sin\phi]$

= 10[0.6×0.8+2.94×0.8]=28.32V

∴ V_2 at load=E_2–Voltage drop=400–28.32=**371.68V**.

Example 30:

A 5 kVA, 200/100 V, 50 Hz, single-phase transformer has

rated secondary voltage on full-load. When the load is removed, the secondary voltage is found to be 110 V. Determine percentage regulation.

Solution :

E_2(no-load)=110V, secondary $V_{2(f.1)}$=100V.

$$\therefore \text{ Percentage regulation } = \frac{E_{2(\text{no-load})} - V_{2(f.l)}}{E_{2(\text{no-load})}} \times 100$$

$$= \frac{110-100}{100} \times 100 = \mathbf{9.091\%}.$$

Example 31:

In a 25 kVA transformer, the iron loss and full-load copper loss are 250 W and 400 W respectively. Calculate the efficiency of the transformer at : (i) half full-load unity p.f., (ii) 3/4th full-load at 0.8 p.f. lagging.

Solution :

Here rating of the transformer = 25 kVA; iron loss = 350 W, full-load copper loss = 400 W.

(i) At half full-load and unity p.f.:

$$\text{Output} = \frac{1}{2} \times \text{kVA rating} \times \text{p.f.} = \frac{1}{2} \times 25 \times 1 = 12.5 \text{ kW}$$

Copper loss at half full-load =$(1/2)^2$×400W=100W

Iron loss (constant at all loads) = 350W

∴ Total loss = (100+350)W = 450W = 0.45 kW

∴ Input=Output+Total loss=(12.5+0.45)kW=12.95kW

$$\therefore \quad \text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{12.5}{12.95} \times 100 = \mathbf{96.525\%}$$

(ii) At 3/4th full load and 0.8 p.f. (lagging):

Output = (3/4)×25×0.8=15 kW

Copper loss = $(3/4)^2$×400W=225 W

Iron loss = 350 W

∴ Total loss = (225+350) W = 575 W=0.575 kW

∴ Input = (15+0.575) kW=15.575 kW

$$\therefore \quad \text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{15}{15.575} \times 100 = \mathbf{96.308\%}$$

Example 32:

A 230/460 V, single-phase transformer has a primary resistance 0.2 ohm, and reactance 0.5 ohm. The corresponding values for the secondary are 0.75 ohm, and 1.8 ohms respectively. Find the secondary terminal voltage, when supplying 10 A at 0.8 p.f. lag.

Solution :

$R_2=r_2+r_1(N_2/N_1)^2=0.75+0.2\times(460/230)^2=1.55\Omega$

$X_2=x_2+x_1(N_2/N_1)^2=1.8+0.5\times(460/230)^2=3.8\Omega$

∴ Voltage drop, $E_2-V_2=I_2(R_2\cos\phi+X_2\sin\phi)$

=10 (1.55×0.8+3.8×0.6)=35.2 V

Hence, secondary terminal voltage,

$V_2=E_2$–Voltage drop = 460 –35.2 V=**424.8 V**.

Example 33:

A 40 kVA transformer with a ratio of 2,000/250 V has a primary winding resistance of 1.15 ohms, and secondary winding resistance of 0.0155 ohm. Calculate (i) the total resistance drop on full-load, and (iii) the total copper loss on full-load.

Solution :

(i) Total resistance in terms of secondary winding.

Full-load secondary current, $I_{2(f.1.)}$=40,000/250=160 A.

(ii) Total resistance drop on full-load,

$= R_2 \times I_{2(f.l)} = 0.0389 \times 160 = 6.23V$

(iii) Total copper loss on full-load,

$W_C = I^2_{2(f.l.)} \times R^2 = (160)^2 \times 0.0389 = \mathbf{996\ W}$.

Example 34:

The reading of direct loading test on a single-phase transformer are :

	Primary side			*Secondary side*	
	$V_1(V)$	$I_1(A)$	$W_1(W)$	$V_2(V)$	$I_2(A)$
(i) No-load condition with resistive load	*220*	*1.5*	*60*	*110*	*0*
(ii) Full-load condition with resistive load	*220*	*15.07*	*3,375*	*104.5*	*30*

Find at full-load its efficiency, and regulation.

Solution :

$$\text{Efficiency} = \frac{\text{Output Power}}{\text{Input Power}} = \frac{V_2 I_2 \cos\phi_2}{W_1} = \frac{104.5 \times 30 \times 1}{3,375}$$

$= \mathbf{0.93\ or\ 93\%}$

(ii) Regulation $= \dfrac{E_2 - V_2}{E_2} \times 100\%$

Here E_2 (no-load voltage)=100V, and V_2(full-load voltage)=104.5V.

$\therefore$ Regulation $= \dfrac{110 - 104.5}{110} \times 100 = \mathbf{5}\%$

Example 35:

The following test data is obtained on a 5 kVA, 220/400V single-phase transformer:

O.C test – 220 V,	*2A, 100 W*	*(on L.V.side)*
S.C. test – 40V,	*11.4A, 200 W*	*(on H.V. side)*

Determine the percentage efficiency and regulation at full-load 0.9 p.f. lag.

Solution :

O.C. test on L.V. side : W_i=100W.

S.C. test on H.V. side : $R_2 = P_{2SC}/(I2SC)^2 = 200/(11.4)^2 = 1.54\Omega$;

$Z_2 = V_{2SC}/I_{2SC} = 40/11.4 = 3.5W$; $X_2 = \sqrt{3.5^2 - 1.54^2} = 3.14\,\Omega$; $I_{2(f.l.)} = 5,000/400 = 12.5A$; output power = 5,000×0.9=4,500 Ω; copper loss at full-load; $W_C = W_{SC} \times (I_{2(f.l.)}/I_{2SC})^2 = 200\ \Omega \times (12.5/11.4)^2 = 240\Omega$; cosϕ=0.9, and sinϕ=0.44.

(i) Efficiency at full-load,

$$\eta = \frac{\text{Output} \times 100\%}{\text{Output} + W_i + W_C} = \frac{4,500 \times 100\%}{4,500 + 100 + 240.5} = \mathbf{93}\%$$

(ii) Regulation $= \dfrac{I_2[R_2 \cos\phi + X_2 \sin\phi] \times 100\%}{E_2}$

$$= \frac{12.5[1.54 \times 0.9 + 3.14 \times 0.44] \times 100}{400} = \mathbf{8.6}\%$$

Example 36:

A 25 kVA, 2,200/200V, 50Hz single-phase transformer has the following resistance and leakage resistances:

$\boldsymbol{r_1 = 0.8\Omega;\ r_2 = 0.009\Omega,\ x_1 = 3.2\Omega,\ x_2 = 0.03\Omega}$

Calculate the equivalent resistance and reactance referred to the secondary.

Solution :

$V_2/V_1 = N_2/N_1 = 2,200/200 = 0.01$

$\therefore$ Equivalent resistance referred to the secondary,

$R_2 = r_2 + r_1(N_2/N_1)^2 = 0.0009 + 0.8(0.1)^2 = \mathbf{0.017\Omega}$

and equivalent reactance referred to the secondary

$X_2 = x_2 + x_1(N_2/N_1)^2 = 0.03 + 3.2(0.1)^2 = \mathbf{0.062\Omega}$

Example 37:

Open-circuit and short-circuit tests on a 5 kVA, 200/40 V, 50 Hz, 1-phase transformer gave the following tests:

O.C test – 220 V, 1A, 100 W (on L.V.side)

S.C. test – 15V, 10A, 85 W (with primary short-circuited)

(i) Draw the equivalent circuited referred to primary; (ii) calculate the approximate regulation of the transformer of 0.8 p.f. laggaging, and leading.

Solution :

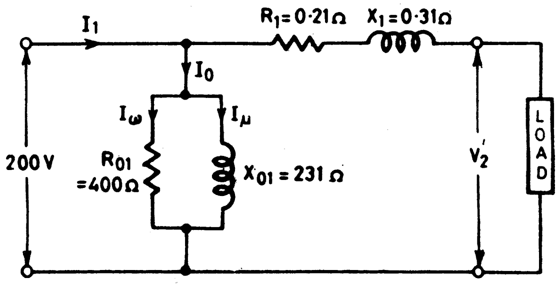
(i) O.C. test on L.V. side : $\cos\phi_0 = W_0/V_{OC}.\ I_0 = 100/200\times 1 = 0.5$; $\sin\phi_0 = 0.866$; $R_{01} = V_{OC}/I_0 \cos\phi_0 = 200/1\times 0.5 = 400\ \Omega$; $X_{01} = V_{OC}/I_0 \sin\phi_0 = 200/1\times 0.866 = 231\ \Omega$.

S.C. test on H.V. side : $R_2 = W_{SC}/I_{sc}^2 + 85/102 = 0.85\ \Omega$; $Z_2 = V_{SC}/I_{SC} = 15/10 = 1.5\ \Omega$; $X_2 = \sqrt{1.5^2 - 0.85^2} = 1.236\ \Omega\cdot$

Full–load secondary current, $I_{2(f.l.)} = 4{,}000/40 = 10$ A,

$\therefore R_1 = R_2\ (N_1/N_2)^2 = 0.85(200/400)^2 = 0.21\ \Omega$; $X_1 = X_2\ (N_1/N_2)^2 = 1.236\times(200/400)^2 = 0.31\ \Omega$.

$\therefore$ equivalent circuit referred to primary is :



(ii) Percentage regulation (referred to secondary) at :

(a) 0.8 p.f. lagging

$$= \frac{I_2(R_2\cos\phi + X_2\sin\phi)\times 100}{E_2}$$

$$= \frac{10\ (0.85\times 0.8 + 1.236\times 0.6)\times 100}{400} = 3.544\%$$

(b) 0.8 p.f. leading

$$= \frac{10\ (0.85\times 0.8 + 1.236\times 0.6)\times 100}{400} = -0.154\%\ .$$

Example : 38

A 75 kVA, single–phase 6,600/230 V transformer gave the followings on S.C. test : voltage = 300 V; current = 11.36 A ; power = 1.5 kW. Determine the percentage regulation, and terminal voltage on full–load, when the p.f. is : (i) 0.8 Lagging, (ii) 0.8 leaing.

Solution :

From S.C. test :$R_1 = P_{SC}/(I_{sc})^2 = 1{,}500/(11.36)^2 = 11.62\ \Omega$; $Z_1 = V_{SC}/I_{SC} = 300/116 = 26.41\ \Omega$; $X_1 = \sqrt{26.41^2 - 11.62^2} = 23.71\ \Omega$

$\therefore R_2 = R_1\ (N_2/N_1)^2 = 11.62\times(230/6{,}600)\ 20.0141\ \Omega$.

$X_2 = X_1\ (N_2/N_1)^2 = 23.71\times(230/6{,}600)^2 = 0.0288\ \Omega$.

(i) Full–load current I_2(f.l) = 75,000/230 = 326.1 A ; $\cos\phi = 0.8$ (lag) ; $\sin\phi = 0.6$; $E_2 = 230$ V.

$$\therefore \text{Regulation} = \frac{I_2(R_2\cos\phi + X_2\sin\phi)\times 100\%}{E^2}$$

$$= \frac{326.1(0.041\times 0.8 + 0.0288\times 0.6)}{230} = 4.05\%$$

Now $$4.05 = \frac{E_2 - V_2}{E_2} \times 100 = \frac{230 - V_2}{230} \times 100$$

$\therefore$ Terminal voltage, V_2 = 230 – 4.05 × 2.3 = 220.7 V.

(ii) At p.f. 0.8 leading, regulation

$$= \frac{326.1\,(0.041 \times 0.8 - 0.0288 \times 0.6) \times 100\%}{230} = -0.85\%$$

Now $$0.85 = \frac{(E_2 - V_2)}{E_2} \times 100 = \frac{230 - V_2}{230} \times 100$$

$\therefore$ Terminal voltage, V_2 = 232 V.

Example : 39

Open–circuit, and short–circuit tests were conducted on a 50 kVA, 6,360/240, 50 Hz, single–phase transformer in order to find its efficiency. The observations during these tests are :

O.C. test : Voltage primary winding = 6,360 V; primary current = 1.0 A, and power input = 2 kW.

S.C. test : Voltage across primary = 180 V; current in secondary winding = 175 A, and power input = 2 kW.

Calculate the efficiency of the transformer, when supplying full–load at p.f. of 0.8 lagging.

Solution :

O.C. test : W_i = 2,000 W ; $I_{2(f.l.)}$ = 50,000/240 = 208.33 A.

S.C. test : I_{2SC} = 175 A ; W_c = 2,000 W.

$$\therefore \text{Cu loss at full-load} = W_c\left(\frac{I_{2(fl)}}{I_{2SC}}\right)^2 = 2{,}000\left(\frac{208.33}{175}\right)^2 = 2{,}833 \text{ W}$$

$$\therefore \text{Efficiency} = \frac{50 \times 10^3 \times 0.8 \times 100\%}{50 \times 10^3 \times 0.8 + 2{,}833} = 95.23\%$$

Example : 40

A 250/500 V single–phase transformer gave the following results :

Short–circuit test with low voltage winding short–circuited : 20 V, 12 A, 100 W ; Open–circuit test : 250 V, 1A, 80 W on low voltage side.

Find the efficiency, when the outputs is 10 A at 500 V at 0.8 p.f. lag.

Solution :

O.C. test : 250 V, 1 A, 8W. Since this test is carried out by applying the rated voltage (250 V) on L.V. side with H.V. side open–circuited, so the wattmeter reading gives the core loss corresponding to normal voltage, i.e.,

Core (or iron) = 80 W.

S.C. test : 20 V, 12 A, 100 W (current on H.V. winding = 12 A and power input = 100 W).

$\therefore$ Copper loss corresponding to 12 A = 100 W.

