SNS COLLEGE OF ENGINEERING

(Autonomous)

DEPARTMENT OF CSE - IoT

COURSE NAME:19EC306 / DIGITAL CIRCUITS

II YEAR/III SEMESTER

UNIT:1- MINIMIZATION TECHNIQUES AND LOGIC GATES

TOPIC:BOOLEAN POSTULATES AND LAWS

Outline

- Interpretation of Boolean Algebra using Logic Operations
- Boolean Algebra and Gates
- Theorems and Proofs

Section 1: Interpretation of Boolean Algebra using Logic Operations

Logic Symbols, 0, 1; and AND, OR Gates.

a =1 => a is true ,

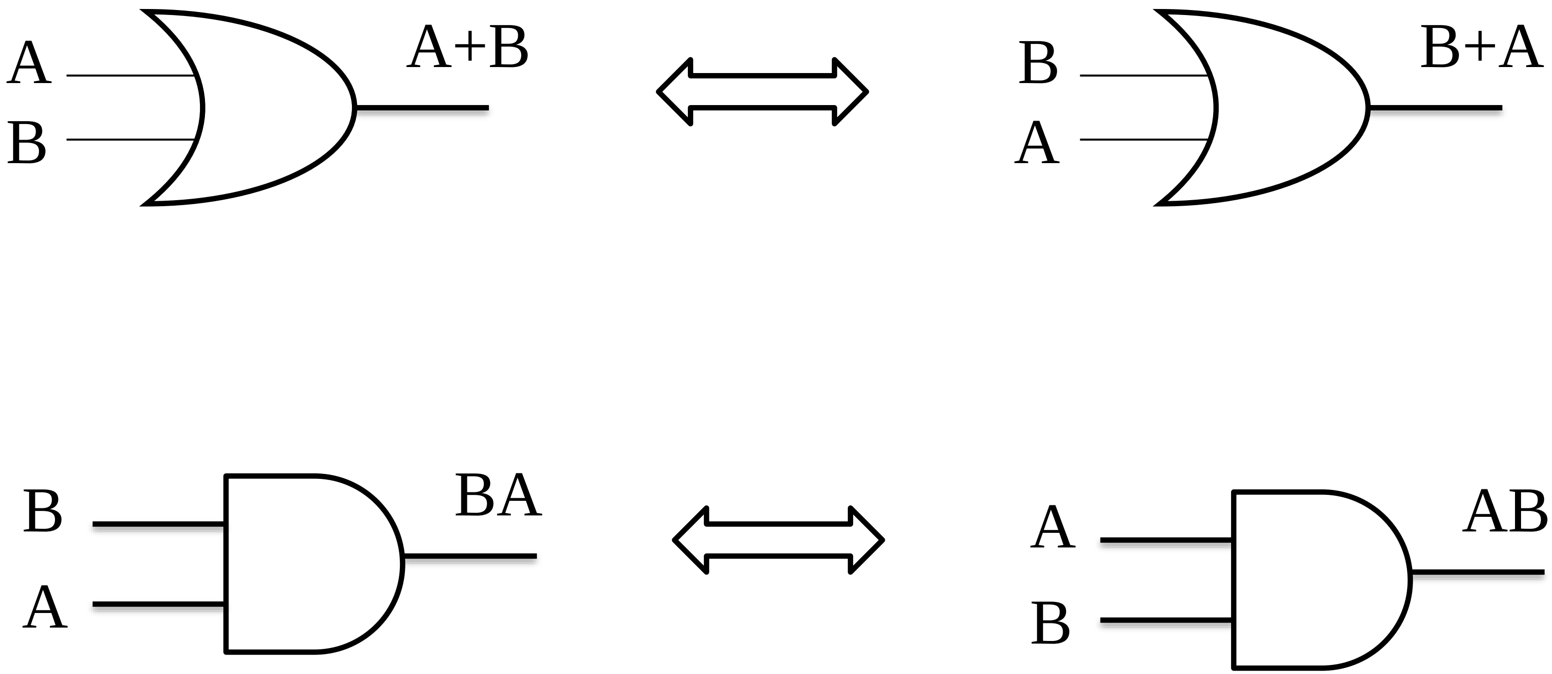
a =0 => a is false.

