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NORMAL FORMS:

Elementary product

The product of variables and their negations is called the elementary product (Product means conjunction).

Eg: $P \wedge Q$, $P \wedge \neg Q$, $P \wedge \neg P$ & $P \wedge Q \wedge \neg R$.

Elementary Sum

The sum of variables and ~~their~~ their negation is called an elementary sum (sum means disjunction)

Eq: $P \vee Q$, $P \vee \neg Q \vee R$.

Factor

Any part of an elementary product (or) elementary sum, which is itself an elementary product (or) sum is a factor of the product (or) sum.

Eq: $Q \vee P$ is a factor of $\neg Q \vee Q \vee P$.

Sum of product

$(P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$

~~Sum~~ Product of Sums

$(P \vee Q) \wedge (P \vee \neg Q) \wedge (\neg P \vee \neg Q)$.

Minterms.

Let P and Q be two statement variable. Construct all possible formula which consist of conjunctions of P (or) its negation & conjunction of Q (or) its negation. None of the formula should contain both a variable & its negation

Delete a formula if it is the commutative of any one of the remaining formula.
Such conjunctions of P and Q are called the min terms of P & Q.

Min terms of 2 & 3 variables.

1) $P\wedge Q$, $P\wedge\neg Q$, $\neg P\wedge Q$, $\neg P\wedge\neg Q$ are minterms in two variables P & Q.

2) $P\wedge Q\wedge R$, $\neg P\wedge Q\wedge R$, $P\wedge\neg Q\wedge R$, $P\wedge Q\wedge\neg R$, $\neg P\wedge\neg Q\wedge R$, $P\wedge\neg Q\wedge\neg R$, $\neg P\wedge Q\wedge\neg R$, $\neg P\wedge\neg Q\wedge\neg R$ are minterms in three variables P, Q & R.

Max terms:

For a given number of variables, the maxterm consists of disjunctions in which each variable or its negation ~~appear~~, but not both, appears only once.

Max terms of 2 & 3 variable.

1. $P\vee Q$, $P\vee\neg Q$, $\neg P\vee Q$, $\neg P\vee\neg Q$ are maxterms of two variables P and Q.

2. $P\vee Q\vee R$, $\neg P\vee Q\vee R$, $P\vee\neg Q\vee R$, $P\vee Q\vee\neg R$, $\neg P\vee\neg Q\vee R$, $P\vee\neg Q\vee\neg R$, $\neg P\vee Q\vee\neg R$, $\neg P\vee\neg Q\vee\neg R$ are maxterms of the three variables P, Q and R.

Disjunctive Normal form (DNF)

A formula which is equivalent to a given formula and which consists of a sum of elementary products is called a disjunctive normal form (DNF). of the given formula

Conjunctive Normal Form (CNF)

A formula which is equivalent to a given formula and which consists of a product of elementary sums is called conjunctive normal form (CNF) of the given formula

Principal Disjunctive Normal form (PDNF) (or) Minterms-normal form (or) Sum of products form

For a given formula, an equivalent formula consisting of disjunctions of ~~minterms~~ minterms only is known as its principal disjunctive normal form. This normal form is also called the sum of products of canonical form. Eg: $P \equiv P \wedge T = P \wedge (Q \vee \neg Q) \equiv (P \wedge Q) \vee (P \wedge \neg Q)$ is PDNF of P.

Principal conjunctive normal form (or) product of Sums canonical form [PCNF]

For a given formula, an equivalent formula consisting of conjunctions of the max-terms only is known as its principal conjunctive normal form. This normal form is also called the product of sums canonical form. Ex: $P \equiv P \vee F \equiv P \vee (Q \wedge \neg Q)$
$\equiv (P \vee Q) \wedge (P \vee \neg Q)$ is the PCNF of P.

) Obtain the PDNF of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

Soln:

$(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R) \Leftrightarrow ((P \wedge Q) \wedge T) \vee ((\neg P \wedge R) \wedge T) \vee ((Q \wedge R) \wedge T)$

$\Leftrightarrow ((P \wedge Q) \wedge (R \vee \neg R)) \vee ((\neg P \wedge R) \wedge (Q \vee \neg Q) \vee (Q \wedge R) \wedge (P \vee \neg P))$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge R \wedge Q) \vee (\neg P \wedge \neg Q \wedge R)$
$\vee (P \wedge Q \wedge R) \vee (\neg P \wedge Q \wedge R)$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

The RHS is the sum of minterms.

