Unit - II

Design of Spur gear & parallel axis Helical gear:

Gear:

A circular and cylindrical shaped body having teeth of uniform formation called gear (or) Toothed gear or Toothed wheel.

Classification of gears:

i) Based on position of shaft axes:

* parallel shaft
  Spur, Helical, Rack & pinion, Herring bone and Internal gears.

* Intersecting shafts:
  Bevel gears & spiral gears.

* Non-parallel and non-Intersecting shaft:
  Worm, Hypoid and Spiral gears.

ii) Based on relative motion of shafts:
Row gears & planetary and differential gears.

iii) Based on position of teeth on the wheel:

* Straight gears.
* Helical gears.
* Herring bone gear
* Curved teeth gear.

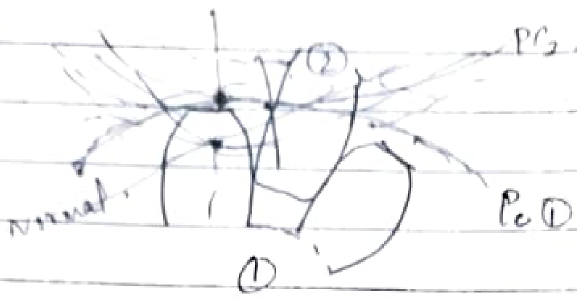
iv) Based on type of gearing:

* External
* Internal
* Rack and pinion.

v) Based on velocity:
Low → Less than 3m/s.
Medium = 3 to 15 m/s.
High = Above 15 m/s.

Design of Spur gear:

Law of gearing:



To achieve constant velocity ratio, at any instant of teeth, the common normal at each point of contact should pass through a pitch point, situated on the line joining the centres of rotation of the pair of mating gears

Types of Gear tooth profile:
- Involute
- cycloidal.

pressure angle:

Normally $14\frac{1}{2}^\circ$ and 20°. full depth

$14\frac{1}{2}^\circ$ composite, 20° stub Involute

Gear materials.

Steels & Alloyed steels.
Cast Iron.
Bronze.
Non-metallics. (Nylon, wood, compressed paper etc).

Gear Manufacturing methods:
- Gear Milling
- Gear Hobbing
- Gear Shaping
- Gear Moulding (Foundry)
- Injection Moulding
- Die casting.
- Investment casting.
- Shell Moulding.

Forces on spur gear:
- Tangential force (F_t)
- Radial Component (F_r).

Tangential Component. (F_t)

$$M_t = F_t \times d/2.$$

$$\boxed{\therefore F_t = \frac{M_t \times 2}{d}}.$$

Radial component (F_r).

$$\boxed{F_r = F_t \cdot \tan\phi}$$ when ϕ = pressure angle.

Power Transmitted (P)

$$\boxed{P = F_t \times V}.$$

Note:

The tangential force on both gear will be same.

Gear Interference:

If 2 gear are not engaged properly by mean of teeth interference one over the other.

It causes problem in power Transmission.

Problem on Design of Spur gear using Lewis and Buckingham Equations: (Type 1)

1. Design a spur gear drive required to transmit 45 kW at a pinion speed of 800 rpm. The velocity ratio is 3.5 : 1. The teeth are 20° full depth involute with 18 teeth on the pinion. Both pinion and gear are made of steel with a maximum safe static stress of 180 N/mm². Assume medium shock conditions.

Given:

power transmitted $P = 45\ kW$.

pinion speed $N_1 = 800 rpm$.

velocity ratio $i = 3.5$.

pressure angle $\phi = 20°$ (Full depth involute).

No of Teeth in pinion $Z_1 = 18$

Max. Safe Static Stress $[\sigma_b] = 180\ N/mm^2$

"Medium Shock Condition"

If static stress given without Centre distance

Solution:

Since both gear and pinion are made of same material the pinion is weaker than gear. So we have to design only "Pinion"

1) Selection of Material:

Given, pinion is made of steel, Assume steel is harded to 200 BHN.

