SNS COLLEGE OF ENGINEERING

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

COURSE NAME : 19EC306 – Digital Circuits

II YEAR / III SEMESTER

Unit I- MINIMIZATION TECHNIQUES AND LOGIC GATES
Topic : De-Morgan‟s Theorem - Principle of Duality - Boolean expression - Minimization of Boolean expressions

Theorems of Boolean Algebra

The following two theorems are used in Boolean algebra.

- Duality theorem
- DeMorgan's theorem

Duality Theorem

This theorem states that the **dual** of the Boolean function is obtained by interchanging the logical AND operator with logical OR operator and zeros with ones. For every Boolean function, there will be a corresponding Dual function.

Let us make the Boolean equations *relations*

that we discussed in the section of Boolean postulates and basic laws into two groups. The following table shows these two groups.

Group1	Group2
$x + 0 = x$	$x.1 = x$
$x + 1 = 1$	$x.0 = 0$
$x + x = x$	$x.x = x$
$x + x' = 1$	$x.x' = 0$
$x + y = y + x$	$x.y = y.x$
$x + y + z = x + y + z$	$x. y. z = x. y .z$
$x. y + z = x.y + x.z$	$x + y. z = x + y . x + z$

DeMorgan's Theorem

This theorem is useful in finding the **complement of Boolean function**. It states that the complement of logical OR of at least two Boolean variables is equal to the logical AND of each complemented variable.

DeMorgan's theorem with 2 Boolean variables x and y can be represented as

$$(x + y)' = x'.y'$$

The dual of the above Boolean function is

$$(x.y)' = x' + y'$$

Example 1

Let us **simplify** the Boolean function, f = p'qr + pq'r + pqr' + pqr

We can simplify this function in two methods.

Method 1

Given Boolean function, f = p'qr + pq'r + pqr' +pqr.

Step 1 − In first and second terms r is common and in third and fourth terms pq is common. So, take the common terms by using **Distributive law**.

$$\Rightarrow f = \; p'q + pq' \;\; r + pq \;\; r' + r$$

Step 2 − The terms present in first parenthesis can be simplified to Ex-OR operation. The terms present in second parenthesis can be simplified to '1' using **Boolean postulate**

$$\Rightarrow f = \; p \oplus q \;\; r + pq \;\; 1$$

Step 3 − The first term can't be simplified further. But, the second term can be simplified to pq using **Boolean postulate**.

$$\Rightarrow f = \; p \oplus q \;\; r + pq$$

Method 2

Given Boolean function, f = p’qr + pq’r + pqr’ + pqr.

Step 1 − Use the **Boolean postulate**, x + x = x. That means, the Logical OR operation with any Boolean variable ‘n’ times will be equal to the same variable. So, we can write the last term pqr two more times.

$$\Rightarrow f = p'qr + pq'r + pqr' + pqr + pqr + pqr$$

Step 2 − Use **Distributive law** for 1st and 4th terms, 2nd and 5th terms, 3rd and 6th terms.

$$\Rightarrow f = qr(p' + p) + pr(q' + q) + pq(r' + r)$$

Step 3 − Use **Boolean postulate**, x + x’ = 1 for simplifying the terms present in each parenthesis.

$$\Rightarrow f = qr(1) + pr(1) + pq(1)$$

Step 4 − Use **Boolean postulate**, x.1 = x for simplifying the above three terms.

$$\Rightarrow f = qr + pr + pq$$

$$\Rightarrow f = pq + qr + pr$$

Any Query????

Thank you……