Principle of Mathematical Induction:

Let $P(n)$ be the given statement.

Base Step: $P(n_0)$ is true for some integer n_0.

Inductive Step: $P(k)$ is true for an arbitrary integer $k \geq n_0$.

Then $P(k+1)$ is true.

Hence $P(n)$ is true for every integer $\mathrm{n} \geq n_0$.

Example 5: Prove by induction $2n^3 + 3n^2 + n \; is \; divisible \; by \; 6, \forall \, n \geq 0$.

Solution:

$Let \; P(n) = 2n^3 + 3n^2 + n \; is \; divisible \; by \; 6, \forall \, n \geq 0$.

Base Step:

$P(0) = 0$ is divisible by 6.

Inductive Step:

Assume $P(k)$ is true for all $k \geq 0$.

$i.e. \, 2k^3 + 3k + k \; is \; divisible \; by \; 6, \forall \, k \geq 0$

$i.e. \, 2k^3 + 3k + k = 6m \; (say), m > 0$. ---------(1)

To Prove: $P(k+1)$ is true.

$i.e. \, 2(k+1)^3 + 3(k+1)^2 + (k+1) \; is \; divisible \; by \; 6$.

Consider $2(k+1)^3 + 3(k+1)^2 + (k+1)$.

$= 2(k^3 + 1 + 3k^2 + 3k) + 3(k^2 + 2k + 1) + (k+1)$

$= 2k^3 + 6k^2 + 6k + 2 + 3k^2 + 6k + 3 + k + 1$

$= (2k^3 + 3k^2 + k) + (6k^2 + 12k + 6)$

$= 6m + 6(k^2 + 2k + 1)$

$= 6(k^2 + 2k + 1 + m)$

Which is divisible by 6.

Thus $P(k+1)$ is true.

Hence $P(n)$ is true.