Now efficiency at 10 A load is required, so :

$$\frac{\text{Copper loss at 10A}}{\text{Copper loss at 12A}} = \left(\frac{10}{12}\right)^2 = \frac{\text{Copper loss at 10A}}{100\text{W}}$$

$\therefore$ Copper loss at 10 A = 100 × $(10/12)^2$ = 69.44 W

Hence, efficiency at 10A, 0.8 p.f. and 500 V

$$= \frac{500 \times 10 \times 0.8}{500 \times 10 \times 0.8 + 80 + 69.44} = \frac{4{,}000}{4{,}149.44} = 0.964 \text{ or } 96.4\%$$

Example : 41

A 10 kVA, 400/230 V, 50 Hz single phase transformer on test gives the following results with instrument connected on H.V. side:

O.C. test : 400V 2A 100W

S.C. test : 25V 25A 80W

Find out the percentage regulation at full–load 0.8 lagging p.f. and efficiency.

Solution

O.C. test : W0 = 100 W.

S.C. test : R_1 = 80 W/$(25)^2$ = 0.128 Ω; Z_1 = 25/25 = 1Ω ; X_1 = $(1– 0.125^2)^{1/2}$ = 0.992 ; cos ϕ = 0.8 ; sin ϕ = 0.6 ; W_c = 80 W.

(i) $$\text{Regulation} = \frac{I_1(R_1\cos\phi + X_1\sin\phi)\times 100\%}{E_1}$$

(ii) $$\text{Efficiency} = \frac{\text{kVA}\times 10^3\times \text{p.f.}\times 100\%}{\text{kVA}\times 10^3\times \text{p.f.}+W_i+W_0}$$

$$= \frac{10\times 10^3\times 0.8\times 100\%}{10\times 10^3\times 0.8+100+80} = 97.79\%$$

Example : 42

Open circuit and short circuits test on a kVA, 200/400 V, 50 Hz, single–phase transformer gave following results :

O.C. test : 200V 1.0 Amp, 100W (carried on LV side)

S.C. test : 15V, 10Amp, 85W (carried on H.V. side)

(i) Draw equivalent circuit referred to L.V. side.

(ii) Determine efficiency at full–load, and half the full–load, and at 0.8 power factor lagging.

Solution :

O.C. test on LV side : V_{OC} = 200 V; I_0 = 1.0 A; P_0 = 100W.

∴ cos ϕ = $P_0/V_{OC}\ I_0$ = 100/200× 100 = 0.5, and sin ϕ_0 = 0.866

R_{01} = V_{OC}/I_0 cos ϕ = 200/1× 0.5 = 400 W

X_{01} = V_{OC}/I_0 sin ϕ = 200/1× 0.866 = 231 Ω

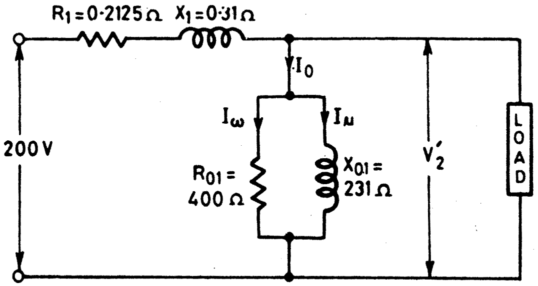
S.C. test on H.V. side : V_{2SC} = 15 V ; I_{2SC} = 10 A; P_{2SC} = 85 W ; R_2 = $P_{2SC}/(I_{2SC})^2$ = $85/10^2$ = 0.85 Ω ; Z_2 = V_{2SC}/I_{2SC} = 15/10 = 1.5 Ω, $X_2 = \sqrt{Z_2^2 - R_2^2} = \sqrt{1.5^2 - 0.85^2}$ = 1.24 Ω.

∴ Resistance and reactacnce referred to L.V. side are :

R_1 = R_2 $(N_1/N_2)^2$ = 0.85 $(1/2)^2$ = 0.2125 Ω.

and X_1 = X_1 $(N_1/N_2)^2$ = 1.24 $(1/2)^2$ = 0.31 Ω.

∴ Equivalent circuit referred to L.V. side is :



(ii) Output at full–load, and 0.8 p.f. = 4,000× 0.8 = 3,200 W,

W_i = 100 ; W_c = $I^2_{2(fl)} \times R_2$ = (4,00/400)2× 0.85 = 85 W.

∴ Efficiency at full–load,

$$\eta_{fl} = \frac{3{,}200\times 100\%}{3{,}200+100+85} = 94.53\%$$

(iii) Output at half–load, and 0.8 p.f. = 0.5× 3,200 W + 1,600 W ; W_i = 100 W; W_c = (0.5)2× 85 W = 21.25 W.

∴ Efficiency at half full–load,

$$\eta_{hfl} = \frac{1{,}600\times 100\%}{1{,}600+100+21.25} = 92.95\%$$

Example : 43

A transformer has its maximum efficiency of 0.98 at 15 kVA at unity power factor. During the day, it is loaded as under :

12 hours 2 kW at power factor 0.5

6 hours 12 kW at power factor 0.8

6 hours 18 kW at power factor 0.9

Find the "all–day efficiency".

Solution :

Output at unity p.f. = 15×1 = 15 kW, and input = 15 = $W_c + W_i$ = 15 + 2 W_c

(similarly At η_{max} $W_i = W_c$)

$$\therefore \eta_{max} = 0.98 = \frac{15}{15 + 2\,W_i} \text{ or } W_i = W_c = 0.153 \text{ kW}$$

All–day output = $2 \times 12 + 12 \times 6 + 18 \times 6$kWh = 204 kWh.

Iron losses in 24 hours = 0.153×24 = 3.672 kWh.

Now 2 kW at 0.5 p.f. = 2/0.5 = 4kVA

12kW at 0.8 p.f. = 12/0.8 = 15 kVA

18 kW at 0.9 p.f. = 18/0.9 = 20 kVA

Copper losses in 12 hours at 2kW, and 0.5 p.f. (or 4 kVA)

$= 12 \times 0.153 \times (4\text{kVA}/15 \text{ kVA})^2 = 0.131$ kWh

Copper losses in 6 hours at 12kW, and 0.8 p.f. (or 15 kVA)

$= 6 \times 0.153 \times (15\text{kVA}/15 \text{ kVA})^2 = 0.918$ kWh

Copper losses in 6 hours at 18kW, and 0.9 p.f. (or 20 kVA)

$= 6 \times 0.153 \times (20\text{kVA}/15 \text{ kVA})^2 = 1.632$ kWh.

$\therefore$ Total copper loss/24 hours = 0.131 + 0.918 + 1.632 = 2.681 kWh

$\therefore$ Input in 24 hours = Output + Copper losses + Iron losses

= 204 + 3.672 kWh

= 210.353 kWh.

$$\therefore \text{All–day efficiency, } \eta_{all-day} = \frac{204}{210.353} \times 100 = 96.98\% .$$

Example : 44

The following test data is obtained in a 5 kVA, 220/440 V single–phase transformer :

O.C. test : 220 V, 2 A, 100W on L.V. side

S.C. test : 40 V, 11.4A, 200 W on H.V. side

Determine : (i) the percentage efficiency, and (ii) regulation at full–load 0.9 p.f. lag.

Solution :

O.C. test : Core loss, W_i = 100 W

S.C. test : R_{01} = 200/(11.4)2 = 1.54 Ω;

$R_{02} = K^2 R_{01} = (440/220)^2 \times 1.54 = 6.16$ Ω;

$Z_{01} = V_{SC}/I_{SC}$ = 40/11.4 = 3.5 Ω

$$\therefore X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(3.5)^2 - (1.54)^2} = 22.7\text{A}$$

Also cos φ = 0.9 , so sin φ = $\sqrt{1 - 0.81} = 0.435$

$\therefore$ Percentage regulation at full–load

$$= \frac{I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi}{V_1} \times 100$$

$$= \frac{[(22.7 \times 1.54 \times 0.9) + (22.7 \times 3.142 \times 0.135)] \times 100}{220} = 73.44\%$$

Copper loss at full–load, $W_c = I_1^2 R_{01} = (22.7)^2 \times 1.54 = 793.5$ W

$\therefore$ Percentage efficiency at full–load

$$= \frac{5{,}000 \times 0.9}{5{,}000 \times 0.9 + 100 + 793.5} \times 100 = 77.44\%$$

Example : 45

A 15 kVA, 2,200/200 V, 50 Hz transformer gave the following test results :

OC (LV side) : 220 V, 2.72 A, 185 W

SC (HV side) : 112 V, 6.3 A, 197 W

Compute : (i) core loss, (ii) full–load copper loss, and (iii) efficiency at full–load, 0.85 logging p.f.

Solution :

(i) O.C. test on L.V. side ; $V_{oc} = 220$ V, $I_0 = 2.72$ A, $P_0 = 185$ W

$\therefore$ Core loss, $W_i = 185$ W

(ii) S.C. test on H.V. side : $P_{2(sc)} = 197$ W ; $I_{sc} = 6.30$ A, full–load current, $I_{2(fl)} = 15{,}00/2{,}200 = 6.82$ A.

$$R_2 = P_{2(sc)}/(I_{sc})^2 = 197/(6.30)^2 = 4.963 \ \Omega$$

$\therefore$ Full–load copper loss, W_c

$$I_{2(fl)}^2 \times R_2 = (6.82)^2 \times 4.963 = 230 \text{ W}$$

(iii) Efficiency,

$$\eta = \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{15{,}000 \times 0.8}{15{,}00 \times 0.8 + 185 + 230}$$

$$= \frac{12{,}000}{12{,}415} = 0.966 \text{ or } 96.6\%$$

Example : 46

A 50 kVA, 6,360/240 V single–phase transformer gave the following test results :

Open–circuit test	*Primary voltage* 6,360 V	*Primary current* 1A	*Primary inputs* 2 kW
Short–circuit test	*Primary voltage* 1,325 V	*Secondary current* 175 A	*Power input* 2kW

Calculate ; (i) the magnetising current and the current corresponding to the normal voltage and frequency, (ii) the parameters of equivalent circuits as referred to high voltage side.

Solution

(i) Core loss, $W_i = 2$kW ; p.f., $\cos \phi_0 = W_i/V_1I_0 = 2\times 103/(6{,}360 \times 1) = 0.314$

$\therefore$ Magnetising current, $I_\mu = I_0 \sin \phi_0 = 1 \times 0.9492 = 0.9492$ A

and working current, $I\omega = I_0 \cos \phi_0 = 1 \times 0.3144$ A.

$$R_0 = V_1/I_\omega = 6{,}340/0.3144 = 22.229 \text{ k}\Omega$$

$$X_0 = V_1/I_\mu = 6{,}340/0.9492 = 6.70 \text{ k}\Omega$$

(ii) S.C. test : $I_{sc} = 175$ A ; $R_1 = 175 \times (240/6{,}360) = 6{,}6037 \ \Omega$

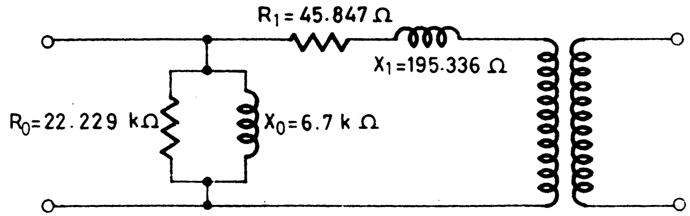
$V_{sc} = 1{,}325$ V, $Z_{sc} = V_{sc}/I_{sc} = 1{,}325/6.6037 = 200.645 \ \Omega$,

p.f., $\cos \phi_2 = W_{sc}/(V_{sc} \times I_{sc}) = 2 \times 10^3 /(1{,}325 \times 6.6037) = 0.2285$

$\therefore$ $R_1 = Z_{sc} \cos \phi_2 = 200.645 \times 0.2285 = 45.847 \ \Omega$

$\therefore$ $X_{sc} = Z_{sc} \cos \phi_2 = 200.645 \times 0.9735 = 195.336 \ \Omega$

$\therefore$ Equivalent circuit referred to H.V. side is :



Example : 47

Test data on a single–phase, 250/500 V, 50 Hz transformaer are:

O.C. test	***20V,***	***1A,***	***80W***	***(carried on L.V. side)***
S.C. test :	***20V,***	***12A,***	***100W***	***(carried on H.L. side)***

(a) draw equivalent circuit referred primary. (b) Find the output power to obtain maximum efficiency at 0.9 lag p.f. (c) Will the answer in (b) above differ, if the p.f. is ; (i) 0.9 lead. or (ii) unity. Justify.

Solution :

(a) O.C. test on L.V. side ; $\cos \phi_0$ = Wα/V1I0 = 80 W/250 V × 1A = 0.32 or ϕ_0 = 71.33°, and $\sin \phi_0$ = 0.94.

Now $I_\omega = I_0 \cos \phi_0 = 1 \times 0.32$ A ; $I_m = I_0 \sin \phi_0 = 1 \times 0.94$ A = 0.94 A.

$$R_{01} = V_1/I_w = 250 \text{ V}/0.32 \text{ A} = 781\ \Omega.$$

$$X_{01} = V_1/I_m = 250 \text{ V}/0.94 \text{ A} = 266\ \Omega.$$

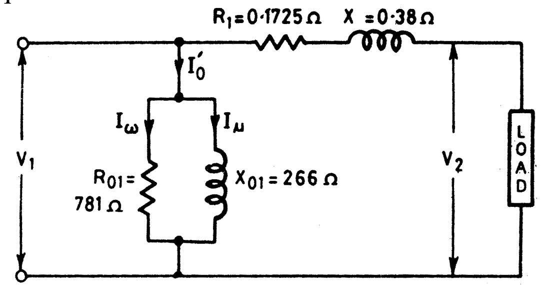
S.C. test pm H.V. side : $Z_2 = 20\text{V}/12\text{A} = 1.67\ \Omega$; $R = W_{2sc}/(I_{2sc})^2$ = $100\text{W}/(12 \text{ A})^2 = 0.69$ W ; $X_2 = \sqrt{1.67^2 - 0.69^2} = 1.52\ \Omega$·

Equivalent resistance, and reactance referred to primary are:

$$R_1 = R_2(N_1/N_2)^2 = 0.69 \times (250/500)^2 = 0.1725\ \Omega.$$

$$X_1 = X_2(N_1/N_2)^2 = 1.52 \times (250/500)^2 = 0.3800\ \Omega.$$

Hence, equivalent circuit is



(b) for $\eta_{max,}$ $W_c = W_i$

∴ For $\eta_{max,}$ $W_c = W_i = 80$ W.

