(1) Demonstrate that R is a valid inference from the premises $P \to Q$, $Q \to R$ and P.

Here given premises are

(1) $P \to Q$

(2) $Q \to R$

(3) P

{1}	1) $P \to Q$	Rule P
{2}	2) P	Rule P
{1,2}	3) Q	Rule T ($P, P \to Q \Rightarrow Q$)
{4}	4) $Q \to R$	Rule P
{1,2,4}	5) R	Rule T ($P, P \to Q \Rightarrow Q$

2. Show that $(P \to Q) \wedge (R \to S), (Q \to M) \wedge (S \to N), \neg(M \wedge N)$ and $(P \to R) \Rightarrow \neg P$.

Given premises are $(P \to Q) \wedge (R \to S)$, $(Q \to M) \wedge (S \to N)$, $\neg(M \wedge N)$ and $(P \to R)$

Conclusion is $\neg P$.

{1}	1) $(P \to Q) \wedge (R \to S)$	Rule P
{1}	2) $P \to Q$	Rule T $(P \wedge Q \Rightarrow P)$
{1}	3) $R \to S$	Rule T $(P \wedge Q \Rightarrow Q)$
{4}	4) $(Q \to M) \wedge (S \to N)$	Rule P
{4}	5) $Q \to M$	Rule T $(P \wedge Q \Rightarrow P)$
{4}	6) $S \to N$	Rule T $(P \wedge Q \Rightarrow Q)$
{1,4}	7) $P \to M$	Rule T $(P \to Q, Q \to R \Rightarrow P$
{1,4}	8) $R \to N$	Rule T $(P \to Q, Q \to R \Rightarrow P$
{9}	9) $P \to R$	Rule P

{1,4,9}	10) $P \rightarrow N$	Rule T ($P \rightarrow a$, $a \rightarrow R \Rightarrow$
{1,4,9}	11) $\neg N \rightarrow \neg P$	Rule T (Taking $\neg$)
{1,4}	12) $\neg M \rightarrow \neg P$	Rule T (Taking $\neg$)
{1,4,9}	13) $(\neg M \vee \neg N) \rightarrow \neg P$	Rule T ($a \rightarrow b$, $c \rightarrow b \Rightarrow (a \vee c) \rightarrow b$)
{1,4,9}	14) $\neg (M \wedge N) \rightarrow \neg P$	Rule T (Demorgan's law)
{15}	15) $\neg (M \wedge N)$	Rule P
{1,4,9,15}	16) $\neg P$	Rule T (P, $P \rightarrow Q \Rightarrow Q$)

3.

Prove that the following argument is valid :

$p \rightarrow \neg q$, $r \rightarrow q$, $r \Rightarrow \neg p$

Given premises are $p \rightarrow \neg q$, $r \rightarrow q$, r

Conclusion is $\neg p$.

{1}	1) r	Rule P
{2}	2) $r \rightarrow q$	Rule P
{1,2}	3) q	Rule T ($P, P \rightarrow Q \Rightarrow Q$)
{4}	4) $p \rightarrow \neg q$	Rule P
{1,2,4}	5) $\neg p$	Rule T ($P \rightarrow \neg Q, Q \Rightarrow \neg P$)