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Dimensionless numbers:

Dimensionless numbers are those numbers which are obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force..

These dimensionless numbers are also called non-dimensional parameters. Some of the important dimensionless numbers are:

1. Reynold's number
2. Froude's number
3. Euler's number
4. Weber's number
5. Mach's number.

Reynold's number: (Re)

Defined as the ratio of inertia force of a flowing fluid and the viscous force of the fluid.

Inertia force (F_i) = Mass × Acceleration of fluid

$= \rho \times V \times \frac{v}{time}$ $\quad Q = AV$

$= \rho \times \frac{V}{time} \times v$

$= \rho \times AV \times v \quad \left[\frac{V}{time} = Q = AV\right]$

$= \rho A v^2$

Viscous force (F_v) = Shear stress × Area

$= \tau \times A$

$= \left(\mu \frac{du}{dy}\right) \times A = \mu \times \frac{v}{L} \times A$

$= \mu \times \frac{V}{L} \times A$

$\therefore \quad Re = \frac{F_i}{F_v} = \frac{\rho A v^2}{\mu \times \frac{v}{L} \times A} = \frac{VxL}{\nu} = \frac{Vxd}{\nu} \quad \left[\nu = \frac{\mu}{\rho}\right]$

$\frac{\rho}{\mu} = \frac{1}{\nu}$

$$\boxed{Re = \frac{VxL}{\nu} = \frac{Vxd}{\nu} = \frac{\rho VL}{\mu} = \frac{\rho Vd}{\mu}}$$

Froude's Number : (Fe)

Defined as the square root of the ratio of inertia force of a flowing fluid to the gravity force.

$$Fe = \sqrt{\frac{Fi}{Fg}}$$

Fg = Mass × Acceleration

$= \rho \times$ Volume $\times g = \rho \times L^3 \times g$

$= \rho \times L^2 \times L \times g = \rho A L g$

$$Fe = \sqrt{\frac{\rho A V^2}{\rho A L g}} = \sqrt{\frac{V^2}{Lg}} = \frac{V}{\sqrt{Lg}}$$

$$\boxed{Fe = \frac{V}{\sqrt{Lg}}}$$

Euler's number : (Eu)

Defined as the square root of the ratio of the inertia force of a flowing fluid to the pressure force.

$$Eu = \sqrt{\frac{Fi}{Fp}}$$

$$= \sqrt{\frac{\rho A V^2}{P \times A}} = \sqrt{\frac{V^2}{\frac{P}{\rho}}} = \frac{V}{\sqrt{\frac{P}{\rho}}}$$

$$\boxed{Eu = \frac{V}{\sqrt{\frac{P}{\rho}}}}$$

Weber's Number : (We)

Defined as the square root of the ratio of the inertia force of a flowing fluid to the surface tension force.

$$We = \sqrt{\frac{Fi}{Fs}}$$

$$= \sqrt{\frac{\rho A V^2}{\sigma \times L}} = \sqrt{\frac{\rho L^2 V^2}{\sigma L}} = \sqrt{\frac{\rho L V^2}{\sigma}}$$

$$= \sqrt{\frac{V^2}{\frac{\sigma}{\rho L}}} = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

$$\boxed{We = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}}$$

Mach's Number: (M)

Defined as the square root of the ratio of the inertia force of a flowing fluid to the elastic force.

$$M = \sqrt{\frac{Fi}{Fe}} \qquad [K = \text{Elastic stress}]$$

$$= \sqrt{\frac{\rho A V^2}{K \times L^2}} = \sqrt{\frac{\rho \times L^2 \times V^2}{K L^2}} = \sqrt{\frac{\rho V^2}{K}} = \sqrt{\frac{V^2}{\frac{K}{\rho}}}$$

where $\sqrt{\frac{K}{\rho}} = C$ .. Velocity of sound in fluid

$$M = \frac{V}{\sqrt{\frac{K}{\rho}}} = \frac{V}{C}$$

$$\boxed{M = \frac{V}{C}} \rightarrow \boxed{C = \sqrt{\frac{K}{\rho}}}$$

MODEL LAWS (OR) SIMILARITY LAWS:

The laws on which the models are designed for dynamic similarity are called model laws or laws of Similarity. Followings are the model laws:

1. Reynold's model law
2. Froude model law
3. Euler model law
4. Weber model law
5. Mach model law

Reynold's model law:

It is the law in which models are based on Reynold's number. Applications: (i) pipe flow (ii) Resistance experienced by sub-marines, airplanes, etc.