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C H A P T E R 13

Keys and Coupling

13.1 Introduction

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

13.2 Types of Keys

The following types of keys are important from the subject point of view :

1. Sunk keys, **2.** Saddle keys, **3.** Tangent keys, **4.** Round keys, and **5.** Splines.

We shall now discuss the above types of keys, in detail, in the following pages.

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13.3 Sunk Keys

The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub or boss of the pulley. The sunk keys are of the following types :

1. ***Rectangular sunk key.*** A rectangular sunk key is shown in Fig. 13.1. The usual proportions of this key are :

Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$

where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.

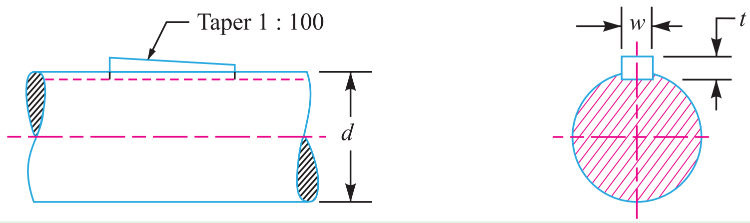


Fig. 13.1. Rectangular sunk key.

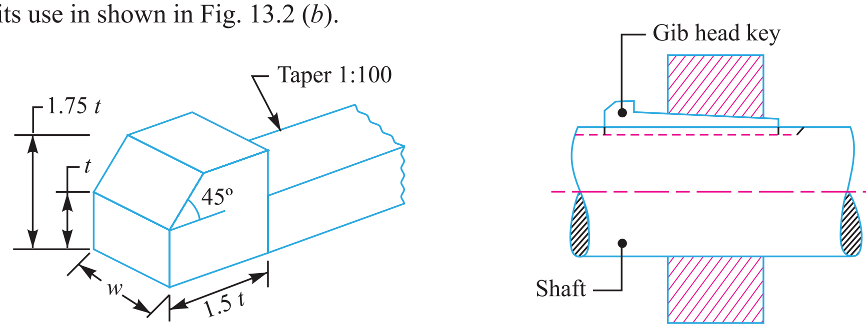
2. ***Square sunk key.*** The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal, *i.e.*

$$w = t = d / 4$$

3. ***Parallel sunk key.*** The parallel sunk keys may be of rectangular or square section uniform in width and thickness throughout. It may be noted that a parallel key is a taperless and is used where the pulley, gear or other mating piece is required to slide along the shaft.

4. ***Gib-head key.*** It is a rectangular sunk key with a head at one end known as ***gib head.*** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig. 13.2 (*a*) and its use in shown in Fig. 13.2 (*b*).

Helicopter driveline couplings.



(*a*) (*b*)

Fig. 13.2. Gib-head key.

The usual proportions of the gib head key are :

Width, $w = d / 4$;

and thickness at large end, $t = 2w / 3 = d / 6$

5. *Feather key.* A key attached to one member of a pair and which permits relative axial movement is known as ***feather key.*** It is a special type of parallel key which transmits a turning moment and also permits axial movement. It is fastened either to the shaft or hub, the key being a sliding fit in the key way of the moving piece.

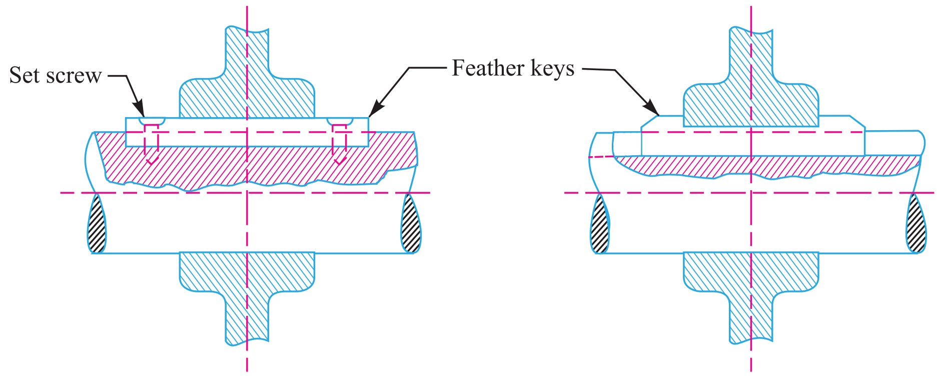


Fig. 13.3. Feather key.

The feather key may be screwed to the shaft as shown in Fig. 13.3 (*a*) or it may have double gib heads as shown in Fig. 13.3 (*b*). The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

The following table shows the proportions of standard parallel, tapered and gib head keys, according to IS : 2292 and 2293-1974 (Reaffirmed 1992).

Table 13.1. Proportions of standard parallel, tapered and gib head keys.

