2. Show that $\neg Q, P \rightarrow Q \Rightarrow \neg P$.

Sol:

| Steps | derivation | Rule | Reason |
|---|---|---|---|
| (1) | $P \rightarrow Q$ | P | Given Premise |
| (2) | $\neg Q \rightarrow \neg P$ | T | (1), $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$ |
| (3) | $\neg Q$ | P | Given Premise |
| (4) | $\neg P$ | T | (2) (3) modus ponens $P, P \rightarrow Q \Rightarrow Q$. |

3. Show that R v S follows logically from the Premises $C \vee D$, $(C \vee D) \rightarrow \neg H$, $\neg H \rightarrow (A \wedge \neg B)$, and $(A \wedge \neg B) \rightarrow (R \vee S)$

Sol:

| Steps | derivation | Rule | Reason |
|---|---|---|---|
| (1) | $(C \vee D) \rightarrow \neg H$ | P | Given Premise |
| (2) | $\neg H \rightarrow A \wedge \neg B$ | P | " |
| (3) | $(C \vee D) \rightarrow (A \wedge \neg B)$ | T | [(1), (2) Hyp. Syllogism $(P \rightarrow Q) \wedge (Q \rightarrow R) \Rightarrow P \rightarrow R$] |
| (4) | $(A \wedge \neg B) \rightarrow R \vee S$ | P | Given Premise |
| (5) | $(C \vee D) \rightarrow R \vee S$ | T | [(3) (4) Hyp. Syllogism] |
| (6) | $C \vee D$ | P | Given Premise |
| (7) | $R \vee S$ | T | [(5), (6) modus ponens $P, P \rightarrow Q \Rightarrow Q$] |

4) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ & $\neg M$.

Solu:

| Steps | derivation | Rule | Reason. |
|---|---|---|---|
| 1 | $P \rightarrow M$ | P | Given premise |
| 2 | $\neg M$ | P | " |
| 3 | $\neg P$ | T | (1),(2) modus tollens $\neg Q, P \rightarrow Q \Rightarrow \neg P$ |
| 4 | $P \vee Q$ | P | Given premise |
| 5 | $Q$ | T | [(3),(4) disjunctive syllogism] $\neg P, P \vee Q \Rightarrow Q$ |
| 6 | $Q \rightarrow R$ | P | Given premise |
| 7 | $R$ | T | [(5),(6) modus ponens $P, P \rightarrow Q \Rightarrow Q$] |
| 8 | $R \wedge (P \vee Q)$ | T | (4),(7) $P, Q \Rightarrow P \wedge Q$ |

5) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

Solu:

| Steps | derivation | Rule | Reason |
|---|---|---|---|
| 1 | $P \vee Q$ | P | Given Premise |
| 2 | $\neg P \rightarrow Q$ | T | [(1), $P \rightarrow Q \Leftrightarrow \neg P \vee Q$] |
| 3 | $Q \rightarrow S$ | P | Given Premise |
| 4 | $\neg P \rightarrow S$ | T | [(2),(3), Hyp. Syllogism] |
| 5 | $\neg S \rightarrow P$ | T | [(4) $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$] |
| 6 | $P \rightarrow R$ | P | Given Premise |
| 7 | $\neg S \rightarrow R$ | T | (5),(6) Hyp. Syllogism |
| 8. | $S \vee R$ | (T) | [(7), $P \rightarrow Q \Leftrightarrow \neg P \vee Q$] |

Problems based on Rule cp (Conditional proof)

6. Show that $R \to S$ can be derived from the Premises $P \to (Q \to S)$, $\neg R \vee P$ & $Q$.

Solu:

Instead of deriving $R \to S$, we shall include R as an additional premise and show S first.

| Steps | derivation | Rule | Reason |
|---|---|---|---|
| 1 | $\neg R \vee P$ | P | Given Premise |
| 2 | R | P | Additional premise |
| 3 | P | T | [1],[2] $\neg P, P \vee Q \Rightarrow Q$ disjunctive syllogism |
| 4 | $P \to (Q \to S)$ | P | Given |
| 5 | $Q \to S$ | T | [3],[4] $P, P \to Q \Rightarrow$ Modus ponens |
| 6 | Q | P | Given premise |
| 7 | S | T | [5],[6] $P, P \to Q \Rightarrow Q$ |
| 8 | $R \to S$ | CP | |

7. Using conditional proof prove that

$\neg P \vee Q$, $\neg Q \vee R$, $R \to S \Rightarrow P \to S$

Solu:

Instead of deriving $P \to S$, we shall include P as an additional Premise and derive S.

| Steps | derivations | Rule | Reason |
|---|---|---|---|
| 1 | $\neg P \vee Q$ | P | Given Premise |
| 2 | P | P | Additional Premise |
| 3 | Q | T | (1),(2) $\neg P, P \vee Q \Rightarrow Q$ disjunctive syllogism |
| 4 | $\neg Q \vee R$ | P | Given Premise |
| 5 | R | T | (3),(4) disjunctive syllogism |
| 6 | $R \rightarrow S$ | P | Given Premise |
| 7 | S | T | (5),(6) Modus ponens |
| 8 | $P \rightarrow S$ | Rule CP | |