UNIT-III

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Dimensional Analysis:

It is a method of dimensions. It is a mathematical technique used in research work for design and for conducting model tests.

It deals with the dimensions of the physical quantities involved. It uses three fixed dimensions in Fluid Mechanics viz Length L, mass M and time T.

Fixed dimensions are also known as fundamental dimensions.

Secondary or derived quantities:

These are those quantities which possess more than one fundamental dimension.

Determine the dimensions of the quantities below (i) Discharge (ii) kinematic viscosity (iii) Force (iv) specific weight (v) Dynamic viscosity.

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Discharge:

$$Q = a \times V$$

$$= L^2 \times \frac{m}{s}$$

$$= L^2 L T^{-1} = L^3 T^{-1}$$

Kinematic viscosity:

$$\nu = \frac{\mu}{\rho}$$

$$\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\frac{F}{A}}{\frac{L/T}{L}} = \frac{\frac{M \times \frac{L}{T^2}}{L^2}}{\frac{L}{T} \times \frac{1}{L}} = \frac{M\frac{L}{T^2} \times \frac{1}{L^2}}{\frac{1}{T}}$$

$$= \frac{ML}{T^2} \times \frac{1}{L} \times T = ML^{-1}T^{-1}$$

$$\rho = \frac{M}{V} = \frac{M}{L^3} = ML^{-3}$$

$$\nu = \frac{ML^{-1}T^{-1}}{ML^{-3}} = ML^{-1}T^{-1}M^{-1}L^{3}$$

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iii) Force = Mass × Acceleration

Mass = M

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$$\text{Acceleration} = \frac{\text{metre}}{\text{sec}^2} = \frac{L}{T^2}$$

$$\therefore \text{Force} = M \times \frac{L}{T^2} = MLT^{-2}$$

(iv) Specific Weight = $\frac{\text{Weight}}{\text{Volume}}$

Weight = Mass × Gravity (acceleration) ;

$$= M \times \frac{m}{\sec^2} = MLT^{-2}$$

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Volume $= L^3$

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$$\text{Sp. Weight} = MLT^{-2}L^{-3}$$

$$= ML^{-2}T^{-2}$$

(v) Dynamic viscosity = $\mu = \frac{\tau}{\frac{du}{dy}} = \frac{\frac{F}{A}}{\frac{m}{\frac{T}{L}}} = \frac{\frac{M \times \frac{L}{T^2}}{L^2}}{\frac{\frac{M}{T}}{L}}$

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$$= \frac{M \times \frac{L}{T^2} \times \frac{1}{L^2}}{\frac{L}{T} \times \frac{1}{L}} = \frac{MT^{-2}L^{-1}}{T^{-1}} = MT^{-2}L^{-1}T^{1}$$

$$= MT^{-1}L^{-1}$$

Dimensional Homogenity :

It means the dimensions of each terms in an equation on both sides equal.

If the dimensions of each term on both sides of an equation are the same, the equation is known as dimensionally homogeneous equation. The powers of fundamental dimensions (i.e. L, M, T) on both sides of the equation will be identical for a dimensionally homogeneous equation

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Prove that $V = \sqrt{2gh}$ is a dimensionally home. equation.

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$$V = \sqrt{2gh}$$

LHS, $V = \frac{L}{T} = LT^{-1}$

RHS $\sqrt{2gh} = \sqrt{\frac{L}{T^2} \times L} = \sqrt{\frac{L^2}{T^2}} = \frac{L}{T} = LT^{-1}$

LHS = RHS, hence it is a dimensionally homogeneous equation.

Methods of dimensional Analysis:

Two methods of dimensional analysis

1. Rayleigh's method and
2. Bukingham's π - theorem.

Rayleigh's method:

This method is used for determining the expression for a variable which depends upon maximum three or four variables only.

Let X is a variable, which depends on X_1, X_2, X_3 variables.