But for full–load, copper losses are 100 W and W_c α I_2

∴ 100 α I_1^2

and 80 α I_1^2

or $(I_2/I_1) = (80/100)^{1/2} = 0.9$

or $I_2 = 0.9\ I_1 = 0.9 \times 12$ A = 10.73 A.

and output power= $V_2I_2 \cos \phi$ – 250 × 10.73 × 0.9 = 2,414 W.

Hence, the given statement is not true.

(c) (i) If p.f. is 0.9 (leading), the output power will be same as above. (ii) If p.f. is unity, then the output power = 250 × 10.73 × 1=2,682.5 W.

Example : 48

A 1–phase , 50 kVA, 2,400/120 V, 50 Hz transformer gave the following results :

O.C. test with instrument on the l.v. side : 120 V, 9.65 A, 386 W.

S.C. test with instrument of the h.v. side : 92V, 20.8 A, 810 W.

Calculate ; (i) the equivalent circuit constants, and draw the equivalent circuit ; (ii) the efficiency, when rated kVA is delivered to a load having a power factor of 0.8 lagging; (iii) the voltage regulation.

Solution :

(i) OC test on l.v. side : V_α = 120V ; I_α = 9.65 A ; P_α = 386 W ; $\cos \phi_0 = P_\alpha/V_\alpha I_\alpha$ = 386/120 × 9.65 = 0.33, and $\sin \phi_0$ = 0.94 ; $R_{02} = V_\alpha/I_0 \cos \phi_0$ = 120/9.65 × 0.33 = 37.68 Ω ; $X_{02} = V_\alpha/I_0 \sin \phi_0$ = 120/9.65 × 0.94 = 13.19 Ω; W_i = 386 W.

S.C. test on h.v. side : $R_1 = P_{sc}/(I_{sc})^2 = 810/(20.8)^2 = 1.87\ \Omega$; $Z_1 = V_{sc}/I_{sc} = 9.2/20.8 = 4.42\ \Omega$; $X_1 \sqrt{Z_1^2 - R_1^2} = \sqrt{4.42^2 - 1.87^2 = 4.00}\ \Omega$.

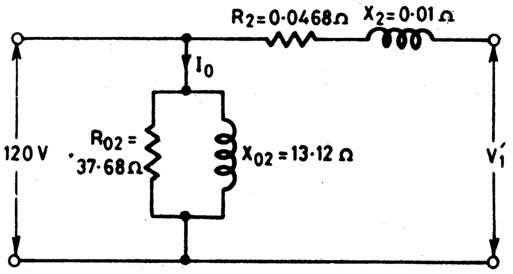
$\therefore$ Parameters wrt. l.v. side are:

$$R_2 = R_1(N_2/N_1)^2 = 1.87 \times (120/2{,}400)^2 = 0.00468\ \Omega$$

$$X_2 = X_1(N_2/N_1)^2 = 4.00 \times (120/2{,}400)^2 = 0.01\ \Omega$$

and $Z_2 = Z_1 \times (N_2/N_1)^2 = 4.42 \times (120/2{,}400)^2 = 0.01105\ \Omega$

Hence, equivalant circuit referred To l.v–side is :



(ii) Output = 50,00 × 0.8 = 40,00 W ; $I_{2(f.l)}$ = 50,000/120 = 416.67 V ; copper losses, $W_c = (I_{2(f.l.)})^2 \times R_2 = (416.67)^2 \times 0.00468 = 812.5$ W.

$$\therefore \eta = \frac{\text{Output} \times 100}{\text{Output} + W_i + W_c} = \frac{40{,}000 \times 100}{40{,}000 + 386 + 812.5} = 97.1\%$$

$$\text{(iii) Regulation} = I_2 \left[\frac{0.00468 \times 0.8 + 0.01 \times 0.6}{120} \right] \times 100\%$$

$$= 3.38\%.$$

Example : 49

A single–phase, 50 Hz, step–up transformer, having trun–ratio N2/N1 = 2, has no–load losses 38.4 watts and no–load power factor 0.22 lag, when the O.C. tests carried out at L.V. side of transformer. The transformer has 10 A secondary rated (full–load) current.

When the transformer is loaded to its full–load current, it is found that :

(i) the secondary terminal voltage drops by 12.5 V from its no–loaded value at 0.8 p.f. (lag), and the voltage regulation (approx) at this load is 2.5% ;

(ii) the secondary terminal voltage drops by 6 volts at unity power factor.

Determine :

(i) Primary, Secondary No–load voltages, and rating of transformer.

(ii) If the S.C. test is carried out at 50% its full-load current by short–circuiting L.V. side, what will be the Voltmeter, and Wattmeter readings?

(iii) At what load, the efficiency of the transformer is maximum ? And find max. efficiency at 0.8 p.f. load.

(iv) Find the no–load current I_0, magnetising component I_m, and iron–loss component I_w if the O.C. test is carried out on L.V. side.

(v) Draw the equivalent of transformer showing all equivalent constants.

Solution :

(i)
$$\%\ \text{V.R.} = \frac{E_2 - V_2}{E_2} \times 100 = \frac{12.5}{E_2} \times 100 = 2.5$$

or $\quad E_2 = 12.5 \times 100/2.5 = 500$ V ;

and $\quad E_1 = E_2\ (N_1/N_2) = 500 \times (1/2) = 250$ V.

$\therefore \quad$ kVA = 500 × 10 = 5,000 VA = 5 kVA.

(ii) Voltage drop = $I_2(R_2 \cos\phi + X_2 \sin\phi)$ kVA

$12.5 = 10\ (R_2 \times 0.8 + X_2 \times 0.6)$ (For 0.8 lag p.f.)

$= 8R_L + 6X_2$

Also $6 = 10\ (R_2 \times 1 + X_2 \times 0) = 10R_2$ (For unity p.f.)

$\therefore$ $R_2 = 6/10 = 0.6\ \Omega$

and $X_2 = \frac{1}{6}[12.5 - 8 \times 0.6] = 1.28\ \Omega$

When S.C. test on l.v. side with 50% full load is carried out, then ammeter reading = $(50/100) \times 10$ A = 5A ; voltmeter reading = $I_{sc} \times Z_{sc} = 5 \times \sqrt{0.6^2 + 1.28^2} = 7.06$ V, and wattmeter reading = $I_{sc}^2 R_2 = 5^2 \times 0.6 = 15$ W.

(iii) W_i = 38.5 W. Efficiency is maximum, when copper loss (W_c) = Iron loss (W_i) = 38.4 W = $I_2^2 R_2$.

$\therefore$ Load, $I_2 = \sqrt{38.4 / 0.6} = 8\text{A}$.

$$\text{Efficiency}, \eta_{max} = \frac{\text{Output}}{\text{Output} + 2W_i} = \frac{500 \times 8 \times 0.8}{500 \times 8 \times 0.8 + 2 \times 38.4} = 97.70\%$$

(iv) When O.C. test is carried out on l.v. side, $E_1 = V_1 = 250$ V ; W_i = 38.4 W ; $\cos\phi_0$ = 0.22 lag ; $\sin\phi_0$ = 0.975.

$\therefore$ $W_i = V_1 I_0 \cos\phi_0$ or $I_0 = W_i / V_1 \cos\phi_0$ = $38.4/250 \times 0.22$ = 0.70 A.

$\therefore$ $I_\mu = I_0 \cos\phi_0 = 0.7 \times 0.22 = 0.15$ A.

$I_\omega = I_0 \sin\phi_0 = 0.70 \times 0.975 = 0.68$ A.

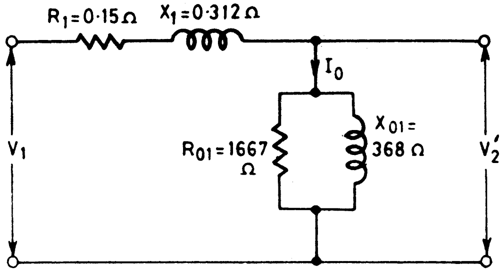
(v) $$R_{01} = \frac{V_1}{I_0 \cos\phi} = \frac{250}{0.15} = 1{,}667\ \Omega$$

$$X_{01} = \frac{V_1}{I_0 \cos\phi} = \frac{250}{0.68} = 368\ \Omega$$

$\therefore$ $R_1 = R_2(N_1/N_2)^2 = 0.6(1/2)^2 = 0.15\ \Omega$

and $X_1 = X_2(N_1/N_2)^2 = 1.28(1/2)^2 = 0.312\ \Omega$

Hence, equivalent circuit referred to primary is :

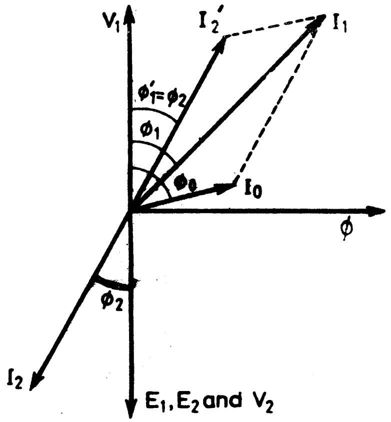


Example : 50

A 50 kVA, 1,000/200 V, 1–phase, step–down transformer takes 4 A at p.f. of 0.2 lagging. When the secondary is open–circuit. Calculate the primary current, and p.f., when a load taking 250 A at a lagging p.f. of 0.8 is connected across the secondary. Assume the voltage drop in the winding to be negligible.

Solution :

Let I_2' represents the component of the primary current to neutralize the damagnetizing effect of the secondary current. Then, ampere–turns due to I_2' must be equal, and opposite to that due to I_2, i.e.,



$I_2' \times 1{,}000 = 250 \times 200$ or $I_2' = 50\text{A}$

Now cos f_2 = 0.8 ; sin f_2 = 0.6 ; cos f_0 = 0.2, and sin f_0 = 0.98.

From figure, it is clear that :

$$I_1 \cos\phi_1 = I_2' \cos\phi_2 + I_0 \cos\phi_0$$

$= 50\times 0.8 + 4\times 0.2 = 40.8$ A ;

and $I_1 \sin \phi_1 = I_2' \sin \phi_2 + I \sin \phi_0$

$= 50\times 0.6 + 4\times 0.98 = 33.92$ A.

∴ Primary current,

$$I_1 = \sqrt{40.8^2 + 33.92^2} = 53.06 \text{ A};$$

and $\tan \phi_1 = \sin \phi_1/\cos \phi_1 = 33.92/40.08 = 0.8463$

or $f_1 = \tan^{-1} (0.8463) = 40.24^o$

∴ P.f. $\cos \phi_1 = \cos 40.24^o = 0.7633$ (lagging).

Example : 51

A 2,200/220 V, 50 Hz, single–phase transformer has exciting current of 0.6 A, and ore loss of 361 W, when Its high voltage side is eergised at rated voltage. Calculate the two components of exciting current. If load current is 60 A at 0.8 lagging p.f., calculate the high voltage side current, and its p.f. Ignore leakage impedane. Draw the phasor diagram.

Solution :

(i) $W_i = 361$ W ; $I_0 = 0.6$ A.

∴ Iron current, $I_\omega = 361/2,200/220 = 0.1641$ A.

and magnetising current,

$$I_\mu = \sqrt{I_0^2 - I_\omega^2} = \sqrt{0.6^2 - 0.1641^2} = 0.5771\text{A}$$

(ii) $I_2 = 60$ A; $\cos \phi_2 = 0.8$; $\sin \phi_2 = 0.6$.

∴ $$I_2' = I_2(N_2/N_1) = 60\,(220/2,200) = 6\text{A}$$

Now $I_\omega = I_0 \cos \phi_0$ or $0.1641 = 0.6 \cos \phi_0$

Or $\cos \phi_0 = 0.1641/0.6 = 0.2735$ and $\sin \phi_0 = 0.9619$.

From figure, it is clear that:

$I_1 \cos \phi_1 = I_2' \cos \phi_2 = I_0 \cos \phi_0$

$= 6\times 0.8 + 0.6\times 0.2735 = 4.964$ A.

$I_1 \sin \phi_1 = I_2' \sin \phi_2 + I_0 \sin \phi_0$

$= 6\times 0.6 + 0.6\times 0.9619 = 4.177$ A.

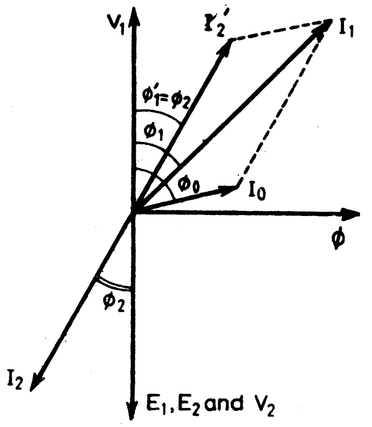
∴ Hv. current,

$$I_1 = \sqrt{4.964^2 + 4.177^2}$$

$= 6.488$ A ;

and p.f., $\cos \phi_1 = \cos \phi_1/I_1 = 4.964/6.488$

$= 0.765$ (lagging).



Example : 52

A 5 kVA, 2000/400 V single–phase transformer has a resistance of 0.12 W and reactance of 0.32 W as referred to L.V. side. Calculate the per unit values of : (i) resistance, (ii) reactance, and (iii) impedance taking rated quantities as the base value on the two sides separately.

