



# *Public key Cryptography*





## Public-Key Requirements



Conditions that these algorithms must fulfil:

It is computationally easy

- for a party B to generate a pair (public- key  $PU_b$ , private key  $PR_b$ )
- for a sender A, knowing the public key and the message to be encrypted, to generate the corresponding ciphertext
- for the receiver B to decrypt the resulting ciphertext using the private key to recover the original message

It is computationally infeasible

- for an adversary, knowing the public key, to determine the private key.
- for an adversary, knowing the public key and a ciphertext, to recover the original message.

The two keys can be applied in either order.





# Rivest-Shamir-Adleman (RSA) Scheme



- RSA is the algorithm used by modern computers to encrypt and decrypt messages. It is an asymmetric cryptographic algorithm.
- Asymmetric means that there are two different keys. This is also called public key cryptography, because one of them can be given to everyone. The other key must be kept private.
- One of the first successful responses to the challenge was Developed in 1977 at MIT by Ron Rivest, Adi Shamir & Len Adleman





# RSA Algorithm



- Plaintext is encrypted in blocks with each block having a binary value less than some number  $n$
- Encryption and decryption are of the following form, for some plaintext block  $M$  and cipher text block  $C$ 
  - $C = M^e \bmod n$
  - $M = C^d \bmod n = (M^e)^d \bmod n = M^{ed} \bmod n$
- Both sender and receiver must know the value of  $n$
- The sender knows the value of  $e$ , and only the receiver knows the value of  $d$
- This is a public-key encryption algorithm with a public key of  $PU = \{e, n\}$  and a private key of  $PR = \{d, n\}$





# Algorithm Requirements



- For this algorithm to be satisfactory for public-key encryption, the following requirements must be met:
  1. It is possible to find values of  $e$ ,  $d$ ,  $n$  such that  $M^{ed} \bmod n = M$  for all  $M < n$
  2. It is relatively easy to calculate  $M^e \bmod n$  and  $C^d \bmod n$  for all values of  $M < n$
  3. It is infeasible to determine  $d$  given  $e$  and  $n$



### Key Generation by Alice

Select $p, q$	$p$ and $q$ both prime, $p \neq q$
Calculate $n = p \times q$	
Calculate $\phi(n) = (p - 1)(q - 1)$	
Select integer $e$	$\text{gcd}(\phi(n), e) = 1; 1 < e < \phi(n)$
Calculate $d$	$d = e^{-1} \pmod{\phi(n)}$
Public key	$PU = \{e, n\}$
Private key	$PR = \{d, n\}$

