SNS COLLEGE OF ENGINEERING
(Autonomous)
DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

19EC502 – TRANSMISSION LINES AND ANTENNAS

III YEAR/ V SEMESTER

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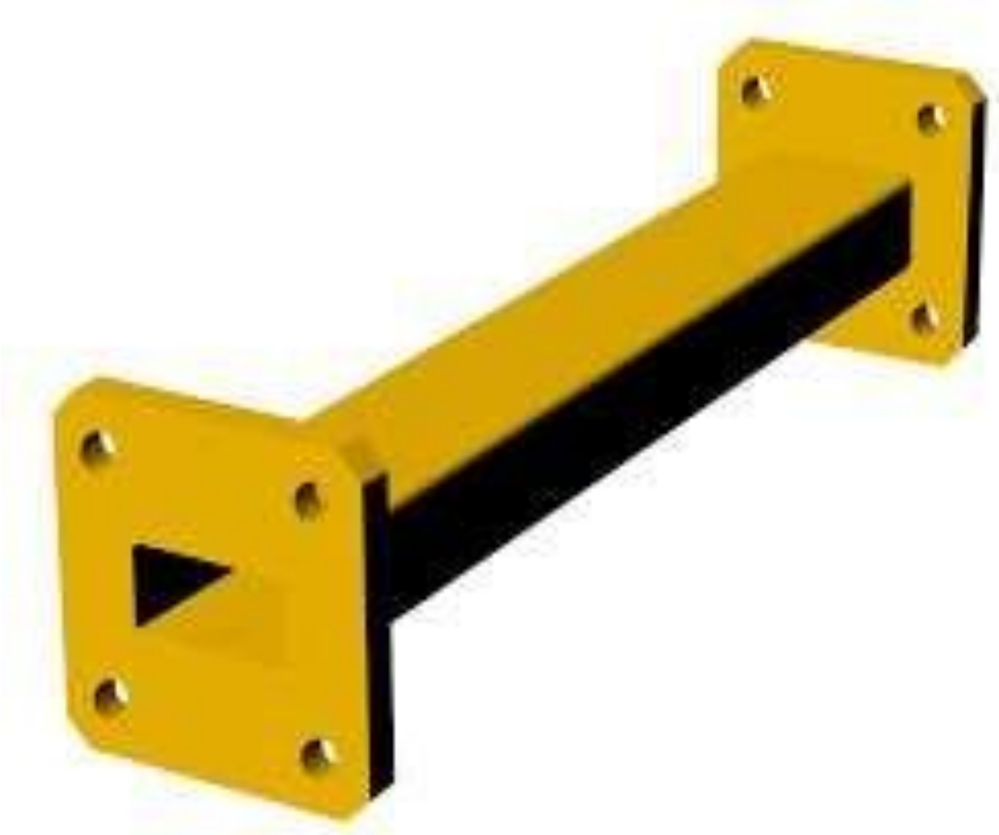
UNIT 2 – GUIDED WAVES

TOPIC 5 – TM WAVES IN RECTANGULAR WAVEGUIDES

CAN YOU RELATE THE FIGURES?



WAVEGUIDES

- A waveguide is a special form of transmission line consisting of a hollow, metal tube.
- The tube wall provides distributed inductance, while the empty space between the tube walls provide distributed capacitance.
- Waveguides are practical only for signals of extremely high frequency, where the wavelength approaches the cross-sectional dimensions of the waveguide.
- Below such frequencies, waveguides are useless as electrical transmission lines.

