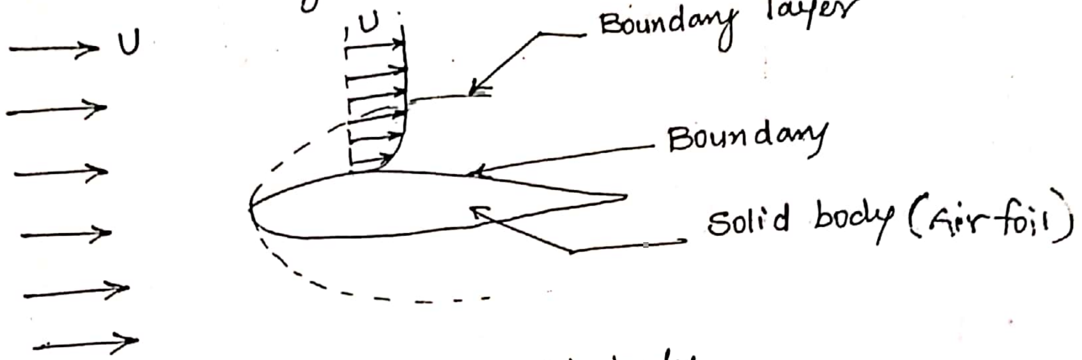
Boundary Layer:

When a real fluid flows past a solid body or a solid wall, the velocity close to the boundary of body will be same as that of the boundary. If the boundary is stationary, the velocity of fluid at the boundary will be zero.



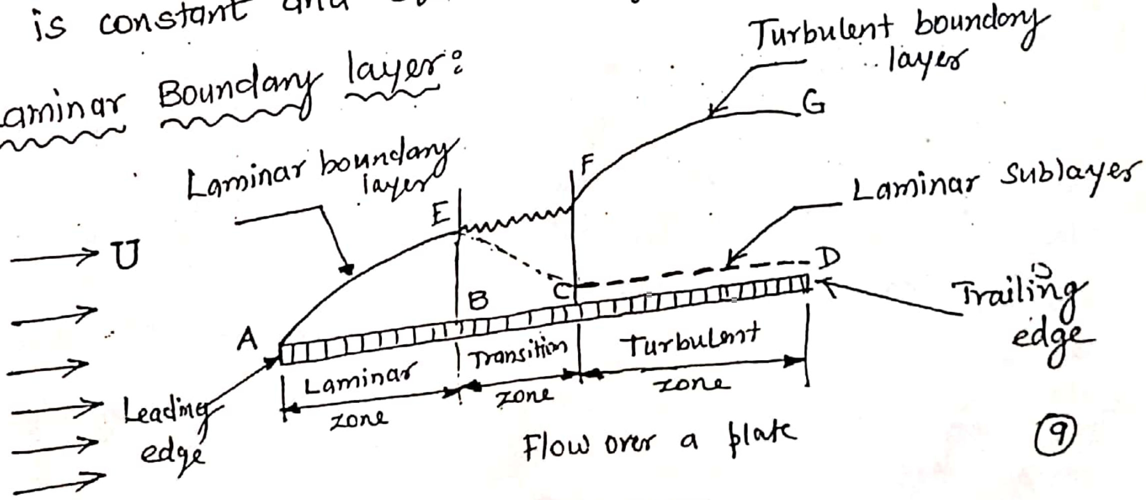
Flow over solid body

Farther away from the boundary, the velocity of fluid increases, will be higher and as a result of this variation, $\frac{du}{dy}$ will exist. The velocity of fluid increases from zero velocity on the stationary boundary to free-stream velocity (U) of the fluid in the direction normal to the boundary.

This change is a narrow region, known as boundary layer. The theory dealing with boundary layer is called as boundary layer theory.

Outside the boundary layer the velocity is constant and equal to free-stream velocity.

Laminar Boundary layer:



Flow over a plate

Near the leading edge of the surface of the plate, where the thickness is small, the flow in the boundary layer is laminar though the main flow is turbulent. This layer of the fluid is said to be laminar boundary layer.

The length of the plate from the leading edge upto which laminar boundary layer exists, is called laminar zone. It can be obtained from Reynolds number (Re) 5×10^5 for the plate.

$$Re = \frac{U \times x}{\nu}$$

Re - Reynold's number = 5×10^5
U - Free-stream velocity of fluid.
ν - kinematic viscosity of fluid

Turbulent Boundary layer:

If the length of the plate is more than the laminar zone, the thickness of boundary layer will go on increasing in the down-stream direction. Then the laminar boundary layer becomes unstable and motion of the fluid is disturbed and irregular, which leads to a transition from laminar to turbulent is called transition zone.