Mathematically, $\boxed{X = f[X_1, X_2, X_3]}$

also

$$\boxed{X = K X_1^a . X_2^b . X_3^c}$$

K - constant; a, b, c - arbitarily powers.

Find the expression for the Power P, developed by a pump depends upon the head H, the discharge Q and specific weight w of the fluid.

Power = Head 'H', Discharge 'Q', specific weight 'w'

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$$\boxed{P = K H^a . Q^b . W^c} \leftarrow (I)$$

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$$\text{Power} = \frac{\overset{MLT^{-2} \times L}{\text{Force} \times \text{distance}}}{\text{time } T} = \frac{\text{Workdone}}{\text{time}} = ML^2T^{-3}$$

$$\text{Head} = L$$

$$\text{Discharge} = \text{Area} \times \text{Velocity} = L^2 \times L \times T^{-1}$$
$$= L^3T^{-1}$$

$$\text{Specific Weight} = \frac{\text{Weight}}{\text{Volume}} = \frac{\text{Mass} \times \text{Acceleration}}{\text{volume}}$$

$$= MLT^{-2}L^{-3}$$
$$= ML^{-2}T^{-2}.$$

$$ML^2T^{-3} = KL^a.(L^3T^{-1})^b.(ML^{-2}T^{-2})^c$$

Power of M: $1 = c$

Power of L: $2 = a + 3b - 2c \rightarrow (1)$

Power of T: $-3 = -b - 2c \rightarrow (2)$

$c = 1$ in (2)

$b = 3 - 2c$
$= 3 - 2$
$\boxed{b = 1}$

$c = 1$ and $b = 1$ in (1)

$a + 3 - 2 = 2$
$a + 1 = 2$
$a = 2 - 1$
$\boxed{a = 1}$

Substituting all in (I)

$P = KH^1 . Q^1 . W^1$

$\boxed{P = KHQW}$

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The time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time period.

Solution:

Time period (t) = Length (L), Acceleration (g)

$$\boxed{t = KL^a . g^b} \longleftarrow \text{I}$$

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Time period = T

Length = L

Acceleration = LT^{-2}

$$T = KL^a . (LT^{-2})^b$$

Power of T $\quad 1 = -2b \longrightarrow$ 1

Power of L $\quad 0 = a + b \longrightarrow$ 2

Taking 1

$$\boxed{b = -\frac{1}{2}}$$

Substituiting $\boxed{b = -\frac{1}{2}}$ in 2

$$0 = a - \frac{1}{2}$$

$$\boxed{a = \frac{1}{2}}$$

Substituting $a = \frac{1}{2}$ and $b = -\frac{1}{2}$ in I

$$t = KL^{\frac{1}{2}} . g^{-\frac{1}{2}}$$

$$\boxed{t = K\sqrt{\frac{L}{g}}}$$

From experiment $K = 2\pi$

$$\therefore \boxed{t = 2\pi\sqrt{\frac{L}{g}}}$$

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Find an expression for the drag force on smooth sphere of diameter D, moving with a uniform velocity V in a fluid of density ρ and dynamic viscosity μ.

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Given data:

Drag Force = Sphere Diameter D, Uniform velocity V, Density ρ, Dynamic viscosity μ

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Solution:

$$\boxed{F = K D^a . V^b . \rho^c . \mu^d} \leftarrow \text{I}$$

Force $= MLT^{-2}$

Diameter $= L$

Velocity $= \cancel{M}LT^{-1}$

Density $= ML^{-3}$

Viscosity $= ML^{-1}T^{-1}$

$$MLT^{-2} = K L^a . (LT^{-1})^b . (ML^{-3})^c . (ML^{-1}T^{-1})^d$$

Power of M $\quad 1 = c + d \rightarrow (1)$

Power of L $\quad 1 = a + b - 3c - d \rightarrow (2)$

Power of T $\quad -2 = -b - d \rightarrow (3)$

[4 Variable = 3 equation not possible for solution]

