Unit-4

Design of Energy Storing Elements

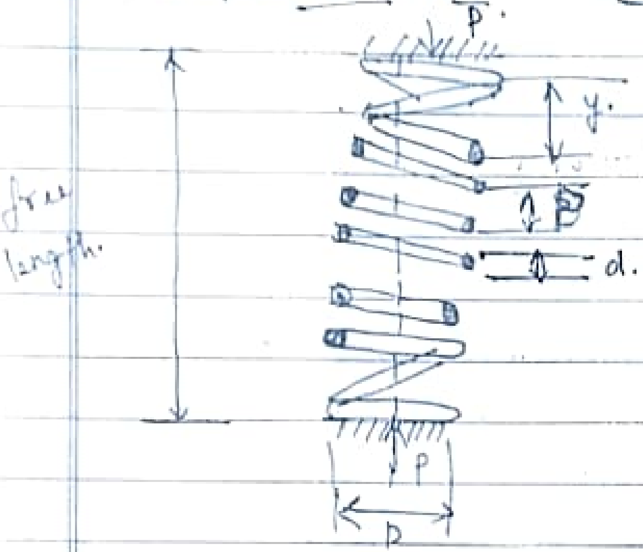
Springs

A spring is an elastic member which deflects under the action of load & regains its original shape after the load is removed.

Classification of Springs

- Helical spring
  - Compression helical spring
  - Tension helical spring
- Conical & volute springs.
- Torsion springs.
  - Helical Torsion spring.
  - Spiral Torsion " "
- Disc spring (or) Belleville spring.
- Laminated (or) leaf spring.

Important Terms used in helical Springs.



p - load in springs, kgf.

y - deflection of spring.

d - dia of wire.

D - mean dia of spring. (or) Pitch dia.

n - no. of coils.

n_t - Total no. of coils.

1) Spring stiffness factor (q)

$$q = \frac{P}{y}$$

2) Spring Index (c).

$$C = \frac{D}{d}$$

3) Free length (L_f).

$$L_f = n_t \times d + y_{max} + 0.15\, y_{max}$$

4) Solid length (L_s).

$$L_s = n_t \times d$$

5) Pitch (P).

$$P = \frac{\text{Free length.}}{\text{No. of active coils.}}$$

$$P = \frac{L_f - L_s}{n} + d.$$

6) Wahl's Stress concentration factor (K_s)

$$\tau = K_s \times \frac{8PD}{\pi d^3} = K_s \times \frac{8PC}{\pi d^2}.$$

$$K_s = \frac{4C-1}{4C-4} + \frac{0.615}{C}.$$

$$K_s = 1 + \frac{1}{2C}$$

(X). C – Spring Index should not be less than 3.

Usually taken as $C = 5-12.$

Industrial springs $C = 6-10.$

7) Deflection of helical springs (y).

$$y = \frac{8PD^3 n}{Gd^4} = \frac{8PC^3 n}{Gd}$$

8) stiffness of spring (q)

$$q = \frac{G d^4}{8 D^3 n} = \frac{G d}{8 C^3 n}.$$

9) Energy stored in spring (U)

$$U = \frac{P \times y}{2}.$$

10) volume of spring (V)

$$V = \pi D n \times \frac{\pi}{4} d^2.$$

Problems on springs.

1) A compression coil spring made of an alloy steel is having the following specifications.

Mean dia of coil = 50 mm.

Wire dia = 5 mm.

No. of. active coils = 20.

If the spring is subjected to an axial load of 500 N. calculate the max. shear stress.

Gn/: — D = 50 mm,

d = 5 mm.

n = 20

P = 500 N.

To find: Max. shear stress (τ)

Spring Index (C) $= \frac{D}{d} = \frac{50}{5} = 10.$

$\boxed{C = 10}$

Shear stress factor (K_S) $= 1 + \frac{1}{2C}$

$= 1 + \frac{1}{2 \times 10}$

$\boxed{K_S = 1.05}$

Max. shear stress (τ)

$$\tau = K_S \times \frac{8 P \cdot D}{\pi d^3}$$

$$= 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3}$$

$\boxed{\tau = 534.7 \text{ MPa}}$

2) Design a helical compression spring for a max. load of 1000N for a deflection of 25mm using the value of spring Index as 5. The max. permissible shear stress for spring wire is 420 MPa & modulus of rigidity is 84 kN/mm².

Given:

load (P) = 1000N.

Spring Deflection (y) = 25 mm.

Spring Index (C) = 5

Max. Shear stress (τ) = 420 MPa.

Modulus of rigidity (G) = 84 kN/mm².