2) Calculation of No of teeth:

No of teeth in pinion $Z_1 = 18$

No of teeth in gear, $Z_2 = i \times Z_1$

$\therefore Z_2 = 3.5 \times 18.$

No of teeth in gear $\boxed{Z_2 = 63.}$

To find module 'm'

3. Calculation of (F_t): (Tangential Load)

From PSG. 8.50, Lewis equation.

$$F_t = \frac{P}{v} \times k_o.$$

$$\text{Velocity } (v) = \frac{\pi \times d_1 \times N_1}{60.}$$

Substituting $\boxed{d_1 = m.Z_1}$ as $m = \frac{d}{Z}$, To find 'm'.

$$\therefore v = \frac{\pi \times m \times Z_1 \times N_1}{60.}$$

$$= \frac{\pi \times m \times 18 \times 800}{60}$$

$\boxed{\text{Velocity } v = 0.754m \text{ m/s.}}$

$k_o =$ ~~1.25~~ 1.5 (for medium shock conditions).

Taking:
Steady - 1.0
Light shock - 1.25
Medium - 1.5
Heavy - 2.0

$$F_t = \frac{45 \times 10^3}{0.754m} \times 1.5.$$

$$\boxed{F_t = \frac{89523}{m.}}$$

4. Calculation of Initial dynamic load : (F_d):

PSG. 8.50, Lewis equation,

Initial dynamic load, $F_d = \frac{F_t}{C_v}$ where C_v - velocity factor

PSG. 8.51,

$C_v = \frac{6}{6+v}$ Assuming gear is Hobbed with $v < 20\,m/s$

Taking $v = 12$.

$\therefore C_v = \frac{6}{6+12} = 6/18$.

$\boxed{C_v = 0.333}$

$F_d = \frac{F_t}{C_v}$

$= \frac{89523}{m} \times \frac{1}{0.333} = \frac{268838}{m}$.

$\boxed{\therefore F_d = \frac{268838}{m}}$

5. Calculation of Beam strength: (F_s) (or) Tooth breakage.

From PSG. 8.50 Lewis equation,

$F_s = [\sigma_b] \times b \times y \times P_c$. where $P_c = \frac{\pi d}{Z}$ PSG. 8.5

$\boxed{\text{We know } m = d/z}$

$\therefore$ Equation may be modified as.

$F_s = [\sigma_b] \times b \times y \times \pi \times m$. $[\sigma_b]$ from PSG. 8.5, Table: 7

Assuming ~~base~~ face width $b = 10m$.

y = Form factor.

Form factor: From PSG. 8.50;

$$y = 0.154 - \frac{0.912}{Z_1} \quad \text{(For 20° Involute)}.$$

$$\therefore y = 0.154 - \frac{0.912}{18}.$$

$$\boxed{y = 0.1033.}$$

Beam Strength $(F_s) = [\sigma_b] \times b \times y \times \pi \times m$.

$$= \overset{\text{30n}}{180} \times 10m \times 0.1033 \times \pi \times m.$$

$$\boxed{(F_s) = 584.14\, m^2}$$

6. Calculation of module (m).

From psg 8.51;

$$F_s \geq F_d.$$

$$584.14 m^2 \geq \frac{268838}{m};$$

$$584.14 m^2 = \frac{268838}{m}.$$

$$\boxed{m = 7.72 \simeq 8\, mm.}$$

From PSG. 8.2 (Choice 1)

7. Calculation of b, d and v:

PSG. 8.22,

Face width (b) = 10 m = 10×8 = $\boxed{80\text{ mm}}$

pitch circle dia (d_1) = $m \cdot z_1$ = 8 × 18 = $\boxed{144\text{ mm}}$

pitch line velocity $v = \frac{\pi d_1 N_1}{60}$

$= \frac{\pi \times 144 \times 10^{-3} \times 800}{60}$

Pitch line velocity $\boxed{v = 6.03\text{ m/s}}$

8. Recalculation of beam Strength:

From PSG. 8.50 Lewis equation.

Beam Strength $F_s = \pi \times m \times b \times [\sigma_b] \times y$.

$= \pi \times 8 \times 80 \times 180 \times 0.1033$

$\boxed{\text{Beam Strength } (F_s) = 37385.45\text{ N}}$

9. Calculation of acurate dynamic load (F_d):

From PSG. 8.51, Buckingham's dynamic load (F_d).

$$F_d = F_t + \left[\frac{0.164V(Cb + F_t)}{0.164V + 1.485\sqrt{Cb + F_t}}\right]$$

$F_t = P/V$ (From PSG. 8.50).