$\boxed{c = 1 - d} \rightarrow$ from (1)

$\boxed{b = 2 - d} \rightarrow$ from (3)

$a = 1 - b + 3c + d$

$= 1 - 2 + d + \cancel{3c} 3(1-d) + d$

$= 1 - 2 + d + 3 - 3d + d$

$\boxed{a = 2 - d}$

$$F = K D^{2-d} . V^{2-d} . \rho^{1-d} . \mu^d$$

$$= K D^2 . V^2 . \rho^1 (D^{-d} . V^{-d} . \rho^{-d} . \mu^d)$$

$$\boxed{F = K \rho D^2 V^2 \left(\frac{\mu}{D V \rho}\right)^d}$$ //

Buckingham's π theorem

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It states, "If there are n variables (independent and dependent variables) in a physical phenomenon and if these variables contains m fundamental dimensions (M, L, T), then the variables are arranged into $(n-m)$ dimensionless terms. Each term is called π-term".

Let X_1 = dependent variable

$X_2, X_3 \ldots X_n$ = independent variables, then

$$\boxed{X_1 = f(X_2, X_3 \ldots X_n)}$$

Can also be written as

$$\boxed{f(X_1, X_2, X_3, \ldots X_n) = 0.}$$

According to Buckingham's π theorem it can be written as

$$\boxed{f_1(\pi_1, \pi_2, \ldots \pi_{n-m}) = 0} \quad (1)$$

∴ Each π term contains $m+1$ variables, where m is the number of fundamental dimensions and also called as repeating variables.

$$\left.\begin{aligned} \pi_1 &= X_2^{a_1} \cdot X_3^{b_1} \cdot X_4^{c_1} \cdot X_1 \\ \pi_2 &= X_2^{a_2} \cdot X_3^{b_2} \cdot X_4^{c_2} \cdot X_5 \\ \pi_{n-m} &= X_2^{a_{n-m}} X_3^{b_{n-m}} \cdot X_4^{c_{n-m}} \cdot X_n \end{aligned}\right\} \text{Equations}$$

Each equation is solved by the principle of dimensional homogenity and values of a_1, b_1, c_1, etc are obtained. Then these values are substituted in equation and values of $\pi_1, \pi_2, \pi_3 \ldots \pi_{n-m}$ are obtained. These values of π's are then substituted in equation (1). Final equation is as follows

$$\pi_1 = \phi\,[\pi_2, \pi_3, \ldots \pi_{n-m}]$$

$$\pi_2 = \phi_1\,[\pi_1, \pi_3, \ldots \pi_{n-m}]$$

Method of selecting repeating variables:
(Rules)

1) No. of repeating variables are equal to the no. of fundamental dimensions of the problem.
2) Dependent variable should not be selected.
3) It should be selected in such a way that one variable contains geometric property, other variable contains flow property and third variable contains fluid property.

Geometric property:
(i) Length (ii) diameter. (iii) height, etc

Flow property:
(i) velocity (ii) acceleration, (iii) discharge

Fluid property:
(i) viscosity (ii) density (iii) specific weight

4) should not form a dimensionless group (should contain all three fundamental dimension)
5) No two repeating variables should have same dimensions.

Examples: (i) d, v, ρ (iii) l, v, ρ
(ii) d, v, μ (iv) l, v, μ.

The resisting force R depends upon the length of the aircraft l, velocity v, air viscosity, μ, air density ρ and bulk modulus of air K. Express the functional relationship between these variables and the resisting force

Solution:

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$$R = f(l, v, \mu, \rho, K)$$

$$f(R, l, v, \mu, \rho, K) = 0$$

Step 1

Total no. of variables = 6 (n)
no. of fundamental dimensions = 3 (m)

No. of dimensionless π-terms $= (n-m) = 6-3 = 3$. Hence

$$f_1(\pi_1, \pi_2, \pi_3) = 0.$$

Each π-term = $m+1$

where m = repeating variables.