= 84×10^3 N/mm².

To find:

Design a helical compression spring.

Solution:

1) Mean diameter of the spring coil.

Wahl's stress factor (K_s) — (PSG D.B 7.100)

$$K_s = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$= \frac{4\times5-1}{4\times5-4} + \frac{0.615}{5}$$

$K_s = 1.31$

Max shear stress (τ). — PSG D.B 7.100.

$$\tau = K_s \times \frac{8PC}{\pi d^2}$$

$$420 = 1.31 \times \frac{8 \times 1000 \times 5}{\pi d^2}$$

$$420 = \frac{16677}{d^2}$$

$$d^2 = 16677/420$$

$$d^2 = 39.7$$

$$\boxed{d = 6.3mm}$$

Mean dia of spring coil $(D) = C \times d$

$= 5 \times 6.3$

$= 31.5$ mm

$$\boxed{D = 32mm}$$

out. dia of spring coil $(D_o) = D + d$

$= 32 + 6.3$

$$\boxed{D_o = 38.3 \text{ mm}}$$

2) Number of turns of coils. (n).

$$y = \frac{8PC^3 n}{G \cdot d}$$

$$25 = \frac{8 \times 1000 \times (5)^3 \times n}{84 \times 10^3 \times 6.3} \Rightarrow$$ ~~25 = 75.58 n~~

$25 = 1.88\ n$

$n = 25 / 1.88$

$n = 13.2 \simeq 14 mm$

$\boxed{n = 14 mm}$

For squared & ground ends,

$n_t = n + 2 = 14 + 2 = 16 mm$ — p.s.g D.B 7.104

$\boxed{n_t = 16 mm}$

3) ~~Free length of spring (L_f)~~

$L_f = n_t \times d + y + ~~0.15y~~$. manipulation

$= 16 \times 6.3 + 25 + ~~0.15 \times 25~~$

$\boxed{L_f = 129.5 mm}$

4) ~~pitch of coil (p).~~

$$p = \frac{L_f}{n_t^1 - 1} = \frac{129.5}{16 - 1}$$

$\boxed{p = 8.6 mm}$

(1.) Problems on helical springs.

8) Design a valve spring of I.C Engine for the following details.

Spring load when valve is closed = 80 N. = W_2

Spring load when valve is open = 100 N. = W_1

Inside guide bush dia = 24 mm (spring constrain to for filament).

outside recess dia = 36mm.

Valve lift = 5 mm.

Max. permissible shear stress = 350 MPa.

Modulus of rigidity = 84 kN/mm². find

1) wire dia 2) spring Index 3) Total no. of coils

4) Solid length of spring 5) free length of spring.

6) pitch of coil. When additional 15% of working deflection is used to avoid complete the coils.

Given: $P_1 = 100N$, $P_2 = 80N$,

Inside bush dia = 24mm, outside recess dia $(D_o) = 36mm$.

Valve lift $(y) = 5mm$,

Shear stress $(\tau) = 350 MPa = 350 N/mm^2$

Modulus of rigidity $(G) = 84 \times 10^3 N/mm^2$

To find:

1) d 2) c 3) n_t 4) L_s 5) L_f 6) p.

Sol:

1). Mean dia of spring coil (D).

D = Inside dia of spring + Dia of spring wire.

D = 24 + d.

Since the ϕ of spring wire is obtained by for max. spr. load (W_1)

∴ Twisting moment (T) = $P_1 \times \frac{D}{2}$ [∵ T = P × R].

$= 100 \times \left(\frac{24+d}{2}\right)$.

$= \frac{2400}{2} + \frac{100d}{2}$

$T = (1200 + 50d)$ N·mm   T = P×R

$T = \pi/16 \times \tau \times d^3$

$1200 + 50d = \frac{\pi}{16} \times 350 \times d^3$

$1200 + 50d = 68.68\, d^3$.   Hit & trial method find 'd'
d = 1, 2, 3, 4, ...

$d = 2.9\text{mm}$

considering Wahl's stress concentration factor (K_S).

$K_S = \frac{4C-1}{4C-4} + \frac{0.615}{C}$ — (PSG D.B 7.100)

$C = \frac{D}{d} = \frac{24+2.9}{2.9}$

$C = 9.27$ mm

$$K_s = \frac{(4\times 2.9)-1}{(4\times 2.9)-4} + \frac{0.615}{9.27}$$

$$\boxed{K_s = 1.46}$$

Max. shear stress (τ)

$$\tau = K_s \times \frac{8\times P\times C}{\pi\times d^2} \quad \text{— (P 304 D.B 7.100).}$$

$$350 = 1.46 \times \frac{8\times 100\times 9.27}{\pi\times d^2}$$

$$\boxed{d = 3.13\ \text{mm}}$$

$D = 24 + d$

$= 24 + 3.13 = 27.13\ \text{mm}.$

Taking larger of two values, $\boxed{d = 3.13\ \text{mm}}$

4) Number of turns of coil (n).

