NUMBER PATTERNS

In drawing scientific conclusions, there are two fundamental processes of reasoning that are commonly used.

One is the process of deduction, which is the process of reasoning from general to particular.

The other process of reasoning is the process of induction, which is the process of reasoning from particular to general. This process may lead to true or false conclusion. To succeed in the art of inductive reasoning one must be good at studying pattern. Observing particular cases or pattern a general statement is usually made. Such a statement is called a conjecture or educated guess. A conjecture remains a conjecture until it is proved or disproved.

Example 1:

From the pattern

$$1.9 + 2 = 11$$
$$12.9 + 3 = 111$$
$$123.9 + 4 = 1111$$
$$1234.9 + 5 = 11111$$

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Write down the n^{th} row and prove the validity of the number pattern.

Solution:

From the given pattern we find the n^{th} row is

$$1.2.3.4.5 \dots n.9 + (n+1) = 1\,1\,1 \dots 1 \ \{n+1\}\text{ones.}$$

$$L.H.S = 1.2.3.4.5 \dots n.9 + (n+1)$$
$$= 9.1.2.3.4.5 \dots n + (n+1)$$
$$= 9\,[1(10)^{n-1} + 2(10)^{n-2} + 3(10)^{n-3} + \dots + (n-1)10 + n.1] + (n+1)$$
$$= (10-1)\,[(10)^{n-1} + 2(10)^{n-2} + 3(10)^{n-3} + \dots + (n-1)10 + n] + (n+1)$$
$$= [10^n + 2.(10)^{n-1} + 3.(10)^{n-2} + \dots + (n-1).10^2 + n.10]$$
$$- [(10)^{n-1} + 2(10)^{n-2} + \dots + (n-2)10^2 + (n-1)10 + n] + (n+1)$$
$$= 10^n + 10^{n-1} + 10^{n-2} + \dots + 10^2 + 10 - n + (n+1)$$
$$= 10^n + 10^{n-1} + 10^{n-2} + \dots + 10^2 + 10 + 1$$
$$= 111 \dots 1 \ \{n+1\}\text{ones.}$$
$$= R.H.S.$$

Example 2:

Using the number pattern

$$1^2 - 0^2 = 1$$

$$2^2 - 1^2 = 3$$

$$3^2 - 2^2 = 5$$

$$4^2 - 3^2 = 7$$

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Make a conjecture about row n and prove the conjecture.

Solution:

From the given number pattern, we find the n^{th} row is

$$n^2 - (n-1)^2 = 2n - 1$$

Therefore the conjecture is

$$n^2 - (n-1)^2 = 2n - 1 \quad \forall n \geq 0$$

$L.H.S = n^2 - (n-1)^2$

$= n^2 - (n^2 - 2n + 1)$

$= 2n - 1$

$= R.H.S.$

Example 3:

Given the pattern

$$9.9 + 7 = 88$$

$$98.9 + 6 = 888$$

$$987.9 + 5 = 8888$$

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Find the formula for the n^{th} row and prove it.

Solution:

Observing the pattern, we find the n^{th} is

$$987\ldots(10-n).9+(8-n)=888\ldots8,\ \{n+1\ eights\} \qquad 1\le n\le 8$$

$$L.H.S = 987\ldots(10-n).9+(8-n)$$

$$= 9.987\ldots(10-n)+(8-n)$$

$$= 9.[9.(10)^{n-1}+8.(10)^{n-2}+\ldots+(11-n).10+(10-n).1]+(8-n)$$

$$= (10-1)\,[9.(10)^{n-1}+8.(10)^{n-2}+\ldots+(11-n).10+(10-n).1]+(8-n)$$

$$= [9.(10)^{n}+8.(10)^{n-1}+7.(10)^{n-2}+\ldots+(11-n).10^2+(10-n).10]$$

$$-[9.(10)^{n-1}+8.(10)^{n-2}+\ldots+(11-n).10+(10-n)]+(8-n)$$

$$= 9.10^n-[10^{n-1}+10^{n-2}+\ldots10^2+10]-(10-n)+(8-n)$$

$$= 10.10^n-[10^n+10^{n-1}+10^{n-2}+\ldots10^2+10]-2$$

$$= 10.10^n-[10^n+10^{n-1}+10^{n-2}+\ldots10^2+10+1]-1$$

$$= 10^{n+1}-\left[\frac{10^{n+1}-1}{10-1}\right]-1$$

$$= 10^{n+1}-\left[\frac{10^{n+1}-1}{9}\right]-1$$

$$= \frac{1}{9}[9.10^{n+1}-10^{n+1}+1-9]$$

$$= \frac{1}{9}[8.10^{n+1}-8]$$

$$= \frac{8}{9}[10^{n+1}-1]$$

$$= 888\ldots8\ \{n+1\ eights\}$$

$$= R.H.S.$$