TYPES

Rectangular waveguides

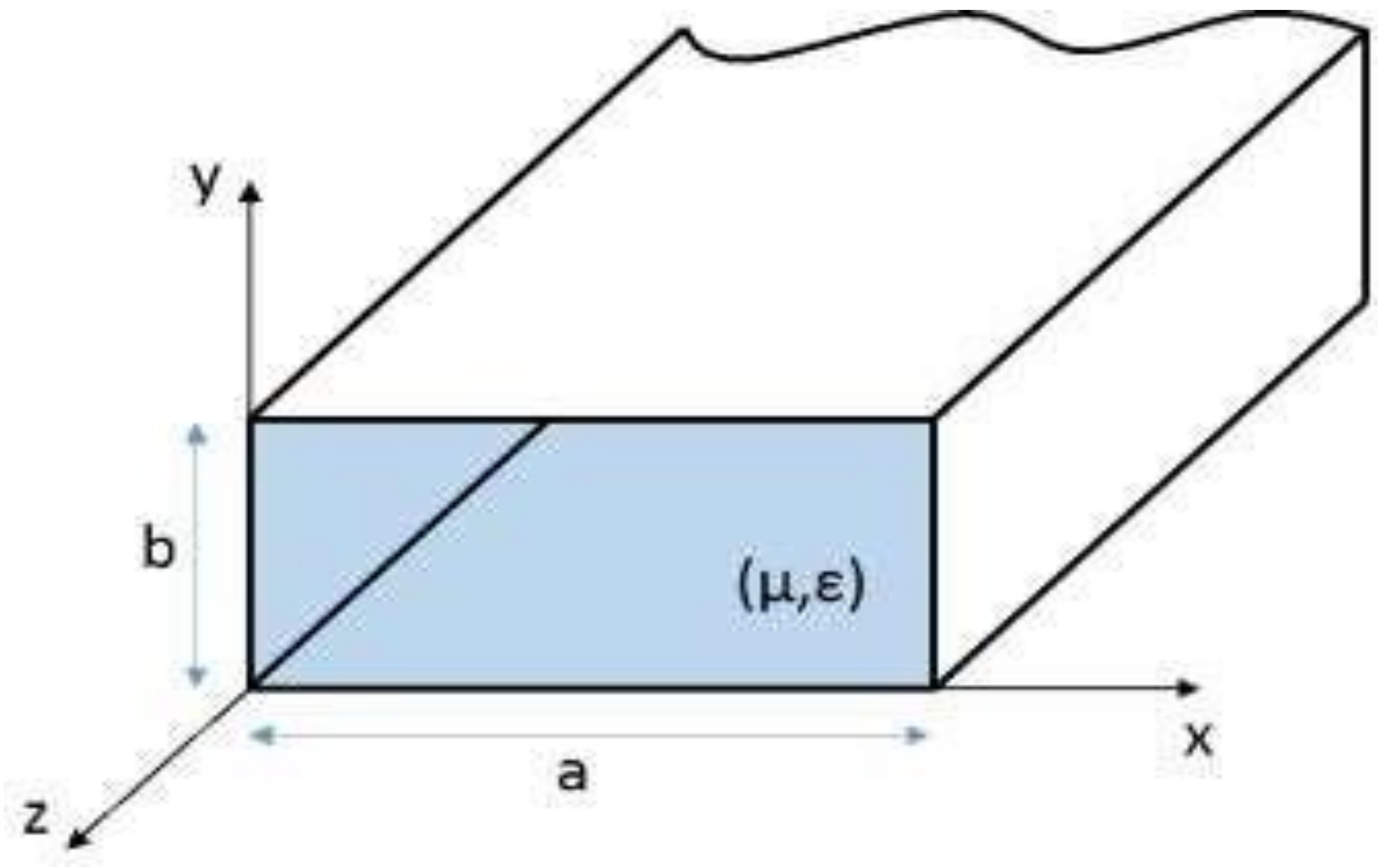
- Most preferred type
- Frequency difference between consecutive modes - smaller

Circular Waveguides

- Less Preferable
- Polarization not maintained – circular symmetry

RECTANGULAR WAVEGUIDE

- Consider a rectangular waveguide of width “a” and breath “b”.
- An EM wave is assumed to propagate in z direction as shown in fig.