$P = P_1 - P_2 = 100 - 80 = 20\ \text{N}.$

$\boxed{P = 20\ \text{N}}$, $\boxed{y = 5\ \text{mm}}$

$$y = \frac{8\times P\times D^3\times n}{G\times d^4} \quad \text{— (P 304 D.B 7.100).}$$

$$5 = \frac{8\times 20\times (27.13)^3\times n}{84\times 10^3\times (3.13)^4}$$

$n = 12.8 \simeq 13$ mm.

$n_t = n + 2$

$= 13 + 2$

$n_t = 15$ mm

Total No. of. coils. $(n_t) = 15$ mm.

$y_{max} = \frac{y}{P} \times P_1$

3) Free length of spring. (L_f).

Deflection of $y = 5$ mm is for 20N.

Max. load $= 100$N is.

$$y_{max} = \frac{5}{20} \times 100$$

$$y_{max} = 25 \text{ mm}$$

$$L_f = n_t \times d + y_{max} + 0.15\, y_{max}$$

$$L_f = 15 \times 3.13 + 25 + 0.15 \times 25$$

$$\boxed{L_f = 75.7 \text{ mm}}$$

4) Solid length of Spring (L_s).

$$L_s = n_t \times d.$$

$$= 15 \times 3.13$$

$$\boxed{L_s = 46.95 \text{ mm}}$$

5) pitch of coil (p).

$$p = \frac{L_f - L_s}{n_t} + d,$$

~~p =~~ ~~$L_f - t$~~ $$p = \frac{\text{free length}}{n_t - 1}$$ (for valve spring)

$$p = \frac{L_f \times 1.15}{15 - 1}$$

$$p = \frac{75.7 \times 1.15}{14}$$

$$p = 6.69\ mm$$

pitch of coil (p) = 7 mm.

4) Design a helical spring for a spring loaded safety valve for the following conditions.

Operating pressure = 1 N/mm²

Max. pressure when valve blows off freely = 1.075 N/mm²

Max. lift of the valve when the pressure is 1.075 N/mm² = 6mm

Dia of valve seat = 100 mm.

Max. shear stress = 400 Mpa.

Modulus of rigidity = 86 kN/mm²

Spring Index = 5.5

Gn/:

Dia of valve seat. (D_1) = 100 mm. $G = 86 \times 10^3$ N/mm²

$P_1 = 1$ N/mm²

$P_2 = 1.075$ N/mm².

$y = 6$ mm.

$\tau_{max} = 400$ N/mm²

$c = 5.5$.

To find: Design a helical spring.

Sol:

1) Mean dia of spring coil (D).

$$P_1 = \frac{\pi}{4} \times (D_1)^2 \times p_1$$

$$= \pi/4 \times (100)^2 \times 1$$

$$\boxed{P_1 = 7853.98 \text{ N}}$$

$$P_2 = \frac{\pi}{4} \times (D_1)^2 \times P_2$$

$$= \frac{\pi}{4} \times (100)^2 \times 1.075$$

$$\boxed{P_2 = 8443.03\ N}$$

$$P = P_2 - P_1 = 8443.03 - 7853.98$$

$$\boxed{P = 589.05\ N}$$

Twisting moment (T)

$$T = P_2 \times \frac{D}{2}$$

$$= 8443.03 \times \frac{5.5 \times d}{2} \qquad 5.5 = D/d.$$

$$T = 23218.33d \text{ ~~N-mm~~}$$

max. twisting moment.

$$T = \frac{\pi}{16} \times \tau \times d^3$$

$$23218.33\,d = \frac{\pi}{16} \times 400 \times d^3$$

$$d^3 = 295.774\,d. \implies d^2 = 295.77$$

$$\boxed{d = 8.6\ mm}$$

$$d = 17.19\ mm$$

$$\boxed{d = 17.2\ mm}$$

Mean dia of coil

$D = 5.5 \times d$

$= 5.5 \times 17.2$

$\boxed{D = 94.6 mm}$

2) Number of turns of coil (n).