Solution :

$(VA)_{base} = 5,000$ VA

L.V. side : V_{base} = 200 V; I_{base} = 5,000/200 = 25A; Z_{base} = 200/25 = 8 Ω ; Z_{lv} = (0.12 + j0.32) Ω

∴ Per unit resistance, $R_{p\mu}$ = 0.12/8 = 0.015 Ω

Per unit resistance, $X_{p\mu}$ = 0.32/8 = 0.04 Ω

Per unit resistance, $Z_{p\mu}$ = (0.015 = j0.04) Ω

H.V. side : V_{base} = 400 V ; I_{base} = 5,000/400 = 12.5 A; Z_{base} = 400/12.5 = 32 Ω ; Z_{hu} = (0.12 + j0.32) $(400/200)^2$ = (0.48+j1.28) Ω.

∴ Per unit resistance, $R_{p\mu}$ = 0.48/32 = 0.015 Ω

Per unit resistance, $X_{p\upsilon}$ = 1.28/32 = 0.04 Ω

Per unit resistance, Z_{pu} = (0.015 + j0.04) Ω

It may be noted that these values are same as calculated on L.V. side.

Example : 53

A single–phase, 200 kVA, 4,000/400 V transfer has a leakage impedance of (0.01 + j0.036) Ω as referred to L.V. side. Calculate ; (i) per unit impedance, (ii) voltage regulation at ; (a) p.f. 0.8 (lag), and (b) 0.8 (lead).

Solution :

(i) $(VA)_{base}$ = 200×1,000 VA; V_{base} = 400 V ; R_1 = 0.01 Ω ; X_2 = 0.036 Ω

∴ Full–load secondary current, Ibase = 200×1,000/400 = 500 A

∴ Per unit resistance, R_{pu} = 500×0.01/400 = 0.012

Per unit reactance, X_{pu} = 500×0.036/400 = 0.045

∴ Per unit impedance, Z_{pu} = (0.0125 = j0.045) Ω.

(ii) (a) Regulation = R_{pu} cos θ + X_{pu} sin θ (for p.f. lagging)

= 0.0125×0.8 + 0.045×0.6 = 0.037 or 3.7%

(b) Regulation = R_{pu} cos θ – X_{pu} sin θ (for p.f. leading)

= 0.0125×0.8 – 0.045×0.6 = – 0.017 or 1.7%.

MAGNETIC CIRCUITS

WORKED EXAMPLES

Example : 1

A conductor of length 150 cm moves at right angles to a uniform magnetic field of flux density 1.5 Wb/m² with a velocity of 60 m/s. Calculate the emf induced in it. Find also the emf induced, if the conductor moves at an angle of 30° to the direction of the field.

Solution :

(i) Here l = 1.5 m; B = 1.5 Wb/m², and v = 60m/s ; θ = 90°, or sin θ = 1

So induced emf

e_L = Blv sin θ volts = 1.5×1.5×60×1 = 135 V.

(ii) θ = 30°, sin θ = 0.5. So induced emf,

e_L' = Blv sin θ volts = 135×0.5 = 67.5 V.

Example : 2

A square coil of 10 cm side having 100 turns is rotated at 1,000 rpm about its axis at 90° to direction of the magnetic field having a flux density of 0.5 Wb/m² (tesla). Calculate the instatneous value of emf, when the plane of the coil is at 30°, and (ii) 45° to the direction of flux.

Solution :

Here B = 1.5 tesla (Wb/m²) ; l = 4×10 cm×100 = 40 m.

and

$$v = r\omega = \left(\frac{1}{2}\times 10\times 10^{-2}\,\text{m}\right)(2\pi\times 1{,}000/60\text{s}) = 10\pi/6\,\text{m/s}$$

∴ Total emf in coil

= Blv sin θ = 1.5×40×(10 π/6) sin θ = 314.2 sin θ V.

(i) With $\theta = 30^o$. emf = 314.2 sin $(90^o - 30^o)$ = 314.2×0.866 = 272.06 V.

[similarly Velocity of the conductor makes $90^o - 30^o = 60^o$ with flux lines]

(ii) With $\theta = 45^o$, emf = 314.2 sin $(90^o - 45^o)$ = 314.2×0.707 = 222.14 V.

[similarly Velocity of the conductor makes $90^o - 45^o = 45^o$ with flux lines].

Example : 3

A coil of 1,500 turns gives rise to a magnetic flux of 2.5 mWb, when carrying a certain current. If this current is completely reversed in 0.2 second. what is the average emf induced in the coil?

Solution :

Here N = 1,500 ; $d\phi = 2.5 - (-2.5) = 5$ mWb 5×10^{-3} mwb (because flux changes from +2.5 mWb to –2.5 mWb due to reversal of current), and dt = 0.2 s.

$\therefore$ Induced emf = L(di/dt) = N $(d\phi/di)$ (di/dt)
[similarly L = $N(\phi/I)$]

$$= N(d\phi/dt)$$

$$= 1{,}500 \times [5 \times 10^{-3}/0.2] = 37.5 \text{ V.}$$

Example : 4

A coil with 250 turns carriers a current of 2 A, and produces a flux of 0.3 mWb. When this current in reduced to zero in 2 milliseconds, the voltage induced in a nearby coil is 60 vols. Calculate ; (i) self inductance of each coil, (ii) mutual inductance between the coils. Assume coefficient of coupling = 0.7.

Solution :

(i) Self inductance,

$$L_1 = N_1\ \phi\ /I1 = 250 \times 0.3\ 10^{-3}/2 + 37.5 \text{ mH.}$$

(ii) Mutually induced emf.

$$e_2 = M(di/dt) = M\frac{(2-0)}{2\times10^{-3}} = M \times 10^3 = 60 \text{ V}$$

or mutual inductance, $M = 60/10^3$ H = 60 mH.

Also $M = 60 = k\ \sqrt{L_1L_2} = 0.7\sqrt{37.5\times L_2}$

Whence, self inductance, L_2 = [(60/0.7)2/37.5] mH = 195.9 mH.

Example : 5

Calculate the inductance of a tortodial coil of 100 turns wound uniformaly on a non-magnetic core ring of mean diameter 140 mm and the cross-sectional area is 750 mm².

Solution :

Here N = 100 ; A = 750 mm² = 7.5×10^{-4} m²;

$$1 = \pi\left(\frac{1}{2}\times140\times10^{-3}\text{m}\right)^2 = 1.539\times10^{-3}\text{m}^2 ;\mu_0 = 4\pi\times 10^{-4}\text{ H/m.}$$

$\therefore$ Assumming air–cored tortoid, then inductacne,

$$L = \frac{\mu_0 AN^2}{l} = \frac{4\pi\times10^{-7}\times10^{-4}\times(100)^2}{1.539\times10^{-3}}.$$

$$= \mathbf{6.124 \times 10^{-2}H \text{ or } 0.6124 \text{ mH.}}$$

Example : 6

A solenoid of 500 turns is would on a former of length 100 cm, and diameter 3 cm. This is placed co–oxially within another solenoid of the same length, and number of turns, but of diameter 6 cm. If the diameter of wire used is 1 mm, and the material is copper, determine the inductance, and resistance of each solenoid $(\rho \text{ for copper} = 50\ \mu\ \Omega\text{ cm})$.

Solution :

Here $l_1 = l_2 = 1_m$; $A_1 = \pi\left(\frac{1}{2}\times 3\times 10^{-2}\,m\right)^2 = 7.07 \times 10^{-4}$ m² ;

$A_2 = \pi\left(\frac{1}{2}\times 6\times 10^{-2}\,m\right)^2 = 2.827\times 10^{-3}\,m^2$; $N_1 = N_2 = 500$;

$\mu_0 = 4\pi\times 10^{-7}$ H/m, and $\rho = 50\times 10^{-6}\,\Omega$ cm $= 5\times 10^{-7}\,\Omega$ m.

$$\therefore L_1 = \frac{\mu_0 A_1 N_1^2}{l_1} = \frac{4\pi\times 10^{-7}\times 7.07\times 10^{-4}\times(500)^2}{1} = 2.22\times 10^{-4}\,H$$

and

$$L_2 = \frac{\mu_0 A_2 N_2^2}{l_2} = \frac{4\pi\times 10^{-7}\times 2.827\times 10^{-3}\times(500)^2}{1} = 8.83\times 10^{-4}\,H$$

Now total length of solenoid wire = π d N ; area of cross–section of wire

$$= \pi\left(\frac{1\times 10^{-3}}{2}\right)^2 = 7.85\times 10^{-7}\,m^2$$

$$\therefore R_1 = \frac{\rho\pi\, d_1 N_1}{7.85\times 10^{-7}} = \frac{5\times 10^{-7}\times\pi\times 3\times 10^{-2}\times 500}{7.85\times 10^{-7}} = 30\,\Omega$$

$$R_2 = \frac{\rho\pi\, d_2 N_1}{7.85\times 10^{-7}} = \frac{5\times 10^{-7}\times\pi\times 6\times 10^{-2}\times 500}{7.85\times 10^{-7}} = 60\,\Omega$$

Example : 7

A long solenoid of cross–section 2 ch2 has a primary winding with 25 turns per cm. At the middle is would a secondary winding of 1000 turns. The primary is suddenly disconnected from the supply when carrying 2A, the current falling to zero in 1/1,000 sec. Calculate the average emf induced in the secondary.

Solution :

Let l m be the length of solenoid, then $N_1 = 2{,}500\, l$; $N_2 = 100$; $A = 2$ cm² $= 2\times 10^{-4}$ m² : $di_1 = 2 - 0 = 2$ A ; $dt = (1/1{,}000)$s $= 1\times 10^{-3}$ s, and $\mu_0 = 4\pi\times 10^{-7}$ H/m.

∴ Induced emf in secondary,

$e_2 = M(di_1/dt) = 6.283\times 10^{-5}\times(2/10^{-3}) = 125$ mV.

Example : 8

A flux of 0.5 mWb is produced by a coil of 900 turns wound on a ring with a current of 3 A in it. Calculate : (i) the inductance of the coil ; (ii) the e.m.f. induced in the coil, when a current of 5 A is switched off, assuming the current to fall to zero in 1 millisecond, and (iii) the mutual inductance between the coils, if a second coil of 600 turns is uniformly wound over the first coil.

Solution :

$\phi = 0.5$ mWb $= 5\times 10^{-4}$ Wb ; $N_1 = 900$; I, 3A ; $N_2 = 600$.

(i) Inductance, $L_1 \frac{N_1\phi}{I_1} = \frac{900\times 5\times 10^{-4}}{3} = 0.15H$

(ii) Self induced emf, $e_M = L_1\frac{di}{dt} = 0.15\times\frac{(5-0)}{1\times 10^{-3}} = 750$ V

(iii) Mutual inductance, $M = N_2\frac{d\phi_1}{dt_1} = \frac{N_2 L_1}{N_1} = \frac{600\times 0.15}{900} = 0.1$ H

Example : 9

A non–magnetic ring having a mean diameter of 30 cm, abd a cross–sectional area of 4 cm2 is uniformaly wound with two coils A, and B one over the other. A has 90 turns, and B has 240 turns. Calculate from first priciple the mutual inductance between the coils. Also calculate the emf induced in A, and B, when a current of 6 in coil is reversed in 0.02 s.

Solution :

Reluctance $S = \dfrac{l}{\mu_0\mu_r A} = \dfrac{\pi D}{\mu_0 A}$

$$= \frac{\pi \times 30 \times \times 10^{-2}}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 1.874 \times 10^{9}\,\text{At/Wb}$$

∴ Inductance of coil A,

$$L_A = \frac{N_A^2}{S} = \frac{90 \times 240}{1.874 \times 10^9} = 11.52\ \mu\text{H}$$

Emf induced in coil A,

$$e_A = L_A \frac{di_1}{dt} = 4.32 \times 10^{-6} \times \left[\frac{6-(-6)}{0.02}\right] = 2.592\ \text{mV}$$

and emf induced in coil B, due to current change in coil A,

$$e_B = M\frac{di_1}{dt} = 11.52 \times 10^{-6} \times \left[\frac{6-(-6)}{0.02}\right] = 6.912\ \text{mV}$$

Example : 10

The combined inductance of the two coils connected in series is 0.75 H, and 0.25 H, depending on the relative directions of currents in the coils. If one of the coils, when isolated, has a self inductance of 0.15 A, then calculate : (i) mutual inductance, and (ii) coefficience of coupling.

Solution :

$L_{additive} = L_1 + L_2 + 2M$

∴ $0.75\ H = 0.15\ H + L_2 + 2M$ ...(i)

$L_{subtractive} = L_1 + L_2 - 2M$

∴ $0.25H = 0.15\ H + L_2 - 2M$ .....(ii)

Adding Eqs. (i), and (ii), we get:

$1.0\ H + 0.3\ H + 2\ L_2$

or $L_2\ (1.0 - 0.3)/2 = 0.35\ H.$ ....(iii)

Eqs. (i), From (i), and (iii), we get :

$0.75\ H = 0.15\ H + 0.35\ H + 2\ M$ or $M = 0.125$ H.

(ii) Coeff. of coupling, $k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{0.125}{\sqrt{0.15 \times 0.35}} = 0.5455$.

Example : 11

Two identical coils, having 1,500 turns each, line in parallel plane such that 70% of the flux produced by coil A links with coil B. A current of 4 in coil A produces a flux of 0.4 m Wb. Calculate the : (1) selfinductanes ; (2) the inductance, and (3) emf induced in coil B, when the current in coil it coil A changes from 4 A to – 4A in 0.02 sec.

Solution :

$N_A = N_B = 1{,}500$; $I_A = 4A$; $\phi_A = 0.04 \times$ mWB $= 4 \times 10^{-5}$ Wb ; $di_A = 4 - (-4) = 8$ A; $dt = 0.02$ s $= 2 \times 10^{-2}$s.

(1) Self inductances,

$$L_A(\text{or } L_B) = \frac{N_A \phi_A}{I_A} = \frac{1{,}500 \times 4 \times 10^{-5}}{4} = 0.015\ \text{H}$$

(2) Flux linking in coil B, $\phi_B = 0.7 \times 4 \times 10^{-5}$Wb.