### Encryption by Bob with Alice's Public Key

Plaintext:	$M < n$
Ciphertext:	$C = M^e \pmod n$

### Decryption by Alice with Alice's Private Key

Ciphertext:	$C$
Plaintext:	$M = C^d \pmod n$



# Example of RSA Algorithm

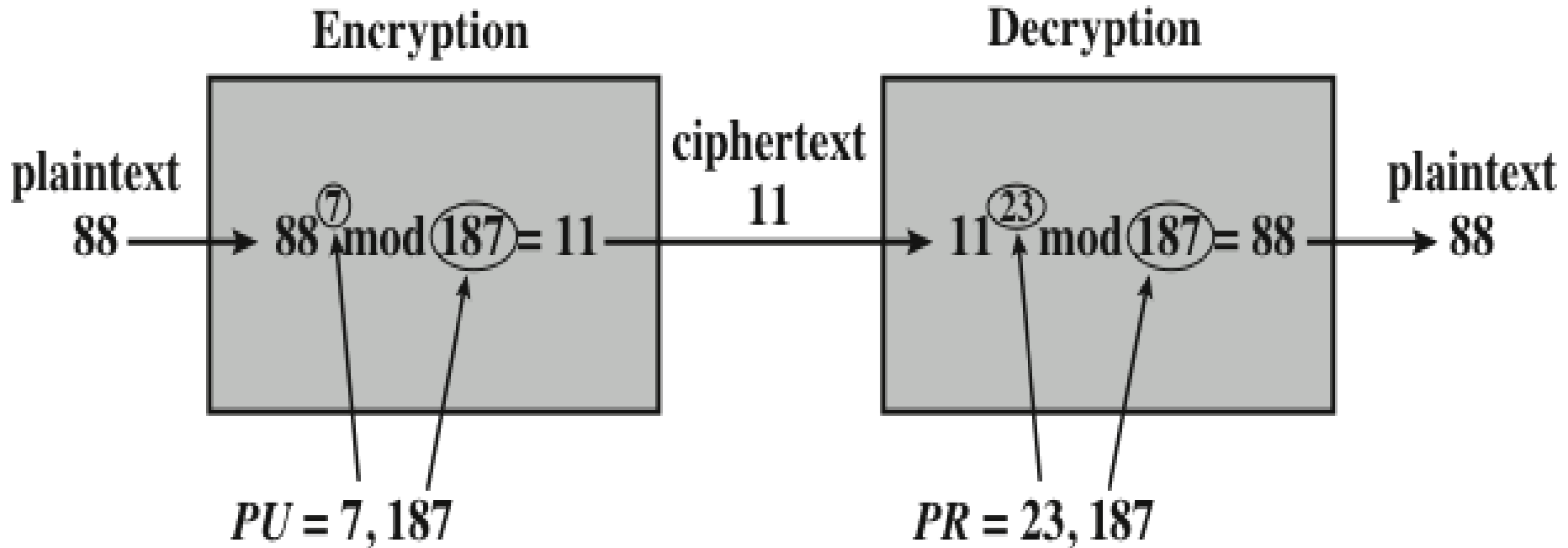
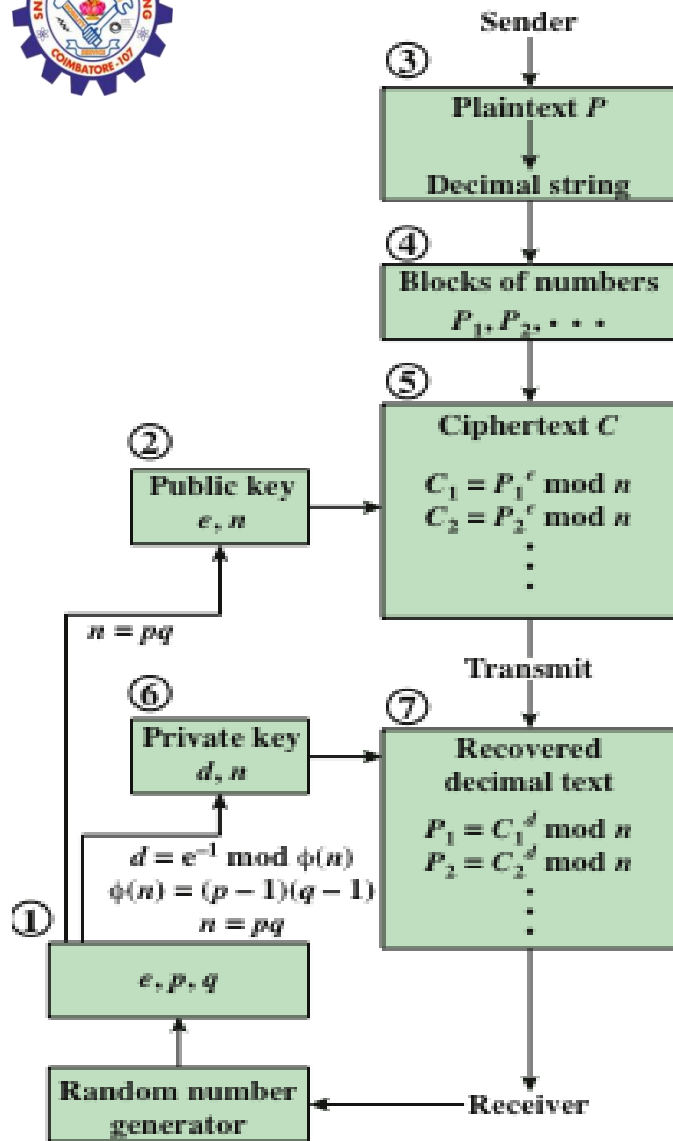
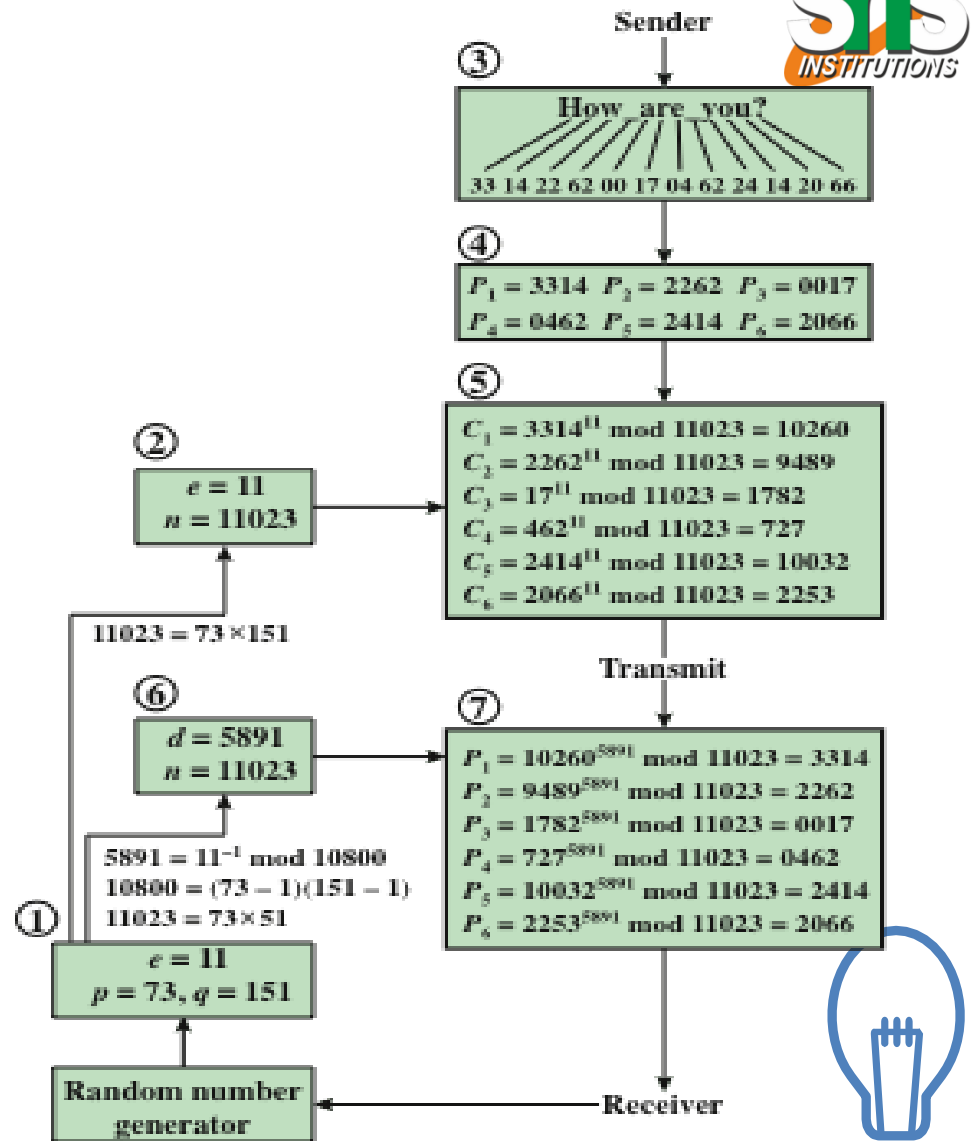


Figure 9.6 Example of RSA Algorithm





(a) General approach



(b) Example





# The Security of RSA



## Chosen ciphertext attacks

- This type of attack exploits properties of the RSA algorithm

## Brute force

- Involves trying all possible private keys

## Mathematical attacks

- There are several approaches, all equivalent in effort to factoring the product of two primes

**Five possible approaches to attacking RSA are:**

## Hardware fault-based attack

- This involves inducing hardware faults in the processor that is generating digital signatures

## Timing attacks

- These depend on the running time of the decryption algorithm





**THANK YOU!!!**