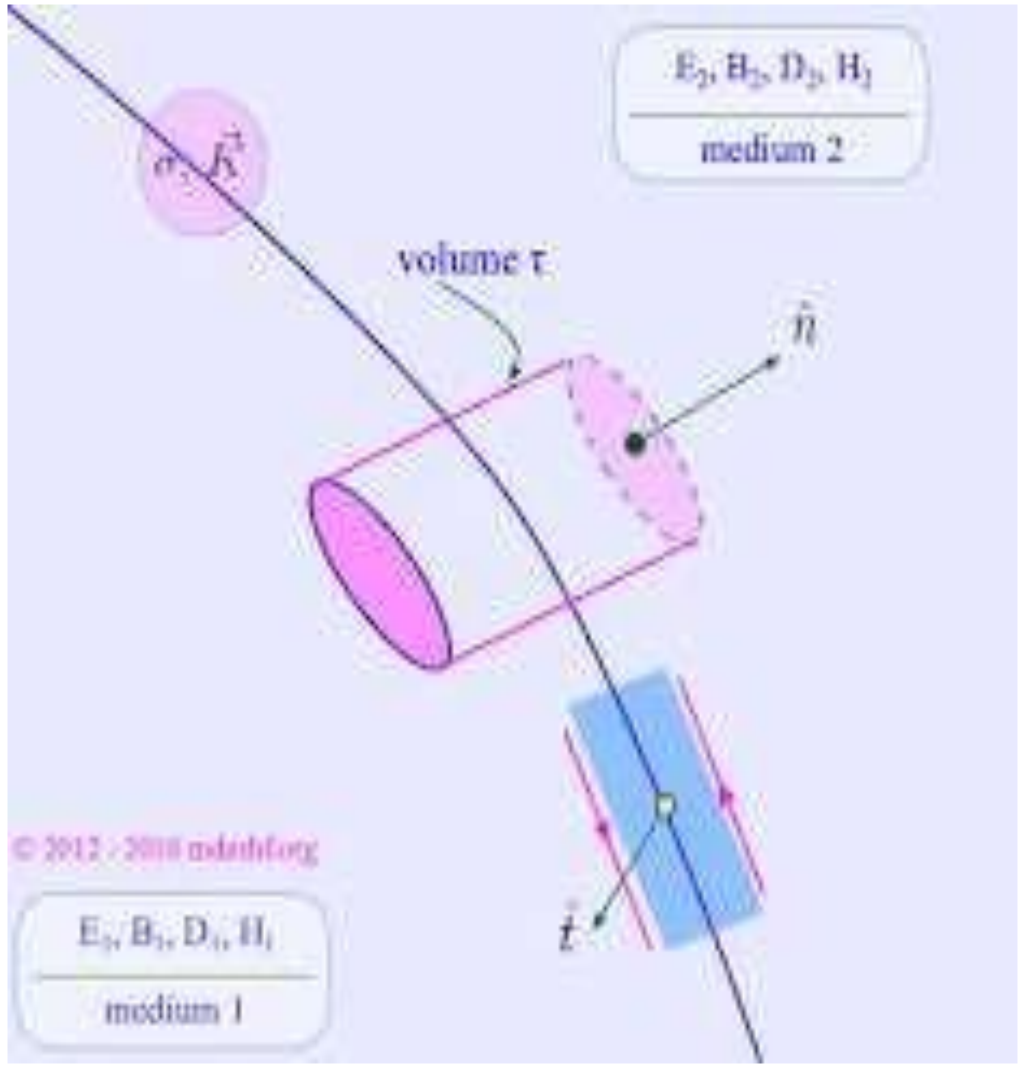


ANALYSIS

- Boundary condition

Ex=Ez=0 at y=0 and y=b

Ey=Ez=0 at x=0 and x=a



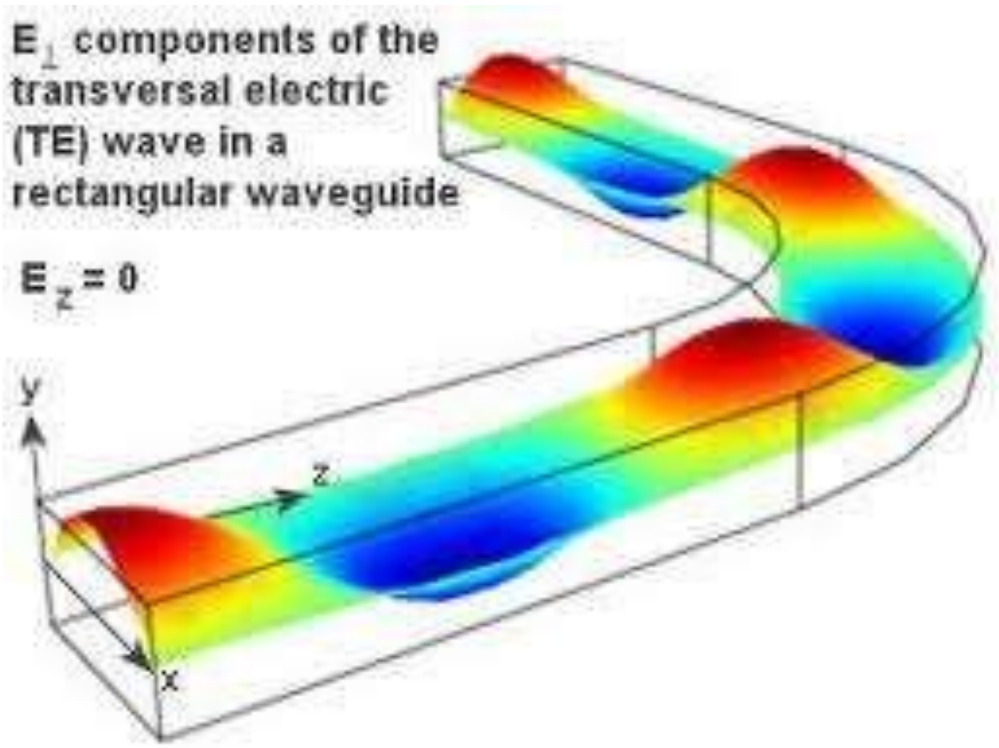
ANALYSIS

- Basic equations from Maxwells curl equations

Maxwell's Equation

$$\nabla^2 E + \omega^2 \mu\varepsilon E = 0$$

$$\nabla^2 H + \omega^2 \mu\varepsilon H = 0$$



ANALYSIS

From $\nabla \times \mathbf{E} = -\mu \dfrac{\partial \mathbf{H}}{\partial t}$, we have

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t} \Rightarrow + j\beta_z E_y = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t} \Rightarrow - j\beta_z E_x = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu \frac{\partial H_z}{\partial t} \Rightarrow \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

ANALYSIS

From $\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$, we get

$$j\omega\varepsilon\mathbf{E} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

ANALYSIS

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \Rightarrow \frac{\partial H_z}{\partial y} + j\beta_z H_y = j\omega\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y \Rightarrow -j\beta_z H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0$$

ANALYSIS

TM WAVES ($H_z = 0$)

H_x & $E_y = 0$

E_z, E_x & H_y will have value.

$$H_y = (C_3 \sin hx + C_4 \cos hx)\, e^{-\bar{\gamma} z}$$

→ B.C can not be applied directly to Evaluate C_3 & C_4.

