Unit-I

Fourier Series

Definition:

Periodic function:

A function $f(x)$ is said to have a period T for all x, $f(x+T) = f(x)$, where T is a +ve constant. The least value of $T \geq 0$ is called the period of $f(x)$.

e.g: $\sin x$, $\cos x$ are periodic function with period 2π.

Fourier series:

If $f(x)$ is a periodic function and satisfies Dirichlet conditions, then it can be represented by an infinite series called Fourier series.

Uses of Fourier Series:

Fourier Series are particularly suitable for expansion of periodic functions. we come across many periodic functions in voltage, current flex density: applied force, potential and electomagnetic force in electricity, hence

Fourier series are very useful in electrical engineering problems.

Deduction: Sum of the Fourier Series

Continuous — Substitute the value directly

Discontinuous — End point: Average values at end points; Middle point: $\frac{L.H.S + R.H.S}{2}$

Problems:

(1) Sum of the Fourier series for

$$f(x) = \begin{cases} x^2, & -\pi \le x \le 0 \\ 0, & 0 \le x \le \pi \end{cases} \quad \text{at } x = \frac{\pi}{2}, \frac{-\pi}{2}.$$

Sol:

$x = \frac{\pi}{2}$ is a continuous point at $(0, \pi)$

$$\therefore f(x) = 0.$$

$x = -\frac{\pi}{2}$ is a continuous point at $(-\pi, 0)$

$$f(x) = x^2$$

$$\Rightarrow f\left(\frac{-\pi}{2}\right) = \left(\frac{-\pi}{2}\right)^2 = \frac{\pi^2}{4}.$$

$$\therefore f(x) = \frac{\pi^2}{4}.$$

(2) Sum the Fourier series for

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2, & 1 < x < 2 \end{cases} \quad \text{at } x=0.$$

Sol: $x=0$ is a discontinuous and end point.

$$\text{Sum of the fourier series} = \frac{f(0)+f(2)}{2}$$

$$= \frac{0+2}{2} = \frac{2}{2}$$

$$\left.\begin{array}{r}\text{Sum of the fourier} \\ \text{series}\end{array}\right\} = 1.$$

(3) Sum of the fourier series for

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ x^2, & \pi < x < 2\pi \end{cases} \quad \text{at } x=\pi.$$

Sol: $x=\pi$ is a discontinuous and middle point.

$$\text{Sum of the fourier Series} = \frac{f(\pi-)+f(\pi+)}{2}$$

$$= \frac{\pi+\pi^2}{2}.$$

Dirichlet condition:

i) $f(x)$ is periodic, single valued and finite

ii) $f(x)$ has a finite no. of finite discontinuous.

iii) $f(x)$ has no infinite discontinuous

iv) $f(x)$ has a finite no. of maxima and minima

Formula for fourier series in $(0, 2\pi)$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

where
$$a_0 = \frac{2}{b-a}\int_a^b f(x)\,dx$$

$$a_n = \frac{2}{b-a}\int_a^b f(x)\cos nx\,dx$$

$$b_n = \frac{2}{b-a}\int_a^b f(x)\sin nx\,dx.$$

Problems:

① Expand $f(x) = x^2$ in $(0, 2\pi)$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.