Theory of Inference

Method of Contradiction

In order to show that a conclusion C follows logically from the premises $H_1, H_2, \cdots, H_m$, we assume that C is false and consider $\neg C$ as an additional premises. If the new set of premises gives contradict value, then the assumption $\neg C$ is true does not hold simultaneously with $H_1 \wedge H_2 \wedge \cdots \wedge H_m$ being true.

Method of Contrapositive

In order to prove $H_1 \wedge H_2 \wedge \cdots \wedge H_m \Rightarrow C$, if we prove $\neg C \Rightarrow \neg (H_1 \wedge H_2 \wedge \cdots \wedge H_m)$ then the original problem follows. This method is called contrapositive method.

Rule of Inference

Rule P : A premise may be introduced at any point in the derivation

Rule T : A formula S may be introduced at any point in a derivation if S is tautologically implied by any one or more of the preceding formulas.

Rule CP : If S can be derived from R and set of premises, then $R \rightarrow S$ can be derived from the set of premises alone.

Implication Rules

1. a) $P, \ P \to Q \ \Rightarrow \ Q$

   b) $\neg Q, \ P \to Q \ \Rightarrow \ \neg P$

   c) $\neg P, \ P \lor Q \ \Rightarrow \ Q$

2. $P \to Q, \ Q \to R \ \Rightarrow \ P \to R$

3. $P, Q \ \Rightarrow \ P \land Q$

   $P \land Q \ \Rightarrow \ P, Q$

   $P, Q \ \Rightarrow \ P \lor Q$

   $P \land \neg Q \ \Leftrightarrow \ \neg (P \to Q)$

   $P \to \neg Q, \ Q \ \Rightarrow \ \neg P$