Shaft diameter (mm) upto and including	*Key cross-section*		*Shaft diameter (mm) upto and including*	*Key cross-section*	
	Width (mm)	*Thickness (mm)*		*Width (mm)*	*Thickness (mm)*
6	2	2	85	25	14
8	3	3	95	28	16
10	4	4	110	32	18
12	5	5	130	36	20
17	6	6	150	40	22
22	8	7	170	45	25
30	10	8	200	50	28
38	12	8	230	56	32
44	14	9	260	63	32
50	16	10	290	70	36
58	18	11	330	80	40
65	20	12	380	90	45
75	22	14	440	100	50

6. *Woodruff key.* The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. 13.4. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.

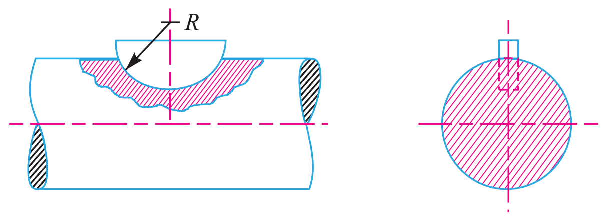


Fig. 13.4. Woodruff key.

The main advantages of a woodruff key are as follows :

1. It accommodates itself to any taper in the hub or boss of the mating piece.

2. It is useful on tapering shaft ends. Its extra depth in the shaft *prevents any tendency to turn over in its keyway.

The disadvantages are :

1. The depth of the keyway weakens the shaft.

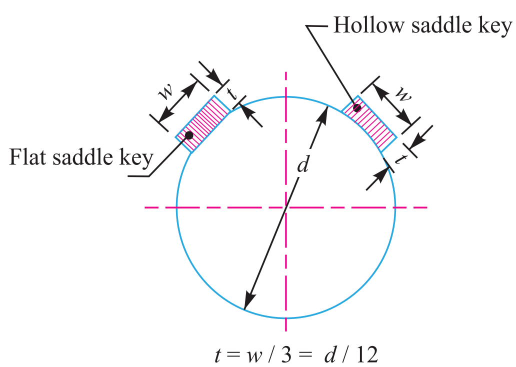
2. It can not be used as a feather.

13.4 Saddle keys

The saddle keys are of the following two types :

1. Flat saddle key, and **2.** Hollow saddle key.

A ***flat saddle key*** is a taper key which fits in a keyway in the hub and is flat on the shaft as shown in Fig. 13.5. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.



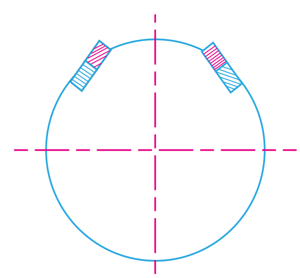


Fig. 13.5. Saddle key. **Fig. 13.6.** Tangent key.

A ***hollow saddle key*** is a taper key which fits in a keyway in the hub and the bottom of the key is shaped to fit the curved surface of the shaft. Since hollow saddle keys hold on by friction, therefore these are suitable for light loads. It is usually used as a temporary fastening in fixing and setting eccentrics, cams etc.

13.5 Tangent Keys

The tangent keys are fitted in pair at right angles as shown in Fig. 13.6. Each key is to withstand torsion in one direction only. These are used in large heavy duty shafts.

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* The usual form of rectangular sunk key is very likely to turn over in its keyway unless well fitted as its sides.

13.6 Round Keys

The round keys, as shown in Fig. 13.7(*a*), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually considered to be most appropriate for low power drives.

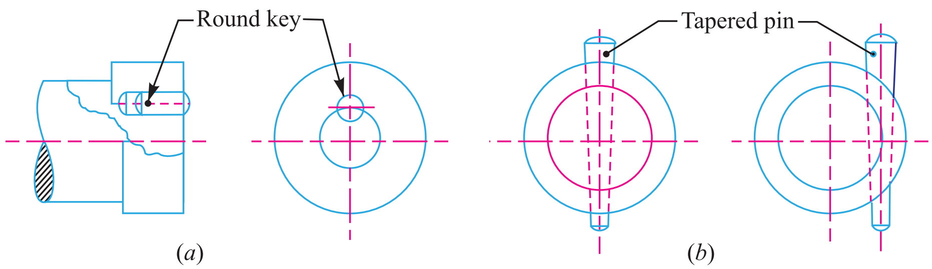


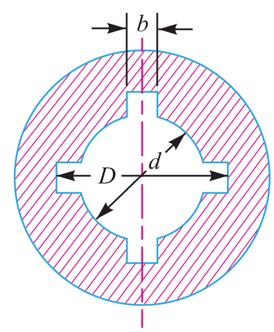
Fig. 13.7. Round keys.

Sometimes the tapered pin, as shown in Fig. 13.7 (*b*), is held in place by the friction between the pin and the reamed tapered holes.

13.7 Splines

Sometimes, keys are made integral with the shaft which fits in the keyways broached in the hub. Such shafts are known as ***splined shafts*** as shown in Fig. 13.8. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.

The splined shafts are used when the force to be transmitted is large in proportion to the size of the shaft as in automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive is obtained.



$D = 1.25\ d$ and $b = 0.25\ D$

Fig. 13.8. Splines.

13.8 Forces acting on a Sunk Key

When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.
2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig. 13.9.