R, l, V, μ, e and K. — total variable

l, V, e — repeating variables.

Step 2

Now it can be written as.

$$\left.\begin{aligned} \pi_1 &= l^{a_1}.\, V^{b_1}.\, e^{c_1}.\, R \\ \pi_2 &= l^{a_2}.\, V^{b_2}.\, e^{c_2}.\, \mu \\ \pi_3 &= l^{a_3}.\, V^{b_3}.\, e^{c_3}.\, K \end{aligned}\right\}$$

Step 3

Each π-term is solved by the principle of homogenity

$\pi_1 = M^0L^0T^0 = L^{a_1}.\, (LT^{-1})^{b_1}.\, (ML^{-3})^{c_1}.\, M.LT^{-2}$

Power of $M = 0 = c_1 + 1$ — ① $\therefore c_1 = -1$

Power of $L = 0 = a_1 + b_1 - 3c_1 + 1$ — ②

Power of $T = 0 = -b_1 - 2$ — ③ $\therefore b_1 = -2$

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Substituting in ②

$a_1 - 2 - 3(-1) + 1 = 0$

$a_1 - 2 + 3 + 1 = 0$

$a_1 + 2 = 0$

$a_1 = -2$

Step 4

Now in $\pi_1 = l^{-2}.\, V^{-2}.\, e^{-1}.\, R.$

$$\boxed{\pi_1 = \frac{R}{l^2V^2e} = \frac{R}{el^2V^2}}$$

Now π_2,

$\pi_2 = l^{a_2}.\, V^{b_2}.\, e^{c_2}.\, \mu$

$\pi_2 = M^0L^0T^0 = L^{a_2}.\, (LT^{-1})^{b_2}.\, (ML^{-3})^{c_2}.\, ML^{-1}T^{-1}$

Power of $M = 0 = c_2 + 1 \rightarrow$ ① $\therefore \boxed{c_2 = -1}$

" " $L = 0 = a_2 + b_2 - 3c_2 - 1 \rightarrow$ ②

" " $T = 0 = -b_2 - 1 \rightarrow$ ③ $\therefore \boxed{b_2 = -1}$

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$a_2 + b_2 - 3c_2 - 1 = 0$

$a_2 - 1 - 3(-1) - 1 = 0$

$a_2 - 1 + 3 - 1 = 0$

$a_2 + 1 = 0$

$\boxed{a_2 = -1}$

Step 4

$\pi_2 = l^{-1} \cdot v^{-1} \cdot \rho^{-1} \cdot \mu$

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$$\boxed{\pi_2 = \frac{\mu}{l v \rho}}$$

Now π_3,

$ML^{-1}T^{-2}$

$\pi_3 = M^0 L^0 T^0 = L^{a_3} \cdot (LT^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot K$

Power of $M = 0 = c_3 + 1 \to (1)$ $\therefore c_3 = -1$

" " $L = 0 = a_3 + b_3 - 3c_3 - 1 \to (2)$

$T = 0 = -b_3 - 2 \to (3)$ $\therefore b_3 = -2$

$c_3 = -1$, $b_3 = -2$ in (2)

$a_3 - 2 - 3(-1) - 1 = 0$

$a_3 - 2 + 3 - 1 = 0$

$a_3 + 0 = 0$

$\boxed{a_3 = 0}$

change in pressure / volume strain

$= \frac{MLT^{-2}}{L^2}$

$= MLT^{-2}L^{-2}$

$= ML^{-1}T^{-2}$

Step 4

$\pi_3 = l^0 \cdot v^{-2} \cdot \rho^{-1} \cdot K$

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$$\boxed{\pi_3 = \frac{K}{v^2 \rho} = \frac{K}{\rho v^2}}$$