$F_t = \frac{45 \times 10^3}{6.03}$

$\boxed{F_t = 7462.68\,N}$

To calculate Deformation factor 'c'.

From PSG 8.53. Table 41 & 42.

C = 11860 e for 20° full depth, steel and steel cutters

e = 0.038 (From Table 42). "e" ⇒ expected error in tooth profiles

∴ C = 11860 × 0.038.

$\boxed{\text{Deformation factor 'C'} = 450.68\ N/mm}$

$$\therefore F_d = 7462.68 + \frac{0.164 \times 6.03 \times 10^3 (80 \times 450.68 + 7462.68)}{0.164 \times 6.03 \times 10^3 + 1.485\sqrt{80 \times 450.68 + 7462.68}}$$

$$F_d = 7462.68 + \frac{4 30 34 910.75}{1298.50}$$

$$F_d = 7462.68 + 33141.80$$

$$\boxed{F_d = 40604.48\ N}$$

10. Check for beam Strength:

Since $F_d > F_s$, the design is unsatisfactory. The dynamic load is greater than beam strength (or) Tooth breakage.

∴ we have to reduce the dynamic load, by ~~secting~~ selecting precision gears.

From PSG. 8.53, Table 42, For module = 8,

$e = 0.019$.

$\therefore C = 11860 \times 0.019$.

$$\boxed{\therefore C = 225.34}$$

Again calculating the dynamic load (F_d), - PSG. 8.51,

$$F_d = 7462.68 + \left[\frac{0.164 V (Cb + F_t)}{0.164 V + 1.485 \sqrt{Cb + F_t}} \right]$$

$$= 7462.68 + \frac{0.164 \times 6.03 \times 10^3 (80 \times 225.34 + 7462.68)}{0.164 \times 6.03 \times 10^3 + 1.485 \sqrt{80 \times 225.34 + 7462.68}}$$

$$= 7462.68 + \frac{3258744.64}{1226.00}$$

$$7462.68 + 26580.11$$

$$\boxed{F_d = 34042.79 \text{ N.}}$$ $\therefore F_d < F_s$ The design is Safe.

11. Calculation of Max. wear load (F_w):

From PSG. 8.51

Wear load, $F_w = d_1 \times b \times Q \times K_w$.

where, Q = Ratio factor. from PSG. 8.51

$$Q = \frac{2i}{i+1} = \frac{2i}{i+1} = \frac{2 \times 3.5}{3.5+1}$$

$$\boxed{Q = 1.555}$$

Wear factor, K_w:

Material Pinion	Gear	K_w for FD $14\frac{1}{2}^\circ$ N/mm^2	K_w for 20° FD N/mm^2
Steel	150 BHN Steel	0.206	0.282
"	250 BHN "	0.673	0.919.
"	400 BHN "	1.869	2.553.
Steel	150 BHN CI.	0.303.	0.414
"	250 BHN "	1.0	1.31
Steel	200 BHN BRONZE	0.503	0.689.
CI	CI.	0.503	0.689.
Non-metal	metal	—	1.4.

$K_w = 0.919\ N/mm^2$, for steel hardened to 250 BHN.

$\therefore F_w = 144 \times 80 \times 1.555 \times 0.919$.

$\boxed{F_w = 16462.6\ N}$

12. Checking for wear:

Hence $F_d > F_w$, design is unsatisfactory. The dynamic load is greater than wear load.

To increase wear load F_w, we can increase the hardness (BHN).

Taking steel hardened to 400 BHN.

$\boxed{\therefore K_w = 2.553\ N/mm^2}$

$F_w = 144 \times 80 \times 1.555 \times 2.553$

$\boxed{F_w = 45733.42\ N}$

Now wear load F_w > dynamic load F_w, i.e. gear tooth have wear capacity and will not wear out. The design is safe.

~~Design of pinion.~~

13. Basic dimensions of pinion and gear. [From PSG. 8.22]

1. Module $m = 8\,mm$
2. Number of teeth $Z_1 = 18$ & $Z_2 = 63$.
3. pitch circle diameter $d_1 = 144\,mm$ &

   $d_2 = m \cdot Z_2 = 504\,mm$.