→ Because tangential component of $H_y \neq 0$ at the surface of the perfect conductors.

ANALYSIS

$$E_z = \frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial x}$$

$$= \frac{1}{j\omega\varepsilon}\frac{\partial}{\partial x}\left[C_3 \sin hx + C_4 \cos hx\right] e^{-\bar{\gamma} z}$$

$$E_z = \frac{h}{j\omega\varepsilon}\left[C_3 \cos hx - C_4 \sin hx\right] e^{-\bar{\gamma} z}$$

Applying B.C I

$$\boxed{C_3 = 0}$$

ANALYSIS

Therefore E_z becomes

$$E_z = -\frac{c_4 h}{j\omega\varepsilon} \sin hx \, e^{-\bar{\gamma} z}$$

Applying B.C II

$E_z = 0$ at $x = a$

$$E_z = -\frac{a c_4}{j\omega\varepsilon} \sin ha \, e^{-\bar{\gamma} z}$$

To make $E_z = 0$ at $x = a$,

$$\boxed{h = \frac{m\pi}{a}}$$

ANALYSIS

Subs

$$E_z = -\frac{m\pi}{a}\frac{C_4}{j\omega\varepsilon}\sin\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\gamma} z}$$

$$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$$

$$H_y = C_4 \cos\left(\frac{m\pi}{a}\right)x\, e^{-\bar{\gamma} z}$$

$\parallel^{ly}$ other fields are obtained.

ANALYSIS

subs $\bar{\gamma} = j\bar{\beta}$ for wave propagation

$$E_z = -\frac{m\pi}{a} \frac{C_4}{j\omega\varepsilon} \sin\left(\frac{m\pi}{a}\right)x \, e^{-j\bar{\beta}z}$$

$$H_y = C_4 \cos\left(\frac{m\pi}{a}\right)x \, e^{-j\bar{\beta}z}$$

$$E_x = \frac{\bar{\beta}}{\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}\right)x \, e^{-j\bar{\beta}z}$$

The above equations are fields of TM waves between parallel planes.