∴ Mutual inductance,

$$M = \frac{N_B \phi_B}{I_A} = \frac{1{,}500 \times 2.8 \times 10^{-5}}{4} 0.0105\ \text{H}$$

(3) Induced emf in coil B, $e_B = \dfrac{M di_A}{dt} = \dfrac{0.0105 \times 8}{2 \times 10^{-2}} = 4.2\ \text{V}$

Example : 12

Coils 'A' and 'B', having 100 and 150 turns respectively, are wound side–by–side on a closed iron circuit of cross–section 125 cm², and mean length 2 m. Determine : (i) self–inductance of each coil ; (ii) mutual inductance between them ; (iii) emf induced in coil 'B', when a current changes from zero to 5A in coil 'A' in 0.02 s. Take relative permeability of iron as 2,000.

Solution :

N_A = 100 ; N_B = 150 ; A = 125 cm² = 1.25×10^{-2} m² ; l = 2 m; di_A = 5 – 0 = 5A ; dt = 0.02 s = 2×10^{-2} s, and μ_r = 2,000.

Reluctance,

$$S=\frac{l}{\mu_0\mu_r A}=\frac{2}{4\pi\times10^{-7}\times2{,}000\times1.25\times10^{-2}}=63{,}662\ \text{AT/Wb}$$

(i) Self inductace, $L_A=\frac{N_A^2}{S}=\frac{(100)^2}{63{,}662}=0.157\ \text{H}$

Self inductance, $L_B=\frac{N_B^2}{S}=\frac{(150)^2}{63{,}662}=0.353\ \text{H}$

(ii) Mutual inductance, $M=\frac{N_A N_B}{S}=\frac{100\times150}{63{,}662}=0.2356\ \text{H}$

(iii) Induced emf, $e_B=\frac{M.di_A}{dt}=\frac{0.2356\times5}{2\times10^{-2}}=58.9\ \text{V}$

Example : 13

Two coils A, and B have self inductances of 10 microhenry, and 40 microhenry respectively. A current of 2 A in coil A produces of flux linkage of 5 μ Wb in coil B. Calculate ; (i) mutual inductance between coils ; (ii) coefficient of magnetic couopling; (iii) average emf induced in coil B, if the current of 1 A in coil is reversed at a uniform rate in 0.1 second.

Solution :

L_A = 10 m H = 1×10^{-5} H ; L_B = 40 μ H = 4×10^{-5} H ; I_A = 2A ; ϕ_B = 5 mWb ; ali_A = 1–(–1) = 2A; dt = 0.1 s = 1×10^{-1} s.

(i) Mutual inductance,

$$M=\frac{\text{Flux linkage in B}}{\text{Current in A}}=\frac{5\mu\text{Wb}}{2\text{A}}=2.5\ \text{mH}$$

(ii) Coeff. of coupling,

$$k=\frac{M}{\sqrt{L_A L_B}}=\frac{2.5\mu\text{H}}{\sqrt{10\mu\text{H}\times40\mu\text{H}}}=0.125\ \text{H}$$

(iii) Average emf,

$$e_B=M\times\frac{di_A}{dt}=2.5\mu\text{H}\times\frac{2\text{A}}{1\times10^{-1}\text{s}}=50\ \text{mV}$$

Example : 14

Two coils, having 30 and 600 turns respectively, are wound side–by–side on a closed iron circuit of section 100 cm², and mean length of 150 cm. A current in first coil grows streadily from zero to 10 A in 0.01s. Find the emf induced in other coil. permeability of iron is 2,000.

Solution :

Here N_1 = 30 ; N_2 = 600 ; A = 100 cm² = $1\times10^{-2}\times$ m² ; l = 150 cm = 1.5 m; μ_r = 2.000 ; di_1 = 10 A ; dt = 0.01 s = 1×10^{-2} s.

Self inductance,

$$L_1=\frac{\mu_0\mu_r AN_1}{l}=\frac{4\pi\times10^{-7}\times2{,}000\times10^{-2}\times(30)^2}{1.5}=0.015\ \text{H}$$

Self inductance, (assuming k = 1) is given by :

$$L_2 = \frac{\mu_0\mu_r AN_2}{l} = \frac{4\pi\times10^{-7}\times2{,}000\times10^{-2}\times(600)^2}{1.5} = 6.032\text{ H}$$

∴ Mutual inductance,

$$M = \sqrt{L_1L_2} = \sqrt{0.015\times6.032} = 0.301\text{ H}$$

∴ Induced emf in coils

$$e_M = M\times\frac{di_A}{dt} = \frac{0.301\times10}{1\times10^{-2}} = 301\text{ V}.$$

Example : 15

Two identical coils X and Y, each with 1,000 turns, lie in parallel planes such that 50% of the flux proceed by one links with the other. A current of 5 A in x produces in it a flux of 50 micro–Wb. If the current in X changes from = 6A to – 6A in 0.01 s, what will be the magnitude of emf induced in the coil Y? Calculate self inductance of each coil ; mutual inductance, and co–efficient of coupling.

Solution :

(i) Self inductance, L (of X or Y) $= N_1\,\phi/I_1 = \frac{1{,}000\times50\times10^{-6}}{5} 0.01$ H.

Flux/ampere is coil = $50\times10^{-6}/5 = 1\times10^{-5}$Wb, so

flux per ampere liked with Y, $\phi_2/I_1 = 0.5\times10^{-5}$Wb.

(ii) Mutual inductance, $M = N_2(\phi_2/I_1) = 1{,}000\times0.5\times10^{-5}$ **= 0.005 H**.

Now $di_1 = 6 - (-6) = 12$ A ; $dt = 0.01$ s.

$$\therefore \text{ Induced emf in Y, } e_M = M\frac{di_A}{dt} = \frac{0.005\times12}{0.01} = 6.0\text{ V}.$$

Example : 16

A coil consists of 750 turns and current of 10 A in the coils gives rise to a magnetic flux of 1,200 µWb. Calculate : (i) the emf induced, and (ii) the energgy stored, when the current is reversed in 0.01s.

Solution :

Here N = 750, I = 10A, ϕ = 1,200 µWb = 1.2×10^{-3} Wb.

Now rate of change of current, $\frac{di}{dt} = \frac{10-(-10)}{0.01} = 2{,}000\text{A/s}$

Also self–inductance, $L = \frac{N\phi}{I} = \frac{750\times120\times10^{-3}}{10} = 0.09\text{ H}$

(i) Self–induced emf, $e_L = L\frac{di}{dt} = 0.09\times2{,}000 = 180\text{ V}$

(ii) Energy stored $= \frac{1}{2}LI^2 = \frac{1}{2}\times0.09\times(10)^2 = 4.5\text{ J}$.

Example : 17

Two 200–turn, air–cored solenoids, 25 cm long have a cross–sectional area of 3 cm² each. The mutual inductance between them is 0.5 µH. Find ; (i) the self inductance of the coils and (ii) the coefficient of coupling.

Solution :

Here $N_1 = N_2 = 200$, L = 25 cm = 0.25 m, A = 3 cm² = 3×10^{-4} m², M = 0.5 µ H = 5×10^{-7} H : $\mu_2 = 1$ (for air)

Now the both coils are identical in length and area of cross–section and also they have same number of turns, so their self–inductances are same.

(i) Self–inductance, $L_1 = L_2 = \frac{\mu_0\mu_r AN^2}{l}$

$$= \frac{(4\pi \times 10^{-7}) \times 1 \times (3 \times 10^{-4}) \times (200)^2}{0.25}$$

= 60.0318×10^{-6} H = **60.0318 μ H**

(ii) Coefficient of coupling,

$$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{M}{L_1} = \frac{0.05\,\mu H}{60.0318\,\mu H}$$

= 8.29×10^{-3} or 0.00829.

Example : 18

A coil of 300 turns and of resistance 10 W is wound uniformly over a steel ring of mean circumfrence 30 cm and cross–sectional area 9 cm². it is connected to a supply of 20 V d.c. If the relative permeability of the rings is 1,500, find : (i) the magnetising force, (ii) the reluctance, (iii) the mmf, and (iv) the flux.

Solution :

(i) Here N = 3000, R_{coil} = 10 Ω, l = 30 cm = 0.3 m ; A = 9 cm² = 9×10^{-4} m² ; μ_r = 1,500 ; $\mu_0 = 4\pi \times 10^{-7}$.

Now, I = V/R_{coil} = 20/10 = 2A

$\therefore$ magnetising force, $H = \frac{N}{l} = \frac{300 \times 2}{0.3} = 2{,}000\ AT/m$.

(ii)Reluctance,

$$S = \frac{l}{A\mu_0\mu_r} = \frac{0.3}{9 \times 10^{-4} \times (4\pi \times 10^{-7}) \times 1{,}5000} = 1.7684 \times 10^9\ AT/Wb$$

(iii) mmf = N_I = 300×2 = 6000 AT.

(iv) Flux, $\phi = \frac{Mmf}{Reluctance} = \frac{NI}{S} = \frac{300 \times 2}{1.7684 \times 10^9}$

= 3.3939×10^{-3} Wb or 3.3939 mWb.

Example : 19

A solenoid of length 1 m, and diameter 10 cm has 5,000 turns. Calculate ; (i) the approximate inductance, and (ii) the energy stored in a magnetic field when a current of 2A flows in the solenoid.

Solution

l = 1m ; N = 5,000 ; r = 5 cm = 0.05 μ; $\mu_0 = 4\pi \times 10^{-7}$; A = π $(0.05)^2$ = $\pi \times 2.5 \times 10^{-3}$ m² ; I = 2A.

(i) Inductance, $L = \frac{\mu_0 A N^2}{l} = \frac{4\pi \times 10^{-7} \times \pi \times 2.5 \times 10^{-3} \times (5{,}000)^2}{1}$

= 246.7×10^{-3} H = 0.247 H.

(ii) Energy stored $= \frac{1}{2} LI^2 = \frac{1}{2} \times 0.247 \times 2^2 = 0.494\ J$.

Example : 20

Two coils A, and b, each with 100 turns, are mounted so that part of the set up one links the other. When the current through coil A is changed from +2 A to –2A in 0.5 second, an e.m.f. pf 8m V is induced in coil B due to 2 A in coil A. Calculate : (i) the mutual inductance between the coils, and (ii) the flux produced in coil B due to 2 A in coil A.

Solution :

$N_A = N_B$ = 100 ; I_A = 2A ; di_A = 2 –(–2) = 4 A ; dt = 0.5 s ; e_M = 8 mV = 8×10^{-3} V.

(i) Now $e_M = M(di_A/dt)$

or $$M = e_M \times \frac{dt}{di_A} = \frac{8 \times 10^{-3} \times 0.5}{4} = 1 \times 10^{-3}\ H$$

(ii) Flux induced in B, $\phi_B = \frac{MI_A}{N_B} = \frac{1 \times 10^{-3} \times 2}{100} = 2 \times 10^{-5}\ Wb$.

Example : 21

Two coils A, and B are wound side–by–side on a paper–tube former. A emf of 0.25 V is induced in coil A, when the flux linking it changes at the rate of I mWb/second. A current of 2 A in coil B causes a flux of 10 mWb to link coil A. Calculate the mutual inductance between the coils.

Solution :

Induced emf, $e_A = N_A \frac{d\phi}{dt} = N_A \times 10^{-3} = 0.25$

$\therefore$ $N_A = 250$

Now $f_A = 10\ \mu\ Wb = 10 \times 10^{-6}\ Wb = 1 \times 10^{-5}$ Wb, and $I_B = 2A$.

$\therefore$ Mutual inductance, $M = N_A\phi_A/I_B = 250 \times 1 \times 10^{-5}/2 = 1.25$ mH.

Example : 22

Two coils A, and B, having 1,200 tuns each, are placed near each other. When the coil b is open–circuited, and the coil A carries a current of 50 A, the flux producexd by coil A is 0.2 Wb, and 30% of this flux links with all the turns of coil B. Calculate the induced emf in coil B on open–circuit, when the current in the coil A is changing at a rate induced emf coil B on open–circuit in the coil A is changing at a rate 1A/s.

Solution :

$\phi_B = (30/100) \times 0.2\ Wb = 0.06\ Wb$; $N_B = 1,200$: $I_A = 50$ A.

$\therefore$ $M = N_B\phi_B/I_A = 1,200 \times 0.06/50 = 1.44$ H

Hence, induced emf in coil B,$e_M = M(di_A/dt) = 1.44 \times 1 = 1.44$ V.

Example : 23

An iron of 20 cm mean diameter having cross–section of 100 cm² is wound with 400 turns of wire. Calculate the exciting current required to estiblish a flux density of 1 Wb/m², μ_r = 1,000.

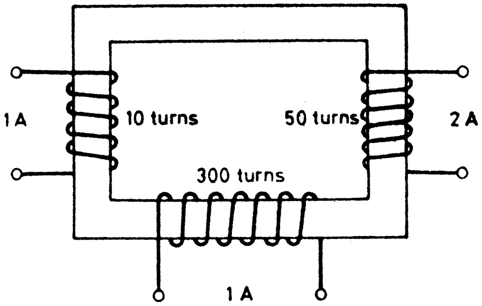
NowAT $I = \frac{1 \times 0.2 \times 10^7}{\mu_0 \mu_r} = \frac{1 \times \pi \times 0.2}{4\pi \times 10^{-7} \times 1,000}$

(i) $I = \frac{1 \times 0.2 \times 10^7}{4 \times 1,000 \times 400} = 1.25A$

(ii) Magnetic energy stored $= \frac{B^2 l_i A}{2\mu_0} = \frac{1^2 \times 0.2\pi \times 0.01}{2 \times 4\pi \times 10^{-7}} =$ 2,500 J.

Example : 24

What is value of the net mmf acting in the magnetic circuit shown below:



Solution :

Net mmf acting in the magnetic circuit

= mmf of [I + II + III] coils = $N_1I_1 + N_2I_2 + N_3I_3$

= 10 × 1 = 300 × 1 = 50 × 2 AT = 410 AT..