4. Centre distance: $a = \frac{m(Z_1 + Z_2)}{2} = 324\,mm$

5. Face width $b = 80\,mm$.
6. Height factor $f_0 = 1$

7. Bottom clearance $C = 0.25\,m = 2\,mm$.

8. Tip diameter:

$$d_{a_1} = (Z_1 + 2f_0)m = (18 + 2 \times 1)8 = 160\,mm.$$

$$d_{a_2} = (Z_2 + 2f_0)m = (63 + 2 \times 1)8 = 520\,mm.$$

9. Root diameter:

$$d_{f_1} = (Z_1 - 2f_0)m - 2C$$

$$= (18 - 2 \times 1)8 - 2 \times 2.$$

$$\boxed{d_{f_1} = 124\,mm.}$$

$$d_{f_2} = (Z_2 - 2f_0)m - 2C.$$

$$= (63 - 2 \times 1)8 - 2 \times 2$$

$$\boxed{d_{f_2} = 484\,mm.}$$

(2) A compressor running at 300 rpm is driven by a 15 kW, 1200 rpm. motor through a $14\frac{1}{2}^\circ$ full depth spur gears. The centre distance is 375mm. The motor pinion is to be of C30 forged steel hardened and tempered, and the driven gear is to be of cast iron. Assuming medium shock condition, design the gear drive completely.

Given:

Speed of Motor $N_1 = 1200$ rpm
Power of Motor $P = 15$ kW.
Speed of Compressor $N_2 = 300$ rpm.
Pressure angle $\phi = 14\frac{1}{2}^\circ$.
Centre distance $a = 375$ mm.

Solution:

Since the pinion and gear materials are different we have to evaluate $[\sigma_{b1}]y_1$ and $[\sigma_{b2}]y_2$ to find weaker element

Gear ratio:

$$i = \frac{N_1}{N_2} = \frac{1200}{300} = \boxed{4 = i}$$

Assuming $Z_1 = 18$.

$$Z_2 = i \times Z_1 = 4 \times 18.$$

$$\boxed{Z_2 = 72.}$$

(Static stress of pinion)

To find $[\sigma_{b1}]\, y_1$: (For pinion). made of C30 forged steel.

From PSG. 8.53,

For $Z = 18 \Rightarrow y_1 = 0.270.$ $\left[\because y_1 = \frac{Y}{\pi}\right.$ PSG. 8.53

Allowable ~~Shear~~ Static stress for C30 steel = $112\, N/mm^2$. (Assuming)

$\therefore [\sigma_{b1}]\, y_1 = 112 \times \frac{0.270}{\pi}$

$$\boxed{[\sigma_{b1}]\, y_1 = 9.625}$$

~~Allowa~~ To find $[\sigma_{b2}]\, y_2$: For gear made of Cast Iron.
(Static stress of gear).

From PSG. 8.53.

For $Z_2 = 72, \Rightarrow Y_2 = 0.360$

Allowable ~~Shear~~ Static stress for CT = $56\, N/mm^2$. (Assuming)
(σ_b) Taken from PSG. 8.5, Table: 7

$[\sigma_{b2}]\, y_2 = \frac{56 \times 0.360}{\pi}$

$$\boxed{[\sigma_{b2}]\, y_2 = 6.417}$$

$[\sigma_{b2}]\, y_2 < [\sigma_{b1}]\, y_1$, i.e. gear is weaker than pinion,

Hence we have to design gear only

Design of gear:

1. Material Selection:

Pinion = C30 Forged steel. (given)
Gear = Cast Iron. (given)

2. To calculate module: 'm'.

As the centre distance is given, we can find module directly with the relation,

$$a = \frac{m(Z_1 + Z_2)}{2} \quad \text{From PSG. 8.22.}$$

$$\therefore 375 = \frac{m(18 + 72)}{2}$$

Module $m = 8.33$. From 8.2 PSG.

Standard $\boxed{\text{module } m = 10\text{mm}}$

3. Calculation of b, d and v:

(PSG. 8.22)

Face width $b = 10 \times m = 100$ mm.

