



TOPIC : 7 – HALF RANGE COSINE SERIES

Half range cosine series in the interval  $(0, \pi), (0, l)$

Formula:  $(0, \pi)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{b-a} \int_a^b f(x) dx$$

$$a_n = \frac{2}{b-a} \int_a^b f(x) \cos nx dx$$

Formula:  $(0, l)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$$

$$a_0 = \frac{2}{l} \int_0^l f(x) dx$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l} dx$$

(1) Find the half range cosine series of  $f(x) = x$  in  $(0, \pi)$ .

Sol:

$$f(x) = x.$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x dx$$

$$= \frac{2}{\pi} \left( \frac{x^2}{2} \right)_0^{\pi}$$



$$a_n = \frac{2}{\pi} \int_0^{\pi} x \cos nx \, dx$$

$$= \frac{2}{\pi} \left[ x \frac{\sin nx}{n} + \frac{\cos nx}{n^2} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[ \frac{(-1)^n}{n^2} - \frac{1}{n^2} \right]$$

$$a_n = \frac{2}{n^2 \pi} [-1 + (-1)^n]$$

$$a_n = \begin{cases} \frac{-4}{n^2 \pi} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$$

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2 \pi} [1 - (-1)^n] \cos nx$$

$$f(x) = \frac{\pi}{2} + \sum_{n=\text{odd}} \frac{-4}{n^2 \pi} \cos nx$$

2. Find the half range cosine series of the function  $f(x) = x(\pi - x)$  in  $(0, \pi)$ .

Sol:

$$f(x) = x\pi - x^2$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} (x\pi - x^2) \, dx$$

$$= \frac{2}{\pi} \left[ \frac{x^2 \pi}{2} - \frac{x^3}{3} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[ \frac{\pi^3}{3} - \frac{\pi^3}{3} \right]$$



$$a_n = \frac{2}{\pi} \int_0^{\pi} x \cos nx \, dx$$

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$$a_n = \frac{2}{\pi} \int_0^{\pi} (x\pi - x^2) \cos nx \, dx$$
$$u = x\pi - x^2 \quad \int dv = \int \cos nx \, dx$$
$$u_1 = \pi - 2x \quad v = \frac{\sin nx}{n}$$
$$u_2 = -2 \quad v_1 = -\frac{\cos nx}{n^2}$$
$$u_3 = 0 \quad v_2 = -\frac{\sin nx}{n^3}$$

$$a_n = \frac{2}{\pi} \left[ (x\pi - x^2) \frac{\sin nx}{n} + (\pi - 2x) \frac{\cos nx}{n^2} + 2 \frac{\sin nx}{n^3} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[ -\pi \frac{(-1)^n}{n^2} - \frac{\pi}{n^2} \right]$$

$$= \frac{2\pi}{\pi n^2} \left[ -(-1)^n - 1 \right]$$

$$= \frac{-2}{n^2} \left[ 1 + (-1)^n \right]$$

$$a_n = \begin{cases} 0 & \text{if } n \text{ is odd} \\ -\frac{4}{n^2} & \text{if } n \text{ is even} \end{cases}$$

$$f(x) = \frac{\pi^2}{6} + \sum_{n=2,4,6,\dots}^{\infty} \frac{-4}{n^2} \cos nx$$

③ Find the half range cosine series of

$$f(x) = (x-1)^2, \quad 0 < x < 1.$$



$$\begin{aligned} a_0 &= \frac{2}{l} \int_0^1 f(x) dx \\ &= 2 \int_0^1 (x-1)^2 dx \\ &= 2 \left[ \frac{(x-1)^3}{3} \right]_0^1 \\ &= \frac{2}{3} \left[ 0 + \frac{1}{3} \right] \end{aligned}$$

$$a_0 = \frac{2}{3}$$

$$\begin{aligned} a_n &= \frac{2}{l} \int_0^1 f(x) \frac{\cos n\pi x}{l} dx \\ &= 2 \int_0^1 (x-1)^2 \frac{\cos n\pi x}{l} dx \end{aligned}$$

$$u = (x-1)^2 \quad \left| \begin{aligned} dv &= \int \frac{\cos n\pi x}{l} dx \\ v &= \frac{\sin n\pi x}{n\pi} \end{aligned} \right.$$

$$u_1 = 2(x-1)$$

$$v = \frac{\sin n\pi x}{n\pi}$$

$$u_2 = 2$$

$$v_1 = -\frac{\cos n\pi x}{n^2\pi^2}$$

$$v_2 = -\frac{\sin n\pi x}{n^3\pi^3}$$

$$a_n = 2 \left[ (x-1)^2 \frac{\sin n\pi x}{n\pi} + 2(x-1) \frac{\cos n\pi x}{n^2\pi^2} - 2 \frac{\sin n\pi x}{n^3\pi^3} \right]$$

$$= 2 \left[ \frac{2}{n^2\pi^2} \right]$$

$$= \frac{4}{n^2\pi^2}$$