BASE – b REPRESENTATIONS

Definition: (Base – b Representations)

If n is a positive integer and $b \geq 2$ and $n = a_k b^k + a_{k-1} b^{k-1} + \cdots + a_1 b + a_0$,

where $a_0, a_1, \ldots, a_k$ are non negative integers then the above expression is called base b of the integer n. We then write $n = (a_k\, a_{k-1} \ldots\, a_1\, a_0)_b$.

Example:

$$(345)_{10} = 3(10)^2 + 4(10) + 5.$$

Note:

(i) The number system with base 2 is called binary system and it has the digits $0, 1$.

(ii) The number system with base 8 is called octal system and it has the digits $0, 1, 2, 3, 4, 5, 6, 7$.

(iii) The number system with base 10 is called decimal system and it has the digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.

(iv) The number system with base 16 is called hexadecimal system and it has the digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ with letters A, B, C, D, E, F .The letters A, B, C, D, E, F denotes the digits $10, 11, 12, 13, 14, 15$ respectively.

Conversion of Binary, Octal, Hexadecimal systems to Decimal system

Example 1:

Express $(101011)_2$ in base 10.

Solution:

$$(101011)_2 = 1(2)^5 + 0(2)^4 + 1(2)^3 + 0(2)^2 + 1(2)^1 + 1(2)^0$$
$$= 32 + 8 + 2 + 1$$
$$= 43.$$

Therefore $(101011)_2 = (43)_{10}$.

Example 2:

Express $(347)_8$ in base 10.

Solution:

$$(347)_8 = 3(8)^2 + 4(8)^1 + 7(8)^0$$
$$= 192 + 32 + 7$$
$$= 231$$

Therefore $(347)_8 = (231)_{10}$.

Example 3

Express $(3AB0E)_{16}$ in base 10.

Solution:

$$(3AB0E)_{16} = 3(16)^4+A(16)^3+B(16)^2+0(16)^1+E(16)^0$$
$$= 3(16)^4+10(16)^3+11(16)^2+0(16)^1+14(16)^0$$
$$= 196608+40960+2816+14$$
$$= 240398.$$

Therefore $(3AB0E)_{16} = (240398)_{10}$.

Conversion of Decimal system to Binary, Octal and Hexadecimal systems

Example 1:

Express $(134)_{10}$ in binary system.

Solution:

$$134 = 67(2) + 0$$
$$67 = 33(2) + 1$$
$$33 = 16(2) + 1$$
$$16 = 8(2) + 0$$
$$8 = 4(2) + 0$$
$$4 = 2(2) + 0$$
$$2 = 1(2) + 0$$
$$1 = 0(2) + 1$$

Therefore $(134)_{10} = (10000110)_2$.

Example 2:

Express $(1543)_{10}$ in octal system.

Solution:

$$1543 = 192(8) + 7$$
$$192 = 24(8) + 0$$
$$24 = 3(8) + 0$$
$$3 = 0(8) + 3$$

Therefore $(1543)_{10} = (3007)_8$.

Example 3:

Express $(15036)_{10}$ in hexadecimal system.

Solution:

$15036 = 939(16) + 12\ (= C)$

$939 = 58(16) + 11\ (= B)$

$58 = 3(16) + 10\ (= A)$

$3 = 0(16) + 3$

Therefore $(15036)_{10} = (3\text{ABC})_{16}$.

We have seen the conversions of base b systems to decimal system and decimal system to base b systems.

Conversion of Binary system to Octal system

To convert a binary system number to octal system, we group the binary digits into blocks of three bits from right to left and adding if necessary initial zeroes at the left most block and replace each group with the corresponding octal digit.

Example 1:

Convert the binary number $(1110011)_2$ into octal digit.

Solution:

Given $1\ 110\ 011$.

We group the digits in blocks of three digits from right to left.

Here the blocks are $001, 110, 011$ (adding zeroes to the left most block to get three digits).

$001 = 0.(2)^2 + 0.(2)^1 + 1.(2)^0 = 1$

$110 = 1.(2)^2 + 1.(2)^1 + 0.(2)^0 = 6$

$011 = 0.(2)^2 + 1.(2)^1 + 1.(2)^0 = 3$

Therefore $(1110011)_2 = (163)_8$.

Example 2:

Convert the binary number $(111010)_2$ into octal digit.

Solution:

Given $111\ 010$.

We group the digits in blocks of three digits from right to left.

Here the blocks are $111, 010$.

$111 = 1(2)^2 + 1(2)^1 + 1(2)^0 = 7$

$010 = 0(2)^2 + 1(2)^1 + 0(2)^0 = 2$

Therefore $(111010)_2 = (72)_8$.

Conversion of Binary system to Hexadecimal system

To convert a binary system number to hexadecimal system, we group the binary digits into blocks of four bits from right to left and adding if necessary initial zeroes at the left most block and replace each group with the corresponding hexadecimal digit.

Example 1:

Convert the binary number $(11111010111100)_2$ into hexadecimal digit.

Solution:

Given $11\ 1110\ 1011\ 1100$.

We group the digits in blocks of four digits from right to left.