$y = \frac{8 \times P \times C^3 \times n}{G \times d}$

$6 = \frac{8 \times 569.05 \times (5.5)^3 \times n}{86 \times 10^3 \times 17.2}$

$n = 11.82 \approx 12$

$\boxed{n = 12}$

$n_t = n + 1$ (spring having both loops)

$= 12 + 1$

$\boxed{n_t = 13}$

Wahl's criterion from soderberg line,

$$\frac{1}{n} = \frac{\tau_m - \tau_a}{\tau_y} + \frac{2\tau_a}{\tau_o} \quad - \text{PSG D.B 7.102.}$$

τ_y - τ_o - Endurance shear stress

Problems on helical spring subjected to var. loading

A helical spring is subjected to a load varying from 400N to 1000N having spring Index of 6 and the design factor of safety is 1.25. The compression of the spring at the maximum load is 30mm. Design the helical compression spring. Take yield stress in shear as 110 N/mm², endurance stress in shear as 350 N/mm² & modulus of rigidity for the spring material as 80 kN/mm².

Gn/.

$P_{min} = 400N$

$P_{max} = 1000N$

$C = 6$

$n = 1.25$

$y = 30mm.$

$\tau_y = 110 \ N/mm^2$

$\tau_o = 350 \ N/mm^2$

$G = 80 \times 10^3 \ N/mm^2$

To find:

Design of helical spring.

Solution/:

1) Dia of spring wire (d).

$$\text{Mean load } (P_m) = \frac{P_{max} + P_{min}}{2}$$

$$= \frac{1000 + 400}{2}$$

$$\boxed{P_m = 700 \text{ N}}$$

$$\text{Amplitude load } (P_a) = \frac{P_{max} - P_{min}}{2}$$

$$= \frac{1000 - 400}{2}$$

$$\boxed{P_a = 300 \text{ N}}$$

$$\text{Direct shear stress } (K_{sh}) = 1 + \frac{0.615}{C}$$

$$= 1 + \frac{0.615}{6}$$

$$\boxed{K_{sh} = 1.1025}$$

Mean shear stress $(\tau_m) = \frac{8 K_{sh} \times P_m \times c}{\pi d^2}$ — PSG D.B 7.102.

$$\tau_m = \frac{8 \times 1.1025 \times 700 \times 6}{\pi \times d^2}$$

$$\tau_m = \frac{11797.45}{d^2}$$

from PSG D.B for value $c = 6$

$K_c = 1.15$ (7.102)

Wahl's stress factor (K_s)

$$K_s = K_{sh} \times K_c$$

$$= 1.1025 \times 1.15$$

$$\boxed{K_s = 1.2678}$$

Amplitude shear stress $(\tau_a) = \frac{8 \times K_s \times P_a \times c}{\pi d^2}$ — PSG D.B 7.102

$$= \frac{8 \times 1.2678 \times 300 \times 6}{\pi \times d^2}$$

$$\tau_a = \frac{5811.51}{d^2}$$

for repeated loading.

$$\frac{1}{n} = \frac{\tau_m - \tau_a}{\tau_y} + \frac{2 \times \tau_a}{\tau_0}$$

$$\frac{1}{1.25} = \frac{\frac{11797.47}{d^2} - \frac{5811.51}{d^2}}{110} + \frac{\frac{2 \times 5811.51}{d^2}}{350}$$

$$d^2 = 109.46$$

$$\boxed{d = 10.462 \text{mm}}$$

2) Mean dia of coil (D)

$$C = D/d$$

$$D = C \times d = 6 \times 10.42.$$

$$\boxed{D = 62.52 \text{ mm}}$$

3) Numbers of Active turns of coil (n).

$$y = \frac{8 \times P_{max} \times C^3 \times n}{G \times d} \quad \text{— PSG D.B 7-100.}$$

$$30 = \frac{8 \times 1000 \times (6)^3 \times n}{80 \times 10^3 \times 10.46} \qquad n = 14.5 \simeq 15$$

$$\boxed{n = 15}$$

Total no. of coils (n_t) = $n+2$

$= 15+2$

$n_t = 17$

4) Solid length of spring (L_s)

$L_s = d\,n + 2d$ (PSG D.B 7.101 table)

$= 10.46 \times 15 + 2 \times 10.46$.

$L_s = 177.82$ mm.

5) Free length of spring (L_f).

$L_f = L_s + y$.

$= 177.82 + 30$

$L_f = 207.82$ mm.

6). pitch of coil (p).

$$p = \frac{L_f - L_s}{n_t} + d.$$

$$= \frac{207.82 - 177.82}{17} + 10.46.$$

$p = 12.22$ mm.

7) Spring stiffness (q).

$$q = \frac{P_{max}}{y} = \frac{1000}{30}$$

$$\boxed{q = 33.33 \ N/mm}$$