We know that,

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

$$f_1\left(\frac{R}{\rho l^2 v^2}, \frac{\mu}{l v \rho}, \frac{K}{\rho v^2}\right) = 0$$

$$\frac{R}{\rho l^2 v^2} = \phi\left[\frac{\mu}{l v \rho}, \frac{K}{\rho v^2}\right] \Rightarrow \boxed{R = \rho l^2 v^2 \phi\left[\frac{\mu}{l v \rho}, \frac{K}{\rho v^2}\right]}$$

Using Bukingham's π- theorem, show that the velocity through a circular orifice is given by $V = \sqrt{2gH}\ \phi\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]$, where H is the head causing flow, D is the diameter of the orifice, μ is co-efficient of viscosity, ρ is the mass density and g is the acceleration due to gravity.

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Solution

$$V = f(H, D, \mu, \rho, g)$$

$$f(V, H, D, \mu, \rho, g) = 0$$

Total no of variables 'n' = 6.

Total no of funda. dim 'm' = 3

$\therefore$ Total no. of dimensionless π-terms $= (n-m) = 6-3 = 3$.

Hence $f_1(\pi_1, \pi_2, \pi_3) = 0$

Selection of repeating variables.

H, g, ρ — repeating variables.

Equations:

$$\left.\begin{aligned} \pi_1 &= H^{a_1} \cdot g^{b_1} \cdot \rho^{c_1} \cdot V \\ \pi_2 &= H^{a_2} \cdot g^{b_2} \cdot \rho^{c_2} \cdot D \\ \pi_3 &= H^{a_3} \cdot g^{b_3} \cdot \rho^{c_3} \cdot \mu \end{aligned}\right\}$$

π- terms

$$\pi_1 = M^0L^0T^0 = L^{a_1} \cdot (LT^{-2})^{b_1} \cdot (ML^{-3})^{c_1} \cdot LT^{-1}$$

$$(ML^{-3})^{c_1} \longrightarrow M^{c_1} L^{-3c_1}$$

Power of $M = 0 = c_1 \rightarrow (1)$

" " $L = 0 = a_1 + b_1 - 3c_1 + 1 \rightarrow (2)$

$T = 0 = -2b_1 - 1 \rightarrow (3)$ $\therefore b_1 = -\frac{1}{2}$

$2b_1 = -1$

$\Rightarrow b_1 = -\frac{1}{2}$

$c_1 = 0$ & $b_1 = -\frac{1}{2}$ in (2)

$a_1 - \frac{1}{2} + 1 = 0$

$$\boxed{a_1 = -\frac{1}{2}}$$

$$\pi_1 = H^{-\frac{1}{2}} \cdot g^{-\frac{1}{2}} \cdot \rho^0 \cdot V^1$$

$$\boxed{\pi_1 = \frac{V}{\sqrt{gH}}}$$

π_2 term

$$\pi_2 = M^0L^0T^0 = L^{a_2} \cdot (LT^{-2})^{b_2} \cdot (ML^{-3})^{c} \cdot L$$

Power of $M = 0 = c \rightarrow$ 1

" " $L = 0 = a_2 + b_2 - 3c + 1 \rightarrow$ 2

" " $T = 0 = -2b_2 \rightarrow$ 3

$c = 0$ & $b = 0$ in 2

$$a_2 + 1 = 0$$

$$\boxed{a_2 = -1}$$

$$\pi_2 = H^{-1} \cdot g^0 \cdot \rho^0 \cdot D$$

$$\boxed{\pi_2 = \frac{D}{H}}$$

π_3 term,

$$\pi_3 = M^0L^0T^0 = L^{a_3} \cdot (LT^{-2})^{b_3} \cdot (ML^{-3})^{c} \cdot ML^{-1}T^{-1}$$