Pitch Dia of pinion $d_1 = m \cdot Z_1 = 10 \times 18 = 180$mm.

pitch O^le dia of gear $d_2 = m \cdot Z_2 = 10 \times 72 = 720$mm.

pitch line velocity $v = \frac{\pi d_2 N_2}{60}$ (considering gear)

$$v = \frac{\pi \times 720 \times 10^{-3} \times 300}{60}$$

$$\boxed{v = 11.31 \text{ m/s.}}$$

4. Calculation of beam strength (F_s) (or) Tooth Breakage:

Beam Strength, $F_s = \pi \cdot m \cdot b \cdot [\sigma_{b2}] y_2$ PSG. 8.50

$\because P_c = \pi d/z$.
$\therefore m = d/z$.

$F_s = \pi \times 10 \times 100 \times 6.417$.

$\boxed{F_s = \text{Beam Strength} = 20160\ N}$

5. Calculation of dynamic load (F_d).

Using Buckingham's dynamic load eqn PSG. 8.51.

$$F_d = F_t \left[\frac{0.164 V (Cb + F_t)}{0.164 V + 1.485 \sqrt{Cb + F_t}} \right]$$

We know, $F_t = P/V$. (PSG. 8.50).

$$F_t = \frac{15 \times 10^3}{11.31}$$ $\boxed{F_t = 1326.26\ N.}$

To calculate Deformation factor 'C'

From PSG. 8.53, Table 41 & 42.

$C = 7850e$, for $14\frac{1}{2}°$ Full depth, Steel & CI.

$e = 0.022$, for module 10 and precision gear.

$\therefore C = 7850 \times 0.022$, $\boxed{C = 172.7\ N/mm}$

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Dynamic load (Fd) $= 1326.26 + \left[\dfrac{11.31\times10^{+3}\times0.164\,(172.7\times100+1326.26)}{0.164\times11.31\times10^{+3}+1.485\sqrt{172.7\times100+1326.26}}\right]$

$= 1326.26 + \left[\dfrac{34493086.9}{2057.34}\right]$

$\boxed{Fd = 18092.12}$

Check for beam strength -

Beam strength Fs > Dynamic load Fd.

∴ The design is satisfactory.

6. Calculation of wear load (Fw).

wear load $Fw = d_1 \times b \times Q \times Kw$. From PSG. 8.51.

Q = Ratio factor $= \dfrac{2i}{i+1}$ PSG. 8.51.

$Q = \dfrac{2\times4}{4+1} = \boxed{1.6 = Q}$

Note:

Kw always less than 1 Mpa for 14.5°

Kw always less than 2Mpa for 20°.

Kw - wear factor $= 1\ N/mm^2$. for steel 250BHN & CI in $14\frac{1}{2}^\circ$ Full depth.

∴ wear load $Fw = 180\times100\times1.6\times1$

$\boxed{Fw = 28800N}$

Wear load Fw > Dynamic load Fd. Hence the design is safe.

Basic dimensions of pinion & gear.
From PSG 8·22

1. Module $m = 10\,mm$.
2. No of teeth: $Z_1 = 18$ & $Z_2 = 72$.
3. pitch c/l dia $d_1 = 180\,mm$ & $d_2 = 720\,mm$.
4. Centre distance $a = 375\,mm$.
5. Face width $b = 100\,mm$
6. Height factor $f_0 = 1$ for $14\frac{1}{2}°$ Full depth.
7. Bottom clearance $C = 0.25\,m = 0.25 \times 10 = 2.25\,mm$.
8. Tip diameter: $d_{a1} = (Z_1 + 2f_0)m = (18 + 2\times1)10 = 200\,mm$.

   $d_{a2} = (Z_2 + 2f_0)m = (72 + 2\times1)10 = 740\,mm$.

9. Root diameter: $d_{f1} = (Z_1 - 2f_0)m - 2C$

$$= (18 - 2\times1)10 - 2\times2.25$$

$$\boxed{d_{f1} = 155.5}$$

$$d_{f2} = (Z_2 - 2f_0)m - 2C$$

$$= (72 - 2\times1)10 - 2\times2.25$$

$$\boxed{d_{f2} = 695\,mm}$$