SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

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Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

COURSE NAME : 19EC306 – Digital Circuits

II YEAR / III SEMESTER

Unit I- MINIMIZATION TECHNIQUES AND LOGIC GATES
Topic : Karnaugh map Minimization

Karnaugh map Minimization

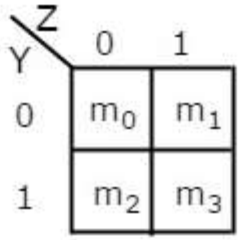
we have simplified the Boolean functions using Boolean postulates and theorems. It is a time consuming process and we have to re-write the simplified expressions after each step.
To overcome this difficulty, **Karnaugh** introduced a method for simplification of Boolean functions in an easy way. This method is known as Karnaugh map method or K-map method. It is a graphical method, which consists of 2^n cells for 'n' variables. The adjacent cells are differed only in single bit position.

K-Maps for 2 to 5 Variables

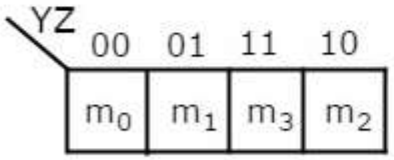
K-Map method is most suitable for minimizing Boolean functions of 2 variables to 5 variables. Now, let us discuss about the K-Maps for 2 to 5 variables one by one.

2 Variable K-Map

The number of cells in 2 variable K-map is four, since the number of variables is two. The following figure shows **2 variable K-Map**.



or



3 Variable K-Map

The number of cells in 3 variable K-map is eight, since the number of variables is three. The following figure shows **3 variable K-Map**.

X \ YZ	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

- There is only one possibility of grouping 8 adjacent min terms.
- The possible combinations of grouping 4 adjacent min terms are {(m_0, m_1, m_3, m_2), (m_4, m_5, m_7, m_6), (m_0, m_1, m_4, m_5), (m_1, m_3, m_5, m_7), (m_3, m_2, m_7, m_6) and (m_2, m_0, m_6, m_4)}.
- The possible combinations of grouping 2 adjacent min terms are {(m_0, m_1), (m_1, m_3), (m_3, m_2), (m_2, m_0), (m_4, m_5), (m_5, m_7), (m_7, m_6), (m_6, m_4), (m_0, m_4), (m_1, m_5), (m_3, m_7) and (m_2, m_6)}.
- If x=0, then 3 variable K-map becomes 2 variable K-map.

4 Variable K-Map

The number of cells in 4 variable K-map is sixteen, since the number of variables is four. The following figure shows **4 variable K-Map**.

WX \ YZ	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

5 Variable K-Map

The number of cells in 5 variable K-map is thirty-two, since the number of variables is 5. The following figure shows **5 variable K-Map**.

V=0

WX \ YZ	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

V=1

WX \ YZ	00	01	11	10
00	m_{16}	m_{17}	m_{19}	m_{18}
01	m_{20}	m_{21}	m_{23}	m_{22}
11	m_{28}	m_{29}	m_{31}	m_{30}
10	m_{24}	m_{25}	m_{27}	m_{26}

Minimization of Boolean Functions using K-Maps

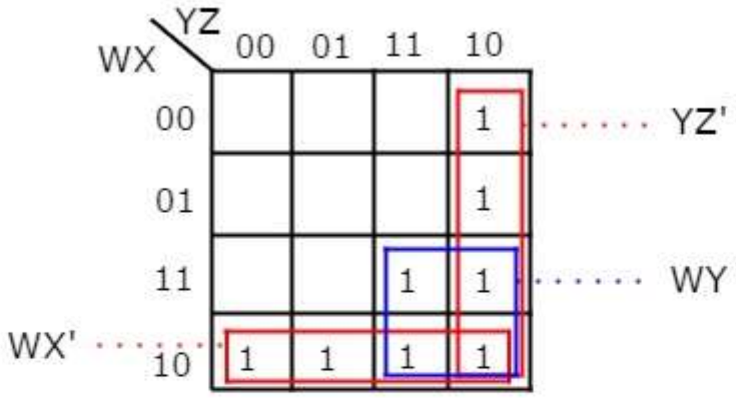
Example

Let us **simplify** the following Boolean function, **f*W,X,Y,Z***
= **WX’Y’ + WY + W’YZ’** using K-map.

The given Boolean function is in sum of products form. It is having 4 variables W, X, Y & Z. So, we require **4 variable K-map**. The **4 variable K-map** with ones corresponding to the given product terms is shown in the following figure.

WX \ YZ	00	01	11	10
00				1
01				1
11			1	1
10	1	1	1	1

There are no possibilities of grouping either 16 adjacent ones or 8 adjacent ones. There are three possibilities of grouping 4 adjacent ones. After these three groupings, there is no single one left as ungrouped. So, we no need to check for grouping of 2 adjacent ones. The **4 variable K-map** with these three **groupings** is shown in the following figure.



Here, we got three prime implicants WX’, WY & YZ’. All these prime implicants are **essential** because of following reasons.

•Two ones **(m_8 & m_9)** of fourth row grouping are not covered by any other groupings. Only fourth row grouping covers those two ones.

•Single one **(m_{15})** of square shape grouping is not covered by any other groupings. Only the square shape grouping covers that one.

•Two ones **(m_2 & m_6)** of fourth column grouping are not covered by any other groupings. Only fourth column grouping covers those two ones.

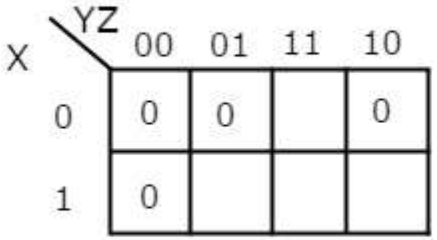
Therefore, the **simplified Boolean function** is

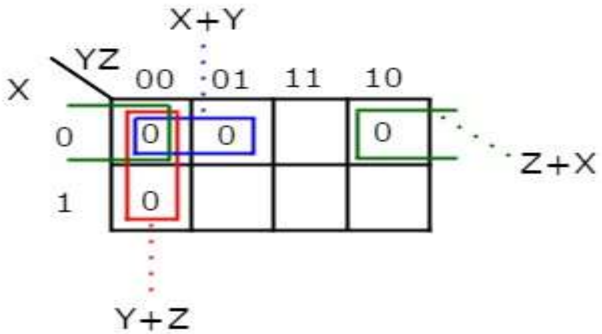
f = WX’ + WY + YZ’

Example

Let us **simplify** the following Boolean function, $f(X,Y,Z)=\prod M(0,1,2,4)$ using K-map.

The given Boolean function is in product of Max terms form. It is having 3 variables X, Y & Z. So, we require 3 variable K-map. The given Max terms are M_0, M_1, M_2 & M_4. The 3 **variable K-map** with zeroes corresponding to the given Max terms is shown in the following figure.





There are no possibilities of grouping either 8 adjacent zeroes or 4 adjacent zeroes. There are three possibilities of grouping 2 adjacent zeroes. After these three groupings, there is no single zero left as ungrouped. The **3 variable K-map** with these three **groupings** is shown in the following figure.

Here, we got three prime implicants X + Y, Y + Z & Z + X. All these prime implicants are **essential** because one zero in each grouping is not covered by any other groupings except with their individual groupings.

Therefore, the **simplified Boolean function** is

f = ***X+Y.Y+Z.Z+X***

Any Query????

Thank you……