id	**a b**	***a OR b***
0	0 0	0
1	0 1	1
2	1 0	1
3	1 1	1

Id	**a b**	***a AND b***
0	0 0	0
1	0 1	0
2	1 0	0
3	1 1	1

Section 2: Boolean Algebra and Gates

P1: Commutative Laws

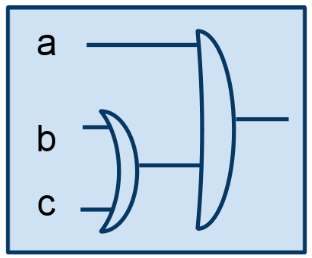


P2: Distributive Laws

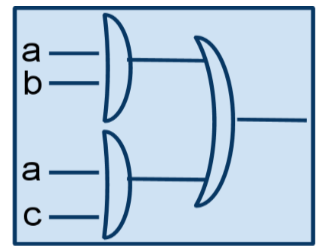
- $a \cdot (b+c) = (a \cdot b)+(a \cdot c)$
- $a+(b \cdot c) = (a+b) \cdot (a+c)$

ID	a	b	c	b+c	a· (b+c)		a· b	a· c	(a· b)+(a· c)
0	0	0	0	0	0		0	0	0
1	0	0	1	1	0		0	0	0
2	0	1	0	1	0		0	0	0
3	0	1	1	1	0		0	0	0
4	1	0	0	0	0		0	0	0
5	1	0	1	1	1		0	1	1
6	1	1	0	1	1		1	0	1
7	1	1	1	1	1		1	1	1

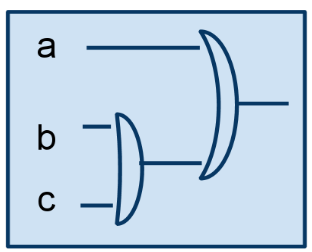
P2: Distributive Laws, cont.



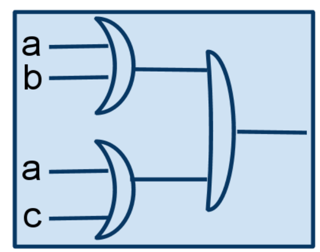
$a\cdot (b+c)$



$(a\cdot b)+(a\cdot c)$



$a+(b\cdot c)$

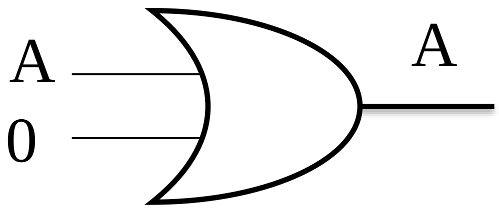


$(a+b)\cdot(a+c)$

P3 Identity

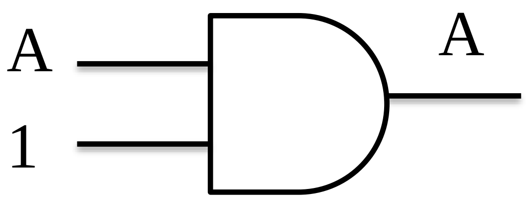
$a+0 = a,$

0 input to OR is passive



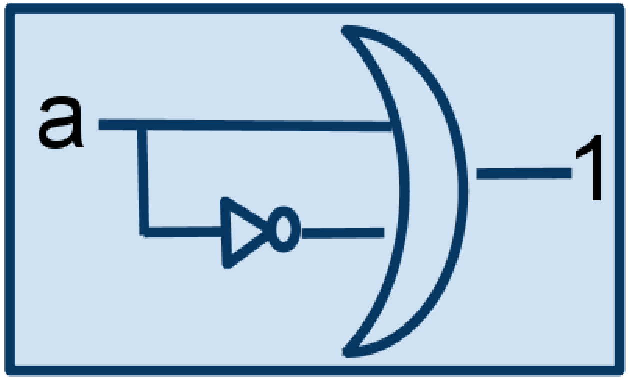
$a \cdot 1 = a,$

1 input to AND is passive

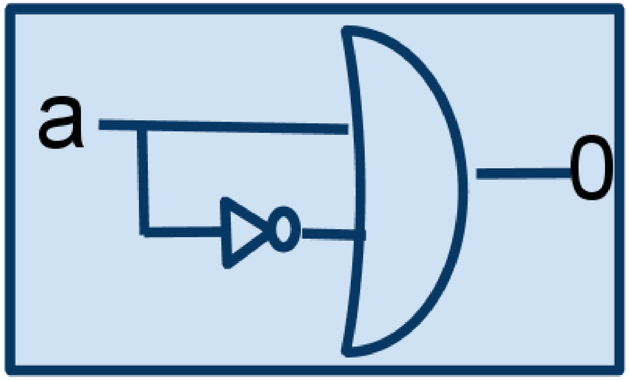


P4 Complement

$a+a' = 1$



$a \cdot a' = 0$



Section 3, Theorems and Proofs

Theorem 1: Principle of Duality

- Every algebraic identity that can be proven by Boolean algebra laws, remains valid if we swap all '+' and '·', 0 and 1.