Power of $M = 0 = +c + 1 \rightarrow$ 1 $\quad c = -1$

" " $L = 0 = a_3 + b_3 - 3c_3 - 1 \rightarrow$ 2

" " $T = 0 = -2b_3 - 1 \rightarrow$ 3 $\quad b_3 = -\frac{1}{2}$

$c = 1$ & $b = -\frac{1}{2}$ in 2

$$a_3 - \frac{1}{2} - 3 - 1 = 0$$

$$a_3 - \frac{1}{2} - 4 = 0$$

$$a_3 = 4 + \frac{1}{2}$$

$$a_3 = \frac{9}{2}$$

$$a_3 = -b_3 + 3c_3 + 1$$

$$= -\left(-\frac{1}{2}\right) - 3 + 1$$

$$= \frac{1}{2} - 2$$

$$= \frac{1-4}{2} = -\frac{3}{2}$$

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$$\pi_3 = H^{-\frac{3}{2}} \cdot g^{-\frac{1}{2}} \cdot \rho^{-1} \cdot \mu$$

$$\pi_3 = H^{-1} \cdot H^{-\frac{1}{2}} \cdot g^{-\frac{1}{2}} \cdot \rho^{-1} \cdot \mu$$

$$\boxed{\pi_3 = \frac{\mu}{H\rho\sqrt{Hg}}}$$

Multiplying by V
& Divide

$$\pi_3 = \frac{\mu V}{H\rho V\sqrt{Hg}} \qquad \left[\because \pi_1 = \frac{V}{\sqrt{Hg}}\right]$$

$$\boxed{\pi_3 = \frac{\mu}{H\rho V} \cdot \pi_1}$$

Now,

$$f_1\left(\frac{V}{\sqrt{Hg}}, \frac{D}{H}, \frac{\mu}{H\rho V} \cdot \pi_1\right) = 0$$

$$\frac{V}{\sqrt{Hg}} = \phi\left[\frac{D}{H}, \pi_1 \cdot \frac{\mu}{H\rho V}\right]$$

$$\boxed{V = \sqrt{2gH}\ \phi\left[\frac{D}{H}, \frac{\mu}{\rho V H}\right]}$$

Using Bukingham's π-Theorem, show the discharge Q consumed by an oil ring is given by (70)

$$Q = Nd^3\phi\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{\rho N^2 d^3}, \frac{w}{\rho N^2 d}\right]$$

where d is the internal diameter of the ring, N is rotational speed, ρ is density, σ is surface tension and w is the specific ~~speed~~ weight of oil.

Solution:

$$Q = f(d, N, \rho, \sigma, \mu, w)$$

$$f(Q, d, N, \rho, \sigma, \mu, w) = 0$$

Total no. of variables $n = 7$

No. of fundamental dimensions $= m = 3$

No. of dimensionless π term $= n - m = 4$

$\therefore f_1 [\pi_1, \pi_2, \pi_3, \pi_4] = 0$

Selection of repeating variables:

d, N, ρ

$$\therefore \left.\begin{aligned} \pi_1 &= d^{a_1}, N^{b_1}, \rho^{c_1} . Q \\ \pi_2 &= d^{a_2}, N^{b_2}, \rho^{c_2} . \mu \\ \pi_3 &= d^{a_3}, N^{b_3}, \rho^{c_3} . \sigma \\ \pi_4 &= d^{a_4}, N^{b_4}, \rho^{c_4} . \omega \end{aligned}\right\} \text{equations}$$

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π_1 term:

$\pi_1 = M^0L^0T^0 = L^{a_1} . (T^{-1})^{b_1} . (ML^{-3})^{c_1} L^3T^{-1}$

Power of $M = 0 = c_1 \rightarrow (1) \therefore c_1 = 0$

" " $L = 0 = a_1 - 3c_1 + 3 \rightarrow (2) \therefore a_1 = -3$

" " $T = 0 = -b_1 - 1 \rightarrow (3) \therefore b_1 = -1$

$\pi_1 = d^{-3} . N^{-1} . \rho^0 . Q$

$$\boxed{\pi_1 = \frac{Q}{d^3 N}}$$

π_2 term:

$\pi_2 = M^0L^0T^0 = L^{a_2} . (T^{-1})^{b_2} . (ML^{-3})^{c_2} ML^{-1}T^{-1}$

Power of $M = 0 = c_2 + 1 \rightarrow (1) \therefore c_2 = -1$

" " $L = 0 = a_2 - 3c_2 - 1 \rightarrow (2) \therefore a_2 = -2$

" " $T = 0 = -b_2 - 1 \rightarrow (3) \therefore b_2 = -1$

$\pi_2 = d^{-2} . N^{-1} . \rho^{-1} . \mu$

$$\boxed{\pi_2 = \frac{\mu}{d^2 N . \rho}}$$

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Π_3 term:

$$\Pi_3 = M^0L^0T^0 = L^{a_3} \cdot (T^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot MT^{-2}$$ $\left| MLT^{-2} \; \frac{N}{m} \right.$

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Power of $M = 0 = c_3 + 1 \rightarrow (1) \therefore c_3 = -1$

" " $L = 0 = a_3 - 3c_3 \rightarrow (2) \; a_3 = -3$

" " $T = 0 = -b_3 - 2 \rightarrow (3) \therefore b_3 = -2$

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$$\Pi_3 = d^{-3} \cdot N^{-2} \cdot \rho^{-1} \cdot \sigma .$$

$$\boxed{\Pi_3 = \frac{\sigma}{d^3 N^2 \rho}}$$

Π_4 term:

$$\Pi_4 = M^0L^0T^0 = L^{a_4} \cdot (T^{-1})^{b_4} (ML^{-3})^{c_4} \cdot ML^{-2}T^{-2}$$

Power of $M = 0 = c_4 + 1 \rightarrow (1) \quad \therefore c_4 = -1$

" " $L = 0 = a_4 - 3c_4 - 2 \rightarrow (2) \therefore a_4 = -1$

" " $T = 0 = -b_4 - 2 \rightarrow (3) \quad \therefore b_4 = -2$

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$$\Pi_4 = d^{-1} \cdot N^{-2} \cdot \rho^{-1} \cdot \omega$$

$$\boxed{\Pi_4 = \frac{\omega}{d N^2 \rho}}$$

Substituting values of Π's in Bukingham's π equation,

$$f_1(\Pi_1, \Pi_2, \Pi_3, \Pi_4) = f_1\left(\frac{Q}{d^3N}, \frac{\mu}{d^2N\rho}, \frac{\sigma}{d^3N^2\rho}, \frac{\omega}{dN^2\rho}\right) = 0$$

$$\frac{Q}{Nd^3} = f_1\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{\rho N^2 d^3}, \frac{\omega}{\rho N^2 d}\right]$$

$$\boxed{Q = Nd^3 \cdot \phi\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{\rho N^2 d^3}, \frac{\omega}{\rho N^2 d}\right]}$$

Model Analysis:

For predicting the performance of the hydraulic structures (such as dams, spill ways, etc.) or hydraulic machines (such as turbines, pumps, etc), before actually constructing or manufacturing, models of the structures or machines are made and tests are performed on them to obtain the desired information.

The study of these models of actual machines is called model analysis. It is actually an experimental method of finding solutions of complex flow problems.

Model:

A model is the small replica of the actual structure or machine.
- undistorted
- distorted

Prototype:

The actual structure or machine is called prototype.

It is not necessary that the models should be smaller than the prototype, they may be larger than the prototype too.

Advantages of dimensional and model analysis:

1. The performance of structure or machine can be easily predicted, in advance.
2. With the help of dimensional analysis, a relationship between the variables influencing a flow problem in terms of dimensionless parameters is obtained. This is used in conducting tests.
3. Merits of alternative designs can be analysed. The most ~~and~~ economical design can be adopted.
4. Used to get useful information about the prototype