Here the blocks are $0011, 1110, 1011, 1100$ (adding zeroes to the left most block to get four digits).

$0011 = 0(2)^3 + 0(2)^2 + 1(2)^1 + 1(2)^0 = 3$

$1110 = 1(2)^3 + 1(2)^2 + 1(2)^1 + 0(2)^0 = 14\ (= E)$

$1011 = 1(2)^3 + 0(2)^2 + 1(2)^1 + 1(2)^0 = 11\ (= B)$

$1100 = 1(2)^3 + 1(2)^2 + 0(2)^1 + 0(2)^0 = 12\ (= C)$

Therefore $(11111010111100)_2 = (3EBC)_{16}$.

Example 2:

Convert the binary number $(1110101)_2$ into hexadecimal digit.

Solution:

Given $111\ 0101$.

We group the digits in blocks of four digits from right to left.

Here the blocks are $0111, 0101$(adding zero to the left most block to get four digits).

$0111 = 0(2)^3 + 1(2)^2 + 1(2)^1 + 1(2)^0 = 7$

$0101 = 0(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 = 5$

Therefore $(1110101)_2 = (75)_{16}$.

Conversion of Octal system to Binary system

To convert an octal system number to binary system, we write each digits from left to right as block of three bits, then grouping all those block of three bits from top to bottom as left to right gives the corresponding binary value.

Example 1:

Convert the octal digit $(3450)_8$ into binary number.

Solution:

Given $(3450)_8$.

We write each digits as a block of three bits.

$3 = 0(2)^2 + 1(2)^1 + 1(2)^0 = 011$

$4 = 1(2)^2 + 0(2)^1 + 0(2)^0 = 100$

$5 = 1(2)^2 + 0(2)^1 + 1(2)^0 = 101$

$0 = 0(2)^2 + 0(2)^1 + 0(2)^0 = 000$

Therefore $(3450)_8 = (011100101000)_2$.

$= (11100101000)_2$.

Example 2:

Convert the octal digit $(12376)_8$ into binary number.

Solution:

Given $(12376)_8$.

We write each digits as a block of three bits.

$1 = 0(2)^2 + 0(2)^1 + 1(2)^0 = 001$

$2 = 0(2)^2 + 1(2)^1 + 0(2)^0 = 010$

$3 = 0(2)^2 + 1(2)^1 + 1(2)^0 = 011$

$7 = 1(2)^2 + 1(2)^1 + 1(2)^0 = 111$

$6 = 1(2)^2 + 1(2)^1 + 0(2)^0 = 110$

Therefore $(12376)_8 = (001010011111110)_2$.

$= (1010011111110)_2$.

Conversion of Hexadecimal system to Binary system

To convert an hexadecimal system number to binary system, we write each digits from left to right as block of four bits, then grouping all those block of four bits from top to bottom as left to right gives the corresponding binary value.

Example 1:

Convert the hexadecimal digit $(3AD)_{16}$ into binary number.

Solution:

Given $(3AD)_{16}$.

We write each digits as a block of four bits.

$$3 = 0(2)^3 + 0(2)^2 + 1(2)^1 + 1(2)^0 = 0011$$

$$\mathrm{A} = 10 = 1(2)^3 + 0(2)^2 + 1(2)^1 + 0(2)^0 = 1010$$

$$D = 13 = 1(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 = 1101$$

Therefore $(3AD)_{16} = (001110101101)_2$.

$= (1110101101)_2$.

Example 2:

Convert the hexadecimal digit $(25)_{16}$ into binary number.

Solution:

Given $(25)_{16}$.

We write each digits as a block of four bits.

$$2 = 0(2)^3 + 0(2)^2 + 1(2)^1 + 0(2)^0 = 0010$$

$$5 = 0(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 = 0101$$

Therefore $(25)_{16} = (00100101)_2$.

$= (100101)_2$.

Example 3:

Find the number of ones in the binary expansion of $2^4 - 1$.

Solution:

$$2^4 - 1 = 15 = 1(2)^3 + 1(2)^2 + 1(2)^1 + 1(2)^0$$

$$= (1111)_2.$$

Hence the number of ones in the binary expansion of $2^4 - 1$ is 4.

Remark:

More generally the number of ones in the binary expansion of $2^n - 1$ is n.

Finding base b values

Example 1:

Find the value of base b if $(1001)_b = 9$.

Solution:

Given that $(1001)_b = 9$.

$\Rightarrow \quad 1(b)^3 + 0(b)^2 + 0(b)^1 + 1(b)^0 = 9$

$\Rightarrow \quad b^3 + 1 = 9$

$\Rightarrow \quad b^3 = 8$

$\Rightarrow \quad b = 2.$

Example 2:

Find the value of base b if $(144)_b = 49$.

Solution:

Given that $(144)_b = 49$.

$\Rightarrow \quad 1(b)^2 + 4(b)^1 + 4(b)^0 = 49$

$\Rightarrow \quad b^2 + 4b + 4 = 49$

$\Rightarrow \quad b^2 + 4b - 45 = 0$

$\Rightarrow \quad (b + 9)(b - 5) = 0$

$\Rightarrow \quad b = -9, 5$

Since $b \geq 2$, we have $b = 5$.