TOPIC:9.- Predicate & Quantifiers

The Predicate Calculus

Consider the following statement :

"Ram is a boy"

In the above statement, "is a boy" is the predicate and the name "ram" is the subject.

If we denote the predicate "is a boy" by B and subject "Ram" by r, then the statement "Ram is a boy" can be represented by as $B(r)$

Example

"Sam is poor and Ram is intelligent"

This given statement can be symbolized as

$P(s) \wedge I(r)$.

Statement Functions

A simple statement function of one variable is defined to be an expression consisting of a predicate symbol and an individual variable.

Example

$M(x)$: x is mortal

A statement function becomes a statement when the variable is replaced by the name of any object.

Definition

Compound statement function is obtained by combining one or more simple statement functions using logical connective.

Example Let $M(x)$: x is a man

and $H(x)$: x is a mortal.

be the 2 simple statement functions

Then we can form compound statement functions as

(i) $M(x) \vee H(x)$

(ii) $M(x) \wedge H(x)$

(iii) $M(x) \rightarrow H(x)$

(iv) $\neg H(x)$

(v) $M(x) \leftrightarrow \neg H(x)$

Definition

A statement function of 2 variables is an expression consisting of a predicate symbol and 2 individual variables.

Example

$G(x, y)$: x is taller than y

In order to obtain a statement, replace x and y by the names of objects.

Quantifiers

Quantifier is one which is used to quantify the nature of variables.

There are 2 important quantifiers which are for "all" and for "some" where "some" means "atleast one".

Universal Quantifer

The quantifer "for all x" is called the universal quantifer. It is denoted by the symbol "$(\forall x$ or $(x))$". The universal quantifer is equivalent to each of the following phrases.

(1) For all x

(2) For every x

(3) For each x

(4) Everything x is such that

(5) Each thing x is such that

Example

(1) "Every apple is red"

For all x, if x is an apple then x is red $\rightarrow (*)$

Now, we will translate it into symbolic form using universal quantifier.

Define $A(x)$: x is an apple

$R(x)$: x is red

$\therefore$ we write $(*)$ into symbolic form as

$$(\forall x)\ (A(x) \rightarrow R(x))$$

(2) "Everything is yellow"

For all x, x is yellow $\rightarrow (*)$

Now, we will translate it into symbolic form using universal quantifier.

Define $Y(x)$: x is yellow.

we write (*) into symbolic form as

$$(\forall x)\,(Y(x))$$

Existential Quantifier

The quantifier for "some x" is called the existential quantifier. It is denoted by the symbol "$(\exists x)$". The existential quantifier is also equivalent to each of the following phrases

(1) For some x
(2) Some x such that
(3) There exists an x such that
(4) There is an x such that
(5) There is atleast one x such that

Example

"Some men are clever"

" there is an x such that x is a man and x is clever" $\longrightarrow$ (*)

Now, we translate it into symbolic form using existential quantifier.

Let $M(x)$: x is a man

and $C(x)$: x is clever