TOPIC : 11 - SOLUTIONS OF LINEAR EQUATIONS OF SECOND AND HIGHER ORDER WITH CONSTANT COEFFICIENTS

Type: (iii)

R.H.S $= \sin(ax+by)$ or $\cos(ax+by)$

Replace $D^2 \to -a^2$

$DD' \to -ab$

$D'^2 \to -b^2$

Solve: $(D^2+2DD'+D'^2)z = \sin(2x-y)$

Sol:

The A.E is $m^2+2m+1=0$

$(m+1)^2=0$

$\Rightarrow m=-1,-1.$

$C.F = f_1(y-x)+x f_2(y-x)$

$$P.I = \frac{1}{D^2+2DD'+D'^2}\sin(2x-y)$$

$$= \frac{1}{-4+2.2-1}\sin(2x-y) \qquad D^2 \to -4,\ DD' \to 2,\ D'^2 \to -1.$$

$$= \frac{1}{-1}\sin(2x-y)$$

$$= -\sin(2x-y)$$

$z = C.F + P.I$

$z = f_1(y-x)+x f_2(y-x) - \sin(2x-y)$

2. Solve: $(D^2-4D'^2)z = \cos 2x \cos 3y$.

Sol.

$$(D^2-4D'^2)z = \frac{1}{2}\left[\cos(2x+3y)+\cos(2x-3y)\right]$$

The A.E is $m^2-4=0$.

$$m^2=4$$
$$m=\pm 2.$$

$$C.F = f_1(y+2x)+f_2(y-2x)$$

$$P.I_1 = \frac{1}{D^2-4D'^2}\cdot\frac{1}{2}\cos(2x+3y) \qquad D^2 \to -4,\ D'^2 \to -9$$

$$= \frac{1}{-4-4(-9)}\cdot\frac{1}{2}\cos(2x+3y)$$

$$= \frac{1}{-4+36}\cdot\frac{1}{2}\cos(2x+3y)$$

$$= \frac{1}{32}\cdot\frac{1}{2}\cos(2x+3y)$$

$$= \frac{1}{64}\cos(2x+3y)$$

$$P.I_2 = \frac{1}{D^2-4D'^2}\,\frac{1}{2}\cos(2x-3y)$$

$$= \frac{1}{-4+36}\cdot\frac{1}{2}\cos(2x-3y) \qquad D^2 \to -4,\ D'^2 \to -9$$

$$= -\frac{1}{64}\cos\cos(2x-3y).$$

$$C.F = f_1(y+2x)+f_2(y-2x)+\frac{1}{64}\cos(2x+3y)$$
$$-\frac{1}{64}\cos(2x-3y).$$

Type: (IV)

$$R.H.S = x \;\&\; y$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots$$

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1. Solve: $(D^2 + 3DD' + 2D'^2)z = x+y$.

Sol:

The A.E is $m^2 + 3m + 2 = 0$

$$(m+1)(m+2) = 0$$

$$\therefore m = -1, -2.$$

$$C.F = f_1(y-x) + f_2(y-2x)$$

$$P.I = \frac{1}{D^2 + 3DD' + 2D'^2}(x+y)$$

$$= \frac{1}{D^2\left[1 + \left(\frac{3DD' + 2D'^2}{D^2}\right)\right]}(x+y)$$

$$= \frac{1}{D^2}\left[1 + \left(\frac{3DD' + 2D'^2}{D^2}\right)\right]^{-1}(x+y)$$

$$= \frac{1}{D^2}\left[1 - \left(\frac{3D'}{D} + \frac{2D'^2}{D^2}\right) + \cdots\right](x+y)$$

$$= \frac{1}{D^2}\left[x+y - \frac{3D'}{D}(x+y)\right]$$

$$= \frac{1}{D^2}\left[x+y - \frac{3}{D}(1)\right]$$

$$P.I = \frac{1}{D^2}\left[x+y-3x\right]$$

$$= \frac{1}{D^2}\left[y-2x\right]$$

$$= \frac{1}{D}\left[yx - \frac{\cancel{2}x^2}{\cancel{2}}\right]$$

$$P.I = \frac{yx^2}{2} - \frac{x^3}{3}$$

$$z = f_1(y-x) + f_2(y-2x) + \frac{yx^2}{2} - \frac{x^3}{3}.$$

2\. Solve: $(D^2 + DD' - 6D'^2)z = x^2y$

Sol: The A.E is $m^2+m-6=0$

$$(m+3)(m-2)=0$$

$$m=-3,\ 2.$$

$$C.F = f_1(y-3x) + f_2(y+2x)$$

$$P.I = \frac{1}{D^2+DD'-6D'^2}\,(x^2y)$$

$$= \frac{1}{D^2\left[1+\left(\frac{DD'-6D'^2}{D^2}\right)\right]}\,(x^2y)$$

$$= \frac{1}{D^2}\left[1+\left(\frac{D'}{D^2} - \frac{6D'^2}{D^2}\right)\right]^{-1} x^2y$$

$$= \frac{1}{D^2}\left[1-\left(\frac{D'}{D^2} - \frac{6D'^2}{D^2}\right)\right] x^2y$$

$$= \frac{1}{D^2}\left[x^2y - \frac{D'}{D^2}(x^2y)\right]$$

$$P.I = \frac{1}{D^2}\left(x^2 y - \frac{x^2}{D}\right)$$

$$= \frac{1}{D^2}\left(x^2 y - \frac{x^3}{3}\right)$$

$$= \frac{1}{D}\left(\frac{x^3}{3} y - \frac{x^4}{12}\right)$$

$$= \frac{x^4}{12} y - \frac{x^5}{60}$$

$$z = f_1(y-3x) + f_2(y+2x) + \frac{x^4}{12} y - \frac{x^5}{60}$$