CANONICAL DECOMPOSITION

Canonical Decomposition

Definition: A canonical decomposition of any positive integer n is of the form $n = p_1^{\alpha_1}.\, p_2^{\alpha_2}\ .\ .\ .\ p_k^{\alpha_k}$, where $p_1^{\alpha_1}, p_2^{\alpha_2}, .\ .\ .\ p_k^{\alpha_k}$ are distinct primes.

Example 1: Find the canonical decomposition of 4312.

Solution: $4312 = 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 \cdot 11 = 2^3 \cdot 7^2 \cdot 11^1$

Example 2: Find the canonical decomposition of 2520.

Solution: $2520 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 7 = 2^3 \cdot 3^2 \cdot 5 . 7^1$

Example 3: Find the (72, 108) using canonical decomposition.

Solution:

$$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$$

$$108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 2^2 \cdot 3^3$$

$$(72, 108) = 2^2 3^2 = 4 * 9 = 36.$$

Example 4: Using recursion, evaluate (18, 30, 60, 75, 132).

Solution:

(18, 30, 60, 75, 132) = ((18, 30, 60, 75), 132)

= (((18, 30, 60), 75), 132)

= ((((18, 30), 60), 75), 132)

= (((6, 60), 75), 132)

= ((6, 75), 132)

= (3, 132) = 3.

Example 5: Using recursion evaluate (14, 18, 21, 36, 48).

Solution:

Consider (14, 18, 21, 36, 48) = ((14, 18), 21, 36, 48)

= (((14, 18), 21), 36, 48)

= ((((14, 18), 21), 36), 48)

= (((2, 21), 36), 48)

= ((1, 36), 48)

= (1, 48)

= 1.

Example 6: Using recursion evaluate (12, 18, 28, 34, 44).

Solution:

Consider (12, 18, 28, 34, 44) = ((12, 18), 28, 34, 44)

= (((12, 18), 28), 34, 44)

= (((12, 18), 28), 34), 44)

= (((6, 28), 34), 44)

= ((2, 34), 44)

= (2, 44)

= 2.