Example : 25

An iron ring 15 cm in diametor, and 10 cm2 in cross–section is wound with 200 turns of wire. For a flux density of 1 Wb/ m², and a permeability of 500, find the exciting current.

Solution :

$l = \pi\ (0.15\ m) = 0.4712\ m$; N = 200; B = 1 Wb/m^2 ; $\mu_r = 500$.

$$\therefore \qquad AT = I \times 200\ H\ l = \frac{Bl}{\mu_0 \mu_r} = \frac{1 \times 0.4712}{4\pi \times 10^{-7} \times 500}$$

$$\text{or} \qquad I = \frac{1 \times 0.4712}{4\pi \times 10^{-7} \times 500 \times 200} A = 3.75\ A$$

Example : 26

An electromagnet has an air–gap of 3 mm, and the flux density in the gap is 1.257 Wb/m^2. Calculate the amp–turns required for the gap.

Solution :

l_g = 3 mm = 3×10^{-3} m ; B = 1.257 Wb/m^2, and $\mu_0 = 4\pi \times 10^{-7}$.

$$\therefore AT\ \text{required} = \frac{B\ l_g}{\mu_0} = \frac{1.257 \times 3 \times 10^{-3}}{4 \times 10^{-7}} = 3{,}001\ AT.$$

Example : 27

An iron ring made up of three parts has l_1 = 10 cm, A_1 = 5 cm^2, l_2 = 8 cm, A_2 = 3 cm^2, l_3 = 6 cm, and A_3 = 2.5 cm^2. it is wound with a coil of 250 turns. Calculate the current required to produce a flux of 0.4 mWb in the ring. μ_1 = 2,670, μ_2 = 1,050, and μ_3 = 650.

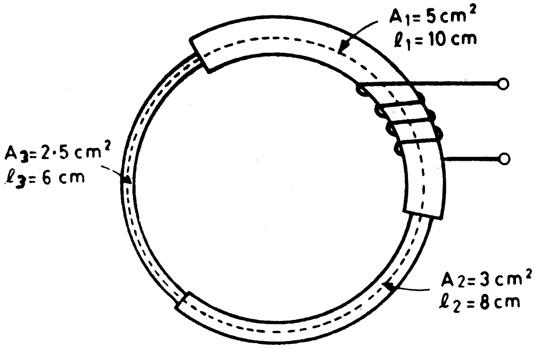
Solution :

Total reluctance (S)

$$= \sum \frac{1}{\mu_0 \mu_r A} = \frac{l_1}{\mu_0 \mu_1 A} + \frac{l_2}{\mu_0 \mu_2 A} = \frac{l_3}{\mu_0 \mu_3 A_3}$$

$$= \frac{1}{4\pi \times 10^{-7}} \left[\frac{0.1}{2{,}670 \times 5 \times 10^{-4}} + \frac{0.08}{1{,}050 \times 3 \times 10^{-4}} + \frac{0.06}{650 \times 2.5 \times 10^{-4}} \right]$$

$$= 5.555 \times 10^5\ AT/Wb.$$



$$\therefore \text{Flux}, \qquad \phi = \frac{\text{Mmf}}{\text{Reluctance}} = \frac{N \times 1}{5.55 \times 10^5}$$

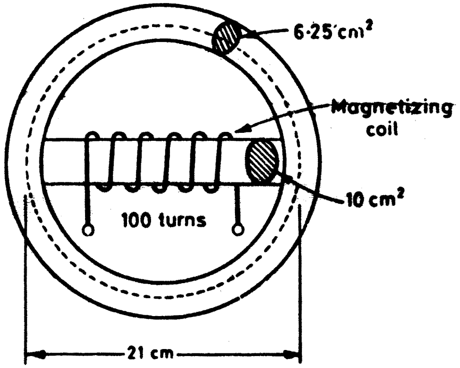
$$\text{or} \qquad 0.4 \times 10^{-3} = \frac{250 \times I}{5.55 \times 10^5}$$

or current, I = 888.8 mA.

Example : 28

A cast steel ring of mean diameter 21 cm is fitted with a cast steel bar as shown. Calculate the current required in the magnetizing coil, having 100 turns, to produce a flux of 1.25 mWb in the cast steel bar. The manetising curve for cast steel is :

B(Wb/m^2)	*1*	*1.1*	*1.18*	*1.25*	*1.33*
H(AT/m)	*900*	*1,050*	*1,200*	*1,450*	*1,650*



Solution :

Here ϕ = 1.25 mWb = 1.25×10^{-3} Wb ; N = 100.

Cast steel bar ; A = 10 cm^2 = 1×10^{-3} m^2 ; l = 0.21 m ; B = ϕ/A = 1.25 × $10^{-3}/1\times10^{-3}$ = 1.25 Wb/m^2. The corresponding value of H is 1,450 AT/m.

$\therefore$ AT required = H×l = 1,450 ×0.21 = 304.5 AT. ....(i)

Sides : A = 6.25 cm^2 = 6.25×10^{-4} m^2 ; $l = \pi d/2 = \pi\times0.21/2$ = 0.3299 m (each) ; flux density, B = (ϕ/2)/A = $(1.25/2)\times10^{-3}/6.25\times10^{-4}$ = 1 Wb/m^2. The corresponding value H is 900 AT/m.

$\therefore$ AT required = H×l = 900 ×0.3299 = 296.91 AT. ....(ii)

Total AT required = 304.5 + 296.91 = 601.41 AT

Hence, current required, I = AT/N = 601.41/100 = 6.014 A.

Example : 29

An iron ring of mean length 100 cm with an air–gap of 2 mm has a winding of 500 turns. The relative permeability of iron is 600. When a current of 3A flows in the winding, determine the flux density. Neglect fringing.

Solution :

Here l_i = 100 cm – 0.2 cm = 99.8 cm = 9.98×10^{-1} m ; l_g = 2 mm = 2×10^{-3} m ; N = 500 turms ; μ_r = 600 ; I = 3 A.

$\therefore$ Flux density, $$B = \mu_0 NI\left[\frac{\mu_r}{l_r} + \frac{1}{l_g}\right]$$

$$= 4\pi\times10^{-7}\times500\times3\left[\frac{600}{9.98\times10^{-1}} + \frac{1}{2\times10^{-3}}\right] \text{Wb/m}$$

= 2.076 Wb/m^2.

Example : 30

Steel ring has mean diameter of 20 cm., a cross–section of 25 cm² and a radial air–gap 0.8 mm cut actoss it. When excited by a current of 1 A through a coil of 1,000 turns wound on the ring core, it produces an air–gap flux of 1 m Wb. Neglecting leakage and fringing, calculate : (i) the relative permeability of steel, and (ii) the total reluctance of the magnetic circuit.

Solution :

Here l_g = 0.8 mm = 8×10^{-4} m, l_i = π (0.2 m) – 8×10^{-4} m = 0.6283 m – 0.0008 m = 0.6275 m ; I = 1A, N = 1,000, ϕ = 1 mWb = 10^{-3} Wb, A = 25 cm^2 = 2.5×10^{-3} m^2.

(i) Total reluctance, S = AT/ϕ = 1×1000 AT/1×10^{-3} Wb = 10^6 AT/Wb.

(ii) Now AT = 1×1,000 $$= \frac{\phi}{A\mu_0}\left[\frac{l_i}{\mu_r} + l_g\right]$$

or

$$1{,}000 = \frac{10^{-3}}{2.5\times10^{-3}\times4\pi\times10^{-7}}\left[\frac{0.6275}{\mu_r} + 8\times10^{-4}\right]$$

$$\frac{0.6275}{\mu_r} = \frac{1{,}000\times2.5\times10^{-3}\times4\pi\times10^{-7}}{1\times10^{-3}} - 8\times10^{-4}$$

$= 3.142\times10^{-3} - 8\times10^{-4} = 2.342\times10^{-3}$

where, μ_r = $0.6275/2.342\times10^{-3}$ = 268.

Example : 31

An iron 100 cm mean diameter, and 10 cm² cross–section has 1,000 turns of copper wire wound on it. If the permeability

of the material is 1,500, and it is required to produce a flux density of 1 Wb/m2 in an air–gap of 2 mm width in the ring, find : (i) reluctance of ring ; (ii) flux required ; (iii) mmf required ; (iv) current produced. Neglect leakage, and fringing.

Solution :

(i) Reluctance (S)

$$= \frac{l_i}{\mu_0 \mu_r A} = \frac{\pi \times 1}{4\pi \times 10^{-7} \times 1{,}500 \times 1 \times 10^{-3}} = 1.67 \times 10^6 \text{AT/Wb}$$

(ii) Flux (ϕ) = B_A = 1 Wb/m² = 1× 10⁻³m² = 1× 10⁻³ Wb.

(iii) Mmf required $= \frac{B}{\mu_0}\left[l_g + \frac{l_i}{\mu_r}\right] = \frac{1}{4\pi \times 10^{-7}}\left[2 \times 10^{-3} + \frac{\pi}{1{,}500}\right]$

$$= \frac{1}{4\pi \times 10^{-7}}\left[4.09 \times 10^{-3}\right] = 3{,}258 \text{ AT}$$

(iv) Current (I)= Mmf/N = 3,258/1,000 = 3,258 A.

Example : 32

A circular iron ring having a cross–sectional area of 10 cm², and a length of 4π cm in iron, has an air–gap of 0.4 π mm made by a saw cut. The relative permeability of iron is 10³, and permeability of free space is 4π× 10⁻⁷ H/m. The ring is wound with a coil of 2,000 turns, and carries 2 mA current. Determine the air–gap flux, meglecting leakage and fringing.

Solution :

A = 10 cm² = 1× 10⁻³ m²; l_i = 4π cm = 0.1257 m ; l_g = 0.4 π mm = 1.257× 10⁻³ m ; μ_r = 10³ ; μ_0 = 4π × 10⁻⁷ H/m ; N = 2,000, I = 2 mA = 2× 10⁻³ A.

$$AT = NI = \frac{\phi}{A\mu_0}\left[\frac{l_i}{\mu_r} + l_g\right]$$

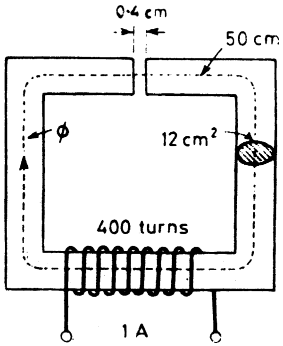
$$\therefore 2{,}000 \times 2 \times 10^{-3} = 4 = \frac{\phi}{1\pi \times 10^{-3} \times 4\pi \times 10^{-7}}\left[\frac{0.1257}{1{,}000} + 1.257 \times 10^{-3}\right]$$

or $$4 = \frac{\phi}{4\pi \times 10^{-10}}\left[1.3827 \times 10^{-3}\right]$$

∴ Flux, $$\phi = \frac{4 \times 4\pi \times 10^{-10}}{1.3827 \times 10^{-3}} = 3.635 \times 10^{-6} \text{Wb}$$

or = 3.635 μ Wb.

Example : 33



An electromagnet as shown has a cross–sectional area of 12 cm². Mean length of iron path is 50 cm. It is excited by two coils, each having 400 turns. When the current by two coils, each having 400 turns. When the current in the coils is 1.0 A, the resulting flux density gives a relative permeability of 1,300. Calculate : (1) reluctance of iron part of the magnetic circuit ; (2) reluctance of the air–gap ; (3) total reluctance ; (4) total flux, and (5) flux density in the air–gap. Neglect leakage and fringing.

Solution :

l_i = 50 cm = 0.5 m ; A = 12 cm² = 1.2× 10⁻³ m² ; l_g = 0.4 cm = 4 × 10⁻³ m; N = 400 + 400 = 800 ; I = 1A ; μ_r = 1,300.

(1) Reluctance, $$S_i = \frac{l_i}{\mu_0 \mu_r A} = \frac{0.5}{4\pi \times 10^{-7} \times 1{,}300 \times 1.2 \times 10^{-3}}$$

= 255,056 AT/Wb.

(ii) Reluctance, $$S_g = \frac{l_g}{\mu_0 A} = \frac{4 \times 10^{-3}}{4\pi \times 10^{-7} \times 1.2 \times 10^{-3}}$$

= 2,652,582 AT/Wb.

(iii) Total reluctance **= S_i + S_g = 2,907,638 AT/Wb.**

(iv) Total flux, $\phi = \frac{AT}{\text{Total reluctance}} = \frac{1 \times 800 \text{ AT}}{2{,}907{,}638 \text{ AT/Wb}}$

= 2.76×10^{-4} Wb.

(v) Flux density in air–gap, B = ϕ A = $2.76 \times 10^{-4} \times 1.2 \times 10^{-3}$ Wb

= 3.3×10^{-7} Wb.

Example : 34

(a) An iron ring wound with 500 turns solenoid produces a flux density 0.94 tesla in the ring carrying a current of 2.4 A. The mean length of iron path is 80 cm, and that of an air–gap is 1 mm. Determine the relative permeability of iron.

(b) Determine the coefficient of self inductance, and energy stored in the above arrangement, if the area of cross–section of ring is 20 cm².

Solution :

(a) Total mmf = NI = 500×2.4 = 1,200 AT.

Mmf for air–gap $= \frac{B \times l_g}{m_0} = \frac{0.94 \times 1 \times 10^{-3}}{4\pi \times 10^{-7}} = 748 \text{ AT}$

∴Mmf for iron path = (1200 – 748) AT = 452 AT

∴ $452 = \frac{Bl_i}{\mu_0 \mu_r} = \frac{0.94 \times 0.8}{4\pi \times 10^{-7} \times \mu_r}$

or permeability of iron, $\mu_r = \frac{0.94 \times 0.8}{4\pi \times 10^{-7} \times 452} = 1{,}324$.

