



Unit - I
Partial Differential Equations

Formation of Partial Differential equations
by Elimination of arbitrary constants

Consider an equation $f(x, y, z, a, b) = 0$ (1)
where a & b denote arbitrary constants.

A p.d.e is formed by eliminating the arbitrary constants that occur in the functional relation between the variables.

Using $\frac{\partial z}{\partial x} = p$, $\frac{\partial z}{\partial y} = q$

1. Form the p.d.e by eliminating the arbitrary constants a & b from $z = ax + by$ (1)

Diff p.w.r.to x we get
 $\frac{\partial z}{\partial x} = a \Rightarrow p = a$

Diff p.w.r.to y we get
 $\frac{\partial z}{\partial y} = b \Rightarrow q = b$

\therefore Eqn (1) becomes, $z = px + qy$



2. Eliminate the arbitrary constants
a & b from $z = (x^2 + a)(y^2 + b)$.

Sol: $z = (x^2 + a)(y^2 + b)$

Diff p.w.r. to x ,

$$p = \frac{\partial z}{\partial x} = 2x(y^2 + b) \Rightarrow \frac{p}{2x} = y^2 + b$$

Diff p.w.r. to y ,

$$q = 2y(x^2 + a) \Rightarrow \frac{q}{2y} = x^2 + a$$

\therefore Eqn ① becomes,

$$z = \frac{p}{2x} \cdot \frac{q}{2y}$$

$$4xyz = pq$$

3. $z = a(x+y) + b$

Sol: $z = a(x+y) + b$

Diff w.r. to x ,

$$p = a \rightarrow \text{①}$$

Diff w.r. to y ,

$$q = a \rightarrow \text{②}$$

From ① & ② we get $pq = a$, $p = q$.



4. Find the partial differential equation of all planes having equal intercepts on the x & y axis.

Sol:

Intercept form of the plane equation is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

Given $a = b$ (Equal intercepts on the x & y axis)

$$\frac{x}{a} + \frac{y}{a} + \frac{z}{c} = 1.$$

Diff. w.r. to x , we get

$$\frac{1}{a} + \frac{1}{c} \frac{\partial z}{\partial x} = 0$$

$$\Rightarrow \frac{1}{a} = -\frac{1}{c} \frac{\partial z}{\partial x} = -\frac{1}{c} p \rightarrow \text{①}$$

Diff. p.w.r. to y , we get

$$\frac{1}{a} + \frac{1}{c} \frac{\partial z}{\partial y} = 0$$

$$\Rightarrow \frac{1}{a} = -\frac{1}{c} \frac{\partial z}{\partial y} = -\frac{1}{c} q \rightarrow \text{②}$$

From ① & ②,

$$-\frac{1}{c} p = -\frac{1}{c} q$$

$$p = q.$$



b. $(x-a)^2 + (y-b)^2 + z^2 = 1.$

Sol:

Diff w.r. to x , we get

$$2(x-a) + 2z \frac{\partial z}{\partial x} = 0$$

$$2(x-a) = -2zp$$

$$x-a = -zp \rightarrow \textcircled{1}$$

Diff p.w.r. to y , we get

$$2(y-b) + 2z \frac{\partial z}{\partial y} = 0$$

$$2(y-b) = -2zq$$

$$y-b = -zq \rightarrow \textcircled{2}$$

Using $\textcircled{1}$ & $\textcircled{2}$, we get

$$(-zp)^2 + (-zq)^2 + z^2 = 1$$

$$z^2 p^2 + z^2 q^2 + z^2 = 1$$

$$z^2 (p^2 + q^2 + 1) = 1.$$



⑥ $Z = (x+a)^3 + (y-b)^2$
Sol:
Diff p.w.r. to x , we get
$$\frac{\partial Z}{\partial x} = 3(x+a)^2 \cdot 1$$
$$\Rightarrow \frac{p}{3} = (x+a)^2$$

Diff p.w.r. to y , we get
$$\frac{\partial Z}{\partial y} = 2(y-b)$$
$$\frac{q}{2} = y-b$$

$$Z = \left(\frac{p}{3}\right)^{3/2} + \left(\frac{q}{2}\right)^2$$
$$Z = \left(\frac{p}{3} \sqrt{\frac{p}{3}}\right)^3 + \left(\frac{q}{2}\right)^2$$
$$Z = \left(\frac{p^{3/2}}{3^{3/2}}\right)^3 + \left(\frac{q}{2}\right)^2$$

⑦ $(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$
Sol:
$$(x-a)^2 + (y-b)^2 = z^2 \cot^2 \alpha$$

Diff w.r. to x we get
$$2(x-a) = 2z p \cot^2 \alpha$$
$$x-a = z p \cot^2 \alpha$$

Diff w.r. to y , we get
$$2(y-b) = 2z q \cot^2 \alpha$$
$$y-b = z q \cot^2 \alpha$$

$$z^2 p^2 \cot^4 \alpha + z^2 q^2 \cot^4 \alpha = z^2 \cot^2 \alpha$$

$$\cancel{z^2 \cot^2 \alpha}, \quad p^2 \cot^2 \alpha + q^2 \cot^2 \alpha = 1$$