



TOPIC : 2 - FORMATION OF PARTIAL DIFFERENTIAL EQUATIONS

Formation of PDE by Elimination of arbitrary constants functions:

$$\phi(u, v) = 0 \Rightarrow \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0.$$

① Eliminate the arbitrary function  $f$  from  $z = f\left(\frac{y}{x}\right)$  form a PDE.  
Sol:  
Given  $z = f\left(\frac{y}{x}\right)$   
Diff p.w.  $x$  to  $x$ , we get  
$$\frac{\partial z}{\partial x} = f'\left(\frac{y}{x}\right) \cdot \frac{-y}{x^2}$$
  
$$\Rightarrow p = f'\left(\frac{y}{x}\right) \cdot \frac{-y}{x^2} \rightarrow \text{①}$$
  
Diff p.w.  $x$  to  $y$ , we get  
$$\frac{\partial z}{\partial y} = f'\left(\frac{y}{x}\right) \cdot \frac{1}{x}$$
  
$$\Rightarrow q = f'\left(\frac{y}{x}\right) \cdot \frac{1}{x} \rightarrow \text{②}$$
  
from ① & ②.  
$$\frac{p}{q} = \frac{-y/x^2 \cdot x}{1/x}$$
  
$$\frac{p}{q} = -\frac{y}{x}$$
  
$$px = -qy$$
  
$$\Rightarrow px + qy = 0.$$



②  $z = f(xy)$

Sol:

$z = f(xy)$   
Diff w.r. to  $x$ , we get

$$\frac{\partial z}{\partial x} = f'(xy) \cdot y$$

$$p = f'(xy) \cdot y$$

Diff w.r. to  $y$ , we get

$$\frac{\partial z}{\partial y} = f'(xy) \cdot x$$

$$q = f'(xy) \cdot x$$

$$\frac{p}{q} = \frac{y}{x}$$

$$px = qy$$

$$\Rightarrow px - qy = 0.$$

③  $\phi(z^2 - xy, \frac{x}{z}) = 0.$

Sol:

Here  $u = z^2 - xy$ ,  $v = \frac{x}{z}$

$$\frac{\partial u}{\partial x} = 2z \frac{\partial z}{\partial x} - y$$
$$= 2zp - y$$

$$\frac{\partial v}{\partial x} = \frac{z \cdot 1 - x \cdot \frac{\partial z}{\partial x}}{z^2}$$
$$= \frac{z - xp}{z^2}$$

$$\frac{\partial u}{\partial y} = 2z \frac{\partial z}{\partial y} - x$$
$$= 2zq - x$$

$$\frac{\partial v}{\partial y} = \frac{z \cdot 0 - x \cdot q}{z^2} = \frac{-xq}{z^2}$$

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$$



$$\Rightarrow \begin{vmatrix} 2zp - y & \frac{z - xp}{z^2} \\ 2zq - x & -\frac{xq}{z^2} \end{vmatrix} = 0$$

$$(2zp - y)\left(-\frac{xq}{z^2}\right) - (2zq - x)\left(\frac{z - xp}{z^2}\right) = 0$$

$$\frac{1}{z^2} [-2zpxq + yxq - 2zqz + 2zqx + xpz - x^2p] = 0$$

$$xyq + xz - 2z^2q - x^2p = 0$$

$$xz = 2z^2q + x^2p - xyq$$

$$xz = x^2p - xyq + 2z^2q$$

4.  $z = f(x^2 + y^2)$ .

Sol:

$$z = f(x^2 + y^2)$$

Diff p.w. r. to  $x$ , we get

$$\frac{\partial z}{\partial x} = f'(x^2 + y^2) \cdot 2x$$

$$p = 2x f'(x^2 + y^2)$$

Diff p.w. r. to  $y$ , we get

$$\frac{\partial z}{\partial y} = f'(x^2 + y^2) \cdot 2y$$

$$q = 2y f'(x^2 + y^2)$$

$$\frac{p}{q} = \frac{2x f'(x^2 + y^2)}{2y f'(x^2 + y^2)}$$

$$\frac{p}{q} = \frac{x}{y}$$

$$py - qx = 0.$$



$$5. z = x f(2x+y) + g(2x+y)$$

Sol:

$$z = x f(2x+y) + g(2x+y)$$

Diff p. w. r. to  $x$ .

$$p = \frac{\partial z}{\partial x} = f(2x+y) + x f'(2x+y) \cdot 2 + g'(2x+y) \cdot 1 \quad \text{--- (1)}$$

$$q = x f'(2x+y) + g'(2x+y) \quad \text{--- (2)}$$

$$r = f'(2x+y) \cdot 2 + x f''(2x+y) \cdot 4 + f''(2x+y) \cdot 2 + g''(2x+y) \cdot 4 \quad \text{--- (3)}$$

$$t = x f''(2x+y) + g''(2x+y) \quad \text{--- (4)}$$

$$s = f'(2x+y) + x f'''(2x+y) + 2g''(2x+y) \quad \text{--- (5)}$$

$$\text{(3)} \Rightarrow r = 2 f'(2x+y) + 4 [x f''(2x+y) + g''(2x+y)]$$

$$r = 4 f'(2x+y) + 4t \quad \text{--- (6)}$$

$$\text{(5)} \Rightarrow s = 2 [x f''(2x+y) + g''(2x+y)] + f'(2x+y)$$

$$s = 2t + f'(2x+y) \quad \text{--- (7)}$$

$$\text{(6)} - 4 \times \text{(7)}$$

$$r = 4 f'(2x+y) + 4t$$

$$4s = 4 f'(2x+y) + 8t$$

$$\begin{array}{r} r \\ - \\ 4s \end{array}$$

$$r - 4s = -4t$$

$$r - 4s + 4t = 0$$



⑥ From the p. de by eliminating  $\phi$  from  
 $\phi(x^2+y^2+z^2, ax+by+cz)=0$ .

Sol: Here  $u = x^2+y^2+z^2$        $v = ax+by+cz$ .

$$\frac{\partial u}{\partial x} = 2x + 2zp \qquad \frac{\partial v}{\partial x} = a + cp$$
$$\frac{\partial u}{\partial y} = 2y + 2zq \qquad \frac{\partial v}{\partial y} = b + cq$$
$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \end{vmatrix} = 0$$
$$\begin{vmatrix} 2x+2zp & a+cp \\ 2y+2zq & b+cq \end{vmatrix} = 0$$
$$2(x+zp)(b+cq) - 2(y+zq)(a+cp) = 0.$$