(b) Self inductance, $L = \frac{N\phi}{I} = \frac{N}{I} \times \left[\frac{NI}{S_i + S_g}\right] = \frac{N^2 \mu_0 A}{(l_i / \mu_r) + l_g}$

$$= \frac{(500)^2 \times 4\pi \times 10^{-7} \times (20 \times 10^{-4})}{(0.8/1{,}324) + (1 \times 10^{-3})} = 0.392 \text{ H}.$$

∴ Energy stored $= \frac{1}{2} LI^2 = \frac{1}{2} \times 0.391 \times (2.4)^2 \text{ J} = 1.129 \text{ J}$

Example : 35

An iron ring of mean dia 100 cm, and area of cross–section 10 cm² is wound with 1,000 turns, and has μ_r = 2,000. Compute : (i) reluctance is ampere–turm per weber ; (ii) flux in webers produced, when the current through coil is 1A ; (iii) flux in the ring, if a saw cur of 1 mm length is cut, and current in the coil is same, and (iv) value of current required to double the flux that is produced in (iii)

Solution :

Length of iron path, l_i = $\pi \times 1$ m : area of cross–section. A = 10 cm² = 1×10^{-3} m² ; number of turns, N = 1,000 ; current I = 1A; μ_r = 2,000 ; μ_0 = $4\pi \times 10^{-7}$.

(i) Reluctance,

$$S = \frac{l_i}{\mu_0 \mu_r A} = \frac{\pi}{4\pi \times 10^{-7} \times 2{,}000 \times 1 \times 10^{-3}} = 1.25 \times 10^6 \text{ AT/Wb}$$

(ii) flux, ϕ = NI/S = $1{,}000 \times 1/1.25 \times 10^6$ = 8×10^{-4} Wb or 0.8 mWb.

(iii) Mmf = 1,000 $= \frac{B}{m_0}\left[\frac{l_i}{\mu_r} + l_g\right] = \frac{B}{4\pi \times 10^{-7}}\left[\frac{\pi}{2{,}000} + 1 \times 10^{-3}\right]$

or flux density, $B = \frac{1{,}000 \times 4\pi \times 10^{-7}}{\left[\pi/2{,}000 + 1 \times 10^{-3}\right]} = 0.4888 \text{ Wb/m}^2$.

∴ Flux, ϕ = B × A = $0.4888 \times 1 \times 10^{-3}$ = 0.4888 mWb.

(iv) New flux, ϕ' = 2×0.4888 mWb.

Hence, magnetizing current required to double the flux

$$= 2 \times 1\text{A} = 2 \text{ A}.$$

Example : 36

An iron ring of 10 cm² cross–section, and mean circumference has a 0.2 cm (or 2 mm) wide saw–cut made in it. A flux of 1 m Wb is required in the air–gap. The leakage factor is 1.2, and iron is such that flux density is 1.2 Wb/m², the relatiive permeability, μ_r = 400. Calculate the number of ampere–turns required.

Solution :

Here area of cross–section, A = 10 cm² = 1×10^{-3} m² ; length of iron path, l_i = 100 cm = 1m; length of air–gap, l_g = 0.2 cm = 2 mm = 2×1^{-3} m; flux, ϕ = 1mWb = 1×10^{-3} Wb ; leakage factor, 1 = 1.2; relative permeability of iron μ_r = 400 ; $\mu_0 = 4\pi \times 10^{-7}$.

$\therefore$AT required

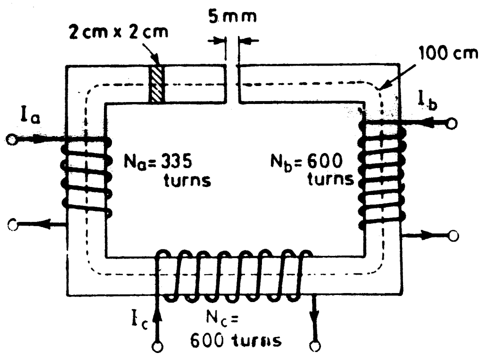
$$= H_{\text{iron}} \times l_i + H_{\text{air-gap}} \times l_g = \frac{Bl_i \times \text{Leakage factor}}{\mu_0 \mu_r} + \frac{Bl_g}{\mu_0}$$

$$= \frac{B}{\mu_0}\left[\frac{l_i \times \text{leakage factor}}{\mu_r} + l_g\right] = \frac{\phi}{A\mu_0}\left[\frac{l_i \times \text{leakage factor}}{\mu_r} + l_g\right]$$

$$= \frac{1 \times 10^{-3}}{1 \times 10^{-3} \times 4\pi \times 10^{-7}}\left[\frac{1 \times 1.2}{400} + 2 \times 10^{-3}\right] \text{AT} = 3{,}979 \text{ AT}$$

Example : 37

A rectangular iron core is shown below. It has a mean length of magnetic path of 100 cm, cross–section of (2cm × 2 cm), relative permeability of 1,400, and an air–gap of 5 mm is cut in the core. The three coils carried by the core have number of turns N_a = 335, N_b = 600, and N_c = 600, and the repective currents are 1.6 A, 4A, and 3A. The directions of the currents are as shown. Find the flux in the air–gap.



Solution :

Total mmf acting in the magnetic circuit (current clockwise)

$$= \sum NI = -355 \times 1.6 + 600 \times 3 = 64 \text{ AT}$$

$$\therefore \quad 64\text{AT} = \frac{\phi}{\mu_0 A}\left[\frac{l_i}{\mu_r} + l_g\right] = \frac{\phi}{4\pi \times 10^{-7} \times (2 \times 2)10^{-4}}\left[\frac{1}{1{,}400} + 5 \times 10^{-3}\right]$$

$$= 1.136 \times 10^{-7} \times \phi$$

$\therefore$ Flux, $\phi = 64/1.136 \times 10^{7} = 5.629 \times 10^{-6}$ Wb or 5.629 μ Wb.

Example : 38

A circular soft iron ring, and a cross–bar of silicon steel is fitted into the former as shown below with relevant data. The cross–sectional area of ring is 12 cm², and that of the cross–bar is 2 cm². Estimate the required ampere–turns to be applied to one–half of the ring to produce a flux density of 1.25 Wb/m² in the other half.

Solution :

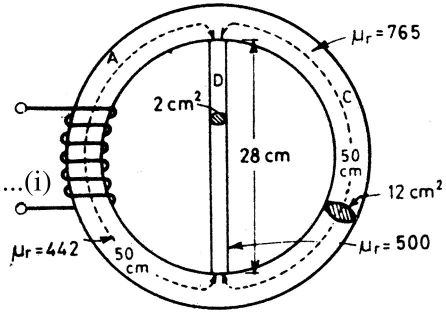
Refer to figure. Now flux figure. Now flux density, B_C = 1.5 Wb/m².

$$\therefore \quad (\text{AT})_C = \frac{B_C \times l_C}{\mu_0 \mu_{r(C)}}$$

$$= \frac{1.5 \times 0.5}{4\pi \times 10^{-7} \times 765}$$

= 780 AT.

= AT for path D.....(i)



$$\therefore H_D = \frac{(AT)_D}{l_D} = \frac{780}{0.28}$$

= 2,785.71 AT/m.

or $B_D = H_D \mu_0 \mu_{r(D)}$ + 2,785.71 × 4π × 10^{-7} × 500

= 1.7503 Wb/m².

∴ $f_D = B_D \times A_D$ = 1.7503 × 2 × 10^{-4} = 3.50 × 10^{-4} Wb.

$f_D = B_C \times A_C$ = 1.50 × 1.2 × 10^{-3} = 1.8 × 10^{-3} Wb.

∴ AT required for A portion $= \frac{\phi_A l_A}{\mu_0 \mu_{r(A)} A_A} = \frac{2.15 \times 10^{-3} \times 50 \times 10^{-2}}{4\pi \times 10^{-7} \times 442 \times 12 \times 10^{-4}} = 1{,}613A$

Hence, total AT required = AT for [A + D (or C)]

= 1,613 + 780 = 2,393 AT.

Example : 39

A cost–steel ring of mean diameter 16 cm has a cross–section of 0.5 cm². It has a saw cut 2 mm wide at one place. Given the magnetization characteristic of the material, calculate how many ampere-turns are required to produce a flux of 0.065 mWb in the ring, if the leakage factor is 1.2.

B(Tesla)	***1.0***	***1.25***	***1.46***	***1.6***
μ_r	***714.3***	***521***	***365***	***246.2***

Solution :

l_{cs} = π × 16 × 10^{-2} m = 0.503 m ; A = 0.5 cm² = 5 × 10^{-5} M² ; l_g = 2 mm = 2 × 10^{-3} m ; ϕ_g = 0.065 mWb = 6.5 × 10^{-5} Wb.

$$\therefore \quad B_g = \frac{\phi_g}{A} = \frac{6.5 \times 10^{-5}}{5 \times 10^{-5}} = 1.30 \text{ Wb/m}^2$$

$$\therefore \text{Corresponding value of } \mu_r = 521 - \frac{0.05}{021} \times (521 - 365) = 484$$

$$\text{Hence, AT required} = \frac{B_g}{\mu_0}\left[\frac{l_{cs} \times \text{Leakage factor}}{\mu_r} + l_g\right]$$

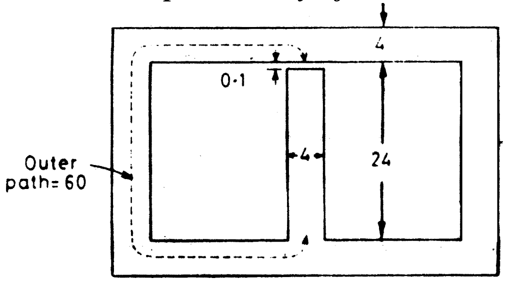
$$= \frac{1.30}{4\pi \times 10^{-7}}\left[\frac{0.503 \times 1.2}{484} + 2 \times 10^{-3}\right]$$

$$= \frac{1.30 \times 10^{7}}{4\pi}\left[1.247 \times 10^{-3} + 2 \times 10^{-3}\right]$$

$$= \frac{1.30 \times 10^{7}}{4\pi}\left[3.247 \times 10^{-3}\right] = 3{,}359.0.$$

Example : 40

A 680 turns coil is wound on the central limb of the steel frame shown below. A total flux of 1.6 m Wb is required in the air–gap. Find the current required throungh the gap. Assume that the gap density is uniform, and there is no leakage. Frame dimensions are given in cm. Take permeability of cast steel as 1,200.



Solution :

Central limb : Ac = 4cm × 4cm = 1.6×10^{-3} m^2 ; l_c = 24 cm = 2.4×10^{-1} m ; $B_c = \phi_c/A_c$ = (1.6 × 10^{-3} = 1 Wb/m^2.

$$\therefore \quad H_c = \frac{B_c}{\mu_0\mu_r} = \frac{1}{4\pi\times10^{-7}\times1{,}200} = 663 \text{AT/m}$$

or $(AT)_c = H_c \times l_c = 663\times2.4\times10^{-1}$ = 159.1 AT. .....(i)

Air–gap: $B_g = B_c$ = 1 Wb/m^2 ; l_g = 0.1 cm = 1×10^{-3} m.

$$\therefore \quad H_g = \frac{B_g}{\mu_0} = \frac{1}{4\pi\times10^{-7}} = 796\times10^3 \text{AT/m}$$

or $(AT)_g = H_g \times l_g = 796\times10^{-3}\times1\times10^{-3}$ = 796 AT. .....(ii)

Outer path : $l_0 = 60\times10^{-2}$ m = 6×10^{-1} m ; $A_0 = 16\times10^{-4}$ = 1.6×10^{-3} m^2.

∴ Flux through each outer path, ϕ_0 = 1.6 mWb/2 = 0.8 m Wb ; $B_0 = \phi_0/A_0$ = 0.8 mWb/1.6 × 1^{-3} = 0.5 Wb/m^2.

$$\text{or} \quad H_0 = \frac{B_0}{\mu_0\mu_r} = \frac{0.5}{4\pi\times10^{-7}\times1{,}200} = 331.6 \text{ AT/m}$$

∴ $(AT)_0 = H_0 \times l_0 = 331.6\times6\times10^{-1}$ = 198.9 AT ....(iii)

∴ Total AT required = 159.1 + 796 + 198.9 = 1,154 AT

Hence current, I = AT/N = 1,154/680 = 1.697 A.

Note :

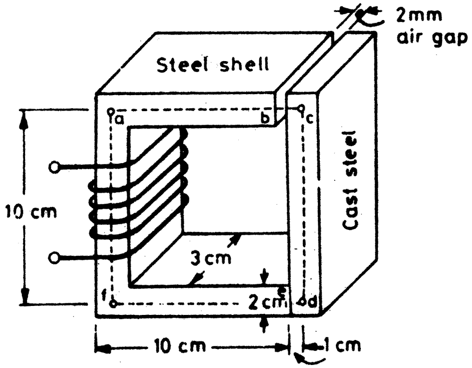
Since there are two parallel paths across the central limb, so flux density of each side path is half that of central limb. Moreover, total mmf is equal to that in the central limb plus either side path.

Example : 41

The relay frame shown below is a typical series magnetic

circuit. In the circuit, the flux is to be 0.3 mWb. Find the current required to establish this flux, if the coil has 100 turns. Form the B-H curve for sheet steel, and cast iron, it is found that a flux density of 0.3 Wb/m^2 required the following magnetisting forces.

Sheet steel, H_{ss} = 200 AT/m ; cast iron, H_{ei} = 1,850 AT/m.



In the above, relay assume that the relay is open so that a 2 mm air–gap is introduced at point b. Find the current to establish a flux of 0.3 mWb in the air-gap.

Solution :

ϕ_g = 0.3 mWb = 3×10^{-4} Wb ; A_g = 3 cm × 2cm = 6 cm^2 = 6×10^{-4} m^2; 1_g = 2 mm = 2×10^{-3} m ; N = 100, 1_{ss} = 10 + 10 + 9.8 cm = 29.8 cm = 0.298 ; l_a = 10 cm = 0.1 m ; H_{ss} = 200 AT/m ; H_a = 1,850 AT/m.