Proof:

- Visible by inspection – all laws remain valid if we interchange all

  '+' and '·', 0 and 1

Theorem 2

Uniqueness of Complement: For every a in B, its complement a' is unique.

Proof: We prove by contradiction.

Suppose that a' is not unique, i.e. a_1', a_2' in B & $a_1' \neq a_2'$.

We have $a_1' = a_1' * 1$ (Postulate 3)

$= a_1' * (a + a_2')$ (Postulate 4)

$= (a_1' * a) + (a_1' * a_2')$ (Postulate 2)

$= 0 + (a_1' * a_2')$ (Postulate 4)

$= a_1' * a_2'$ (Postulate 3).

Likewise, we can also prove the same with a_2', i.e.

$a_2' = a_1' * a_2'$.

Consequently, we have $a_1' = a_2'$, which contradicts our initial assumption that $a_1' \neq a_2'$.

Theorem 3

Boundedness: For all elements a in B, a+1=1; a*0=0.

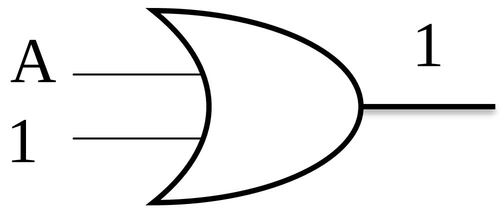
Proof:

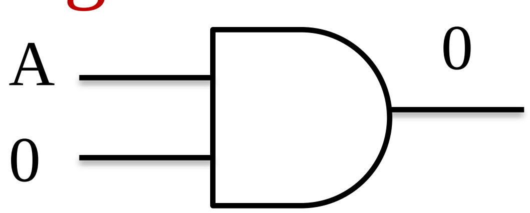
$$\begin{aligned} a+1 &= 1*(a+1) && \text{(Postulate 3)} \\ &= (a + a')*(a+1) && \text{(Postulate 4)} \\ &= a + a'*1 && \text{(Postulate 2)} \\ &= a + a' && \text{(Postulate 3)} \\ &= 1 && \text{(Postulate 4)} \end{aligned}$$

Comments:

'1' dominates as input in OR gates.

’0' dominates as input in AND gates.





Theorem 4

Statement:

- The complement of element 1 is 0 and vice versa, i.e.

$$0' = 1,\ 1' = 0.$$

Proof:

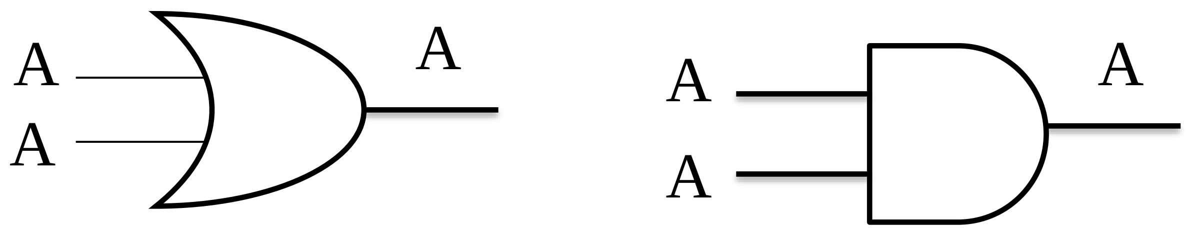
$$0 + 1 = 1 \text{ and } 0 * 1 = 0 \text{ (Postulate 3)}$$

Thus 0’= 1, 1’= 0 (Postulate 4 and Theorem 2)

Theorem 5: Idempotent Laws

Statement: For every a in B,

$$a + a = a \quad \text{and} \quad a * a = a.$$



Proof:

$$\begin{aligned}
a + a &= (a + a) * 1 && \text{(Postulate 3)} \\
&= (a + a) * (a + a') && \text{(Postulate 4)} \\
&= a + (a*a') && \text{(Postulate 2)} \\
&= a + 0 && \text{(Postulate 4)} \\
&= a && \text{(Postulate 3)}
\end{aligned}$$

Thank you

11/09/23