Hence RHS is required PDNF.

2) Obtain the PCNF of the formula $S: (\neg P \to R) \wedge (Q \leftrightarrow P)$ and hence obtains its PDNF.

Soln:

PCNF:

$$S \Leftrightarrow (\neg P \to R) \wedge ((Q \to P) \wedge (P \to Q))$$

$$\Leftrightarrow (P \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q)$$

$$\Leftrightarrow (P \vee R \vee F) \wedge (\neg Q \vee P \vee F) \wedge (\neg P \vee Q \vee F)$$

$$\Leftrightarrow ((P \vee R) \vee (Q \wedge \neg Q)) \wedge ((\neg Q \vee P) \vee (R \wedge \neg R)) \wedge ((\neg P \vee Q) \vee (R \wedge \neg R))$$

$$\Leftrightarrow [(P \vee Q \vee R) \wedge (P \vee \neg Q \vee R)] \wedge [(P \vee \neg Q \vee R) \wedge (P \vee \neg Q \wedge \neg R)] \wedge [(\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)]$$

$$S \Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \wedge \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$$

The RHS is the product of sum form.

Hence RHS is the required PCNF of S.

PCNF of $\neg S \Leftrightarrow (P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$

PDNF of $S \Leftrightarrow \neg$ (PCNF of $\neg S$)

$$\Leftrightarrow (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$$

3) Obtain PDNF and PCNF of $P \to ((P \to Q) \wedge (\neg Q \vee \neg P))$ by truth table method.

Solu:

Let $S : P \to ((P \to Q) \wedge (\neg Q \vee \neg P))$

Truth table:

P	Q	$P \to Q$	$\neg Q \vee \neg P$	$(P \to Q) \wedge (\neg Q \vee \neg P)$	S	Minterm	Maxterm
T	T	T	F	F	F	-	$\neg P \vee \neg Q$
T	F	F	T	F	F	-	$\neg P \vee Q$
F	T	T	T	T	T	$\neg P \wedge Q$	-
F	F	T	T	T	T	$\neg P \wedge \neg Q$	-

Consider the truth value T

$\therefore$ Minterms are $\neg P \wedge Q$, $\neg P \wedge \neg Q$

$\therefore$ PDNF of S: $(\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$

PDNF of $\neg S$: $(P \wedge Q) \vee \cancel{\neg P}\,(P \wedge \neg Q)$

PCNF of $S = \neg$ (PDNF of $\neg S$)

$\Rightarrow$ PCNF of S : $(\neg P \vee \neg Q) \wedge (\neg P \vee Q)$

4) Obtain the Principal disjunctive normal form of $(P \wedge Q) \vee (\neg P \wedge R)$. 1) Using truth table 2) without using truth table.

Soln.

P	Q	R	$\neg P$	$P \wedge Q$	$\neg P \wedge R$	$(P \wedge Q) \vee (\neg P \wedge R)$	Minterms
T	T	T	F	T	F	T	$P \wedge Q \wedge R$
T	T	F	F	T	F	T	$P \wedge Q \wedge \neg R$
T	F	T	F	F	F	F	–
T	F	F	F	F	F	F	–
F	T	T	T	F	T	T	$\neg P \wedge Q \wedge R$
F	T	F	T	F	F	F	–
F	F	T	T	F	T	T	$\neg P \wedge \neg Q \wedge R$
F	F	F	T	F	F	F	–

The PDNF is

$$(P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$$

(2) without using truth table

$$(P \wedge Q) \vee (\neg P \wedge R) \Leftrightarrow ((P \wedge Q) \wedge T) \vee ((\neg P \wedge R) \wedge T))$$

$$\Leftrightarrow ((P \wedge Q) \wedge (R \wedge \neg R)) \vee ((\neg P \wedge R) \wedge (Q \vee \neg Q))$$

$$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R \vee (\neg P \wedge R \wedge Q) \vee (\neg P \wedge R \wedge \neg Q$$

$$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$$

is the required PDNF