∴ $B_g = f_g/A_g = 3\times10^{-4}$ = 0.5 Wb/m^2.

$$\therefore \text{ AT required } = \frac{B_g l_g}{\mu_0} + H_{ss}\times l_{ss} + H_{ci}\times l_{ci}$$

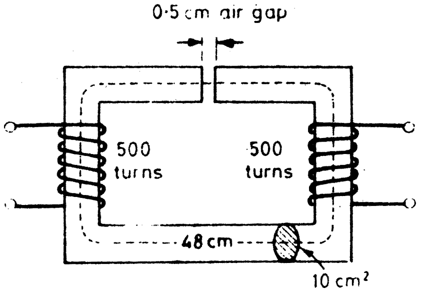
$$= \frac{0.5\times2\times10^{-3}}{4\pi\times10^{-7}} + 200\times0.298 + 1{,}850\times0.1$$

= 795.8 + 59.6 + 185 AT = 1,040.4 AT

Hence, current requried, $I = \dfrac{\text{Mmf}}{\text{Turns}} = \dfrac{1,040.4}{100} = 10.4\ \text{A}$

Example : 42

An electromagnet of form shown below is exicted by two coils, each having 500 turns. When the exciting current is 0.8 A, and the resultant flux density gives permeability of 1,250, calculate : (i) total reluctance, and (ii) flux produced 0.5 cm air–gap.



Solution :

l_i = 48 cm = 0.48 m ; l_g = 0.5× 10^{-3} m; A = 10 cm² = 1× 10^{-3} m²; μ_r = 1,250 ; I = 0.8 A ; N = 500

(i) Reluctance, $$S = \frac{1}{\mu_0 A}\left[l_g \frac{l_i}{\mu_r}\right]$$

$$= \frac{1}{4\pi \times 10^{-7} \times 10^{-3}}\left[5\times 10^{-3} + \frac{0.48}{1,250}\right]$$

= 4.285× 10^6 AT/Wb.

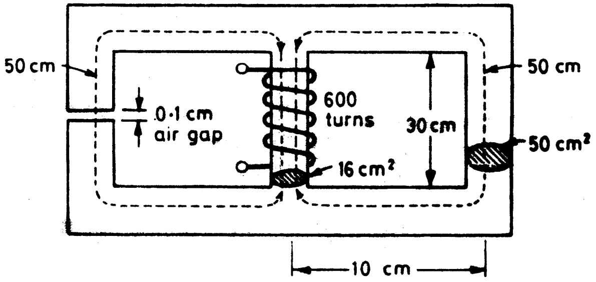
(ii) Flux, $$\phi = \frac{\text{Mmf}}{\text{Reluctance}} = \frac{NI}{S} = \frac{2\times 500 \times 0.8}{4.285 \times 10^6}$$

[similarly there are 2 coils]

= 1.867× 10^{-4} Wb.

Example : 43

A magnetic core made up of an alloy has the dimensions as shown in figure below :



Calculate the exciting current required to produce a flux of 1 mWb in the air–gap. Neglect leakage, and fringing. B/H curve is as follows :

B(Tesla)	***1***	***1.2***	***1.42***	***1.5125***	***1.6***
H(AT/m)	***200***	***400***	***1791.4***	***3,300***	***6,000***

Solution :

Side limb with air–gap : B = fg/A =1× 10^{-3} Wb/0.1× 10^{-2} m² = 1 Wb/m² = 1 Wb/m². The corresponding value of H = 200 AT/m.

∴ AT for left limb

$$= H_s l_s + \frac{B l_g}{\mu_0} = 200 \times \left(50 \times 10^{-2}\right) + \frac{1 \times 0.1 \times 10^{-2}}{4\pi \times 10^{-7}}$$

= 100 + 795.8 = 895.8 A = AT for right limb

= H_r× 150× 10^{-2} or H_r = 1,1,791 AT/m .....(i)

The coressponding value of B_r = 1.42 tesla. So

ϕ_r = 1.42× 50× 10^{-4} Wb = 7.1 mWb

Central limb : Total flux, fc = 1 + 7.1 = 8.1 mWb. So

B_c = 8.1 m ×Wb/16× 10^{-4} m² = 506 tesla.

The corresponding value of H(assuming straight curve between B = 1 to 5.06 tesla)

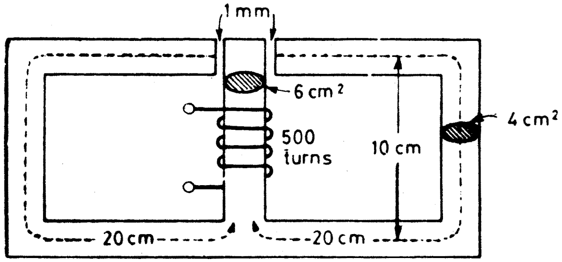
$$= 200 + \frac{(5.06-1)(6,000-200)}{(1.6-1)} = 39,447 \text{ AT/m}$$

$\therefore$ At for central limb = $H_c . l_c$ = 39,447×0.3 = 11,834 AT ...(ii)

Hence, exciting current, I = AT/N = 11,834/600 = 19.7 A .

Example : 44

A magnetic circuit shown below is constructed of wrought iron :



Thr cross–section of central limb is 6 cm2, and of each outer limb is 4 cm2. If the coil is wound with 500 turns, calculate the exciting current required to set up a flux of 0.9 mWb in the central limb. B-H curve of wrought iron are :

B(Wb/m2)	***1.125***	***1.5***
H(AT/m)	***500***	***2,000***

Solution :

Cental limb : $\phi = 0.9\times10^{-3}$ Wb; $A_1 = 6\times10^{-4}$ m²; l_1 = 10 cm = 0.1 m; $B_1 = \phi_1/A_1 = 0.9\times10^{-3}/6\times10^{-4}$ = 1.5 Wb/m².

$\therefore$AT required = $H_1 l_1$ = 2,000×0.1 = 200 AT.

Outer limb : $\phi = \frac{1}{2}\times0.9\times10^{-3}$ Wb : $A_2 = 4\times10^{-4}$ m²; L_2 = 20 cm = 0.2 m; $B_2 = 0.45\times10^{-3}/4\times10^{-4}$ = 1.125 Wb/m.

From B-H curves, H2 = 500 AT/m.

$\therefore$AT required = $H_2 l_2$ = 500×0.2 = 100 AT. ...(ii)

Air–gap : $B_g = B_2$ = 1.125 Wb/m² ; $l_g = 1\times10^{-3}$ m

$$\therefore \text{AT required} = \frac{B_g l_g}{\mu_0} = \frac{1.125\times10^{-3}}{4\times10^{-7}} = 895 \text{ AT} \quad ...(iii)$$

$\therefore$ Total AT required = 200 + 100 + 895 = 1,195 AT.

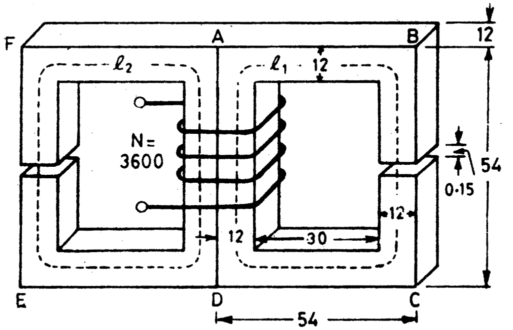
Hence, exciting current (I)

= AT/N = 1,195/500 = 2.39 A.

Example : 45

A magnetic circuit composed of two branches which are similar and in parallel, is shown below. The dimensions are given in cm. The permeability of the steel used is 700 at the operating point. A coil of 3,600 turns is placed on the central core.

Neglecting leakage, and fringing, calculate the flux density in the air–gaps, when the coil carries a current of 1.6 ampers.



Solution :

Central limb AD : Let B be the flux density (in Wb/m2) required ; l_{AD} = 54 – 6 = 42 cm = 0.42 m ; μ_r = 700.

$$\therefore \text{ AT required} = \frac{B l_{AD}}{\mu_0 \mu_r} = \frac{B\times0.42}{4\times10^{-7}\times700} = 477.5 \text{ B AT} \quad ....(i)$$

Side limb : l_{ANCD} = 42 + 42 + 42 = 126 cm = 1.26 m ; l_g = 0.15 cm = 1.5×10^{-3} m; flux density = B/2 (since then are parallel paths).

$$\therefore \text{ AT required} = \frac{B/2}{\mu_0}\left[\frac{l_{\text{ABCD}}}{\mu_r} + l_g\right]$$

$$= \frac{B}{2\times4\pi\times10^{-7}}\left[\frac{1.26}{700} + 1.5\times10^{-3}\right]$$

= 1,313 B AT. ...(iii)

∴ Total AT required = 477.5 B + 1,313 B = 1,790.5 B AT

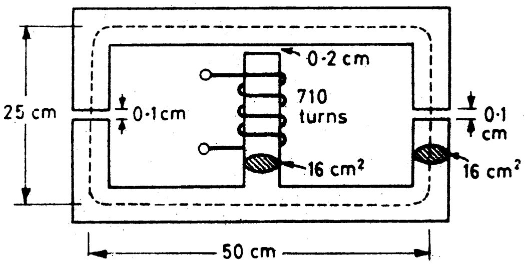
∴ 1,790.5 B = NI = 3,600×1.6 = 5,760

or B = 5,760/1,790.5 = 3.217 Wb/m².

Hence, flux density in the air–gap, = B/2 = 3.217/2 = 1.608 Wb/m².

Example : 46

A 710 turns coil is wound on the central limb of the cast steel symmetrical frame of uniform cross–section (16 cm²) as shown below :



Calculate the current required to produce flux of 1.8 mWb is an air–gap of 0.2 cm length. Magnetising curve for cast iron is :

H(AT/m)	***300***	***500***	***700***	***900***	***1,100***
B(Wb/m²)	***0.1***	***0.45***	***0.775***	***1.00***	***1.13***

Solution :

Central limb : $B_g = f_g/A = 1.8\times10^{-3}/16\times10^{-4}$ = 1.125 Wb/m².

Corresponding value of

$$H_e = 900 + \frac{0.125}{0.130}\times200 = 1{,}092 \text{ AT/m}.$$

∴ AT for central limb

$$= H_c l_c + \frac{B_g l_g}{\mu_0} = 1{,}092\times0.25 + \frac{1.125\times2\times10^{-3}}{4\pi\times10^{7}}$$

= 273 = 1,790 = 2,063 AT.

Side limb : B_s = 1.125/2 = 0.5625 Wb/m² (since there are 2 side limbs). The corresponding value

$$\text{of } H_s = 500 + \frac{0.1125}{0.325}\times200 = 569 \text{ AT/m}$$

$$\therefore \text{ AT for side limb } H_s l_s + \frac{B_s l_{gg}'}{\mu_0}$$

$$= 569\times(0.25+0.25) + \frac{0.5625\times0.1\times10^{-2}}{4\pi\times10^{-7}}$$

= 569×0.75 + 447.6 AT = 874 AT ....(ii)

∴ Total AT required = 2,063 + 874 = 2,937 AT.

Hence, current, I = 2,937/710 = 414 A.

Example : 47

Determine the force necessary to separate two surfaces with 100 cm² of contact area, when the flux density normal to the surface is 1 Wb/m².

Solution :

Here A = 100 cm² = 10^{-2} m2 ; B = 1 Wb/m².

$$\therefore \quad F = \frac{B^2 A}{2\mu_0} = \frac{1^2 \times 10^{-2}}{2 \times 4\pi \times 10^{-7}} N = 3{,}980\ N$$

Example : 48

In a telephone receiver, the cross–section of pole is 1 cm×0.5 cm. The flux between the poles and the diaphragm is 0.006 mWb. What is the force acting on the diapharagm ?

Solution :

Here ϕ = 0.006 mWb = 6×10^{-6} Wb ; A = 1 cm×0.5 cm = 0.5 cm^2 = 5×10^{-5} m^2.

$$\therefore \quad B = \frac{\phi}{A} = \frac{6\times10^{-6}}{5\times10^{-5}} = 0.12\ Wb/m^2$$

∴ Force acting on the diaphragm,

$$F = \frac{B^2 A}{2\mu_0} = \frac{(0.12)^2 \times 5\times10^{-5}}{2\times4\pi\times10^{-7}} = 0.286\ N\ \text{per pole}$$

∴ Force acting on both poles = 0.286×2 = 0.572 N.

Example : 49

A horse–shoe magnet is formed of a bar of wrought iron 45 cm long having cross–section of 6 cm². Exciting coils of 500 turns are placed on each limb and connected in seies. Determine the exciting currents necessary for the magnet to lift a load of 50kg, assuming that the load makes close contact with the magnetic. Relative permeability of iron is 800.

Solution :

Force of attraction at each pole,

F = 60/2 = 30 kg = 30×9.81 = 294.3 N

area of cross section, A = 6 cm^2 = 6×10^{-4} m^2.

$$\text{Now} \quad F = \frac{B^2 A}{2\mu_0}\ \text{newton}$$

$$\therefore\ 294.3 = \frac{B^2 \times 6\times10^{-4}}{2\times4\pi\times10^{-7}} \text{ or } B^2 = \frac{294.3\times2\times4\pi\times20^{-7}}{6\times10^{-4}} = 1.232$$

or B = $(1.232)^{1/2}$ = 1.11 Wb/m^2.

$$\text{Also} \quad H = \frac{B}{\mu_0\mu_r} = \frac{1.11}{4\pi\times10^{-7}\times800} = 1{,}104\ AT/m$$

Now length of the path = 45 cm 0.45 m.

∴ AT required = 1,104×0.45 = 496.8

Also total number of turns 2×500 = 1,000

$$\therefore \text{ Current required } = \frac{496.8}{1{,}000} = 0.968\ A\ .1$$