In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

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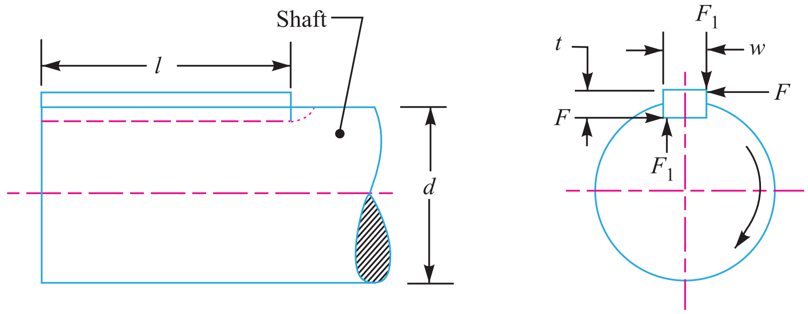

Fig. 13.9. Forces acting on a sunk key.

13.9 Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig. 13.9.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft,

l = Length of key,

w = Width of key.

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

A little consideration will show that due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

$$F = \text{Area resisting shearing} \times \text{Shear stress} = l \times w \times \tau$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \qquad ...(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

∴ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad ...(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \qquad \text{...[Equating equations } (i) \text{ and } (ii)]$$

or

$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \qquad ...(iii)$$

The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress. Therefore from equation (*iii*), we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

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In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \qquad \text{...}(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \qquad \text{...}(v)$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations (*iv*) and (*v*), we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\therefore \qquad l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau} \qquad \text{... (Taking } w = d/4) \qquad \text{...}(vi)$$

When the key material is same as that of the shaft, then $\tau = \tau_1$.

$$\therefore \qquad l = 1.571\, d \qquad \text{... [From equation } (vi)]$$

Example 13.1. *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa.*

Solution. Given : d = 50 mm ; τ = 42 MPa = 42 N/mm^2 ; σ_c = 70 MPa = 70 N/mm^2

The rectangular key is designed as discussed below:

From Table 13.1, we find that for a shaft of 50 mm diameter,

Width of key, w = 16 mm **Ans.**

and thickness of key, t = 10 mm **Ans.**

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800\, l \text{ N-mm} \qquad \text{...}(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\,(50)^3 = 1.03 \times 10^6 \text{ N-mm} \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$l = 1.03 \times 10^6 / 16\,800 = 61.31 \text{ mm}$$

Now considering crushing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\, l \text{ N-mm} \qquad \text{...}(iii)$$

From equations (*ii*) and (*iii*), we have

$$l = 1.03 \times 10^6 / 8750 = 117.7 \text{ mm}$$

Taking larger of the two values, we have length of key,

l = 117.7 say 120 mm **Ans.**

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Example 13.2. *A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.*

Solution. Given : d = 45 mm ; σ_{yt} for shaft = 400 MPa = 400 N/mm^2 ; w = 14 mm ; t = 9 mm ; σ_{yt} for key = 340 MPa = 340 N/mm^2; $F.S.$ = 2

Let l = Length of key.

According to maximum shear stress theory (See Art. 5.10), the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\ (45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\ 775\ l$$

$$\therefore \quad l = 1.8 \times 10^6 / 26\ 775 = 67.2 \text{ mm}$$

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\ 213\ l$$

$$\ldots \left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$$\therefore \quad l = 1.8 \times 10^6 / 17\ 213 = 104.6 \text{ mm}$$

Taking the larger of the two values, we have

l = 104.6 say 105 mm **Ans.**

13.10 Effect of Keyways

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft. It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

where e = Shaft strength factor. It is the ratio of the strength of the shaft with keyway to the strength of the same shaft without keyway,

w = Width of keyway,

d = Diameter of shaft, and

$$h = \text{Depth of keyway} = \frac{\text{Thickness of key } (t)}{2}$$

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It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation :

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

where k_θ = Reduction factor for angular twist.

Example 13.3. *A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.*

Solution. Given : P = 15 kW = 15 × 10^3 W ; N = 960 r.p.m. ; d = 40 mm ; l = 75 mm ; τ = 56 MPa = 56 N/mm^2 ; σ_c = 112 MPa = 112 N/mm^2

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\, w$$

$$\therefore \quad w = 149 \times 10^3 / 84 \times 10^3 = 1.8 \text{ mm}$$

This width of keyway is too small. The width of keyway should be at least d / 4.

$$\therefore \quad w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm}$$ **Ans.**

Since $\sigma_c = 2\tau$, therefore a square key of w = 10 mm and t = 10 mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \qquad \ldots (\because h = t/2)$$

$$= 1 - 0.2\left(\frac{10}{20}\right) - \left(\frac{10}{2 \times 40}\right) = 0.8125$$

∴ Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\,(40)^3\ 0.8125 = 571\ 844 \text{ N}$$

and shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\ 000 \text{ N}$$

$$\therefore \quad \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\ 000}{571\ 844} = 1.47$$ **Ans.**

13.11 Shaft Coupling

Shafts are usually available up to 7 metres length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.