Further, down stream the transition zone, The boundary layer is turbulent and continues to grow in thickness. This layer of boundary is called turbulent boundary layer.

Laminar Sub-layer:

This is the region in turbulent boundary layer zone, adajacent to the solid surface, where the velocity variation is influenced only by viscous effect. Velocity variation is assumed to be linear and so the $\frac{du}{dy}$ = constant. Therefore, the shear stress in the laminar sub-layer would be constant and equal to the boundary shear stress.

$$\tau_0 = \mu \frac{u}{y}$$

Assistant Professor / Mechanical Engg
SVS College of Engg., Cbe-109

Boundary layer thickness (δ)

It is defined as the distance from the boundary of the solid body, measured in y-direction to the point, where the velocity of the fluid is approximately equal to 0.99 times the free-stream (U) velocity of the fluid.

"1904, German Scientist Ludwig Prandtl"

Displacement Thickness (δ^*) (Another method of defining boundary layer)

It is defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for the reduction in flow rate on account of boundary layer formation.

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

(or)

It indicates the distance by which the external free stream is effectively displaced due to the retardation of fluid layers in the boundary layer.

Momentum thickness (θ)

- reduction of momentum of the flowing fluid

$$\theta = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u}{U}\right] dy$$

Energy Thickness (δ^{**})

- reduction of in kinetic energy

$$\delta^{**} = \int_0^{\delta} \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy$$

Find the displacement thickness, the momentum thickness, and energy thickness for the velocity distribution in the boundary layer given by $\frac{u}{U} = \frac{y}{\delta}$, where u is the velocity at a distance y from the plate and u = U at y = δ where δ = boundary layer thickness. Also calculate the value of $\frac{\delta^*}{\theta}$.

(46)

Given data:

$$\frac{u}{U} = \frac{y}{\delta}$$

Solution :

i) Displacement thickness :

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right) dy$$

$$= \int_0^\delta \left(1-\frac{y}{\delta}\right) dy$$

$$= \left[y - \frac{y^2}{2\delta} \right]_0^\delta$$

$$= \delta - \frac{\delta^2}{2\delta} = \delta - \frac{\delta}{2} = \frac{\delta}{2}$$

ii) Momentum thickness :

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right) dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1-\frac{y}{\delta}\right) dy$$

$$= \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2} \right]_0^\delta$$

$$= \frac{\delta^2}{2\delta} - \frac{\delta^3}{3\delta^2} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{3\delta - 2\delta}{6}$$

$$\theta = \frac{\delta}{6}$$

(iii) Energy thickness :

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1-\frac{u^2}{U^2}\right) dy$$

$$= \int_0^\delta \frac{y}{\delta}\left(1-\frac{y^2}{\delta^2}\right) dy$$

$$= \int_0^\delta \left(\frac{y}{\delta} - \frac{y^3}{\delta^3}\right) dy$$

$$= \left[\frac{y^2}{2\delta} - \frac{y^4}{4\delta^3}\right]_0^\delta$$

$$= \left[\frac{\delta^2}{2\delta} - \frac{\delta^4}{4\delta^3}\right] = \frac{\delta}{2} - \frac{\delta}{4} = \frac{4\delta - 2\delta}{8} = \frac{2\delta}{8} = \frac{\delta}{4}$$

iv) $\frac{\delta^*}{\theta}$

$$= \frac{\delta/2}{\delta/6} = \frac{\delta}{2} \times \frac{6}{\delta} = 3$$

Total drag

= (Drag due to laminar for length(A) + Drag due to turbulent for entire length) − Drag due to turbulent for length (A)

Lam

$\frac{4.91x}{\sqrt{Re_x}}$

$C_D = \frac{1.328}{\sqrt{Re_L}}$

Tur

$\delta = \frac{0.372}{(Re_x)^{1/5}}$

$C_D = \frac{0.074}{(Re_L)^{1/5}}$

Formula:

* $Re_L = \frac{\rho U L}{\mu}$; $F_D = \frac{1}{2} \rho U^2 A \times C_D$; $\tau_0 = \frac{0.332 \rho U^2}{\sqrt{Re_L}}$

for $Re_L < 5 \times 10^5$

* $\delta = \frac{4.91x}{\sqrt{Re_x}}$ (~~$\frac{1.328}{\sqrt{Re_L}}$~~)

* $C_D = \frac{1.328}{\sqrt{Re_L}}$

$> 5\times10^5$ & $< 10^7$

$\delta = \frac{0.37x}{(Re_x)^{\frac{1}{5}}}$

$C_D = \frac{0.072}{(Re_L)^{\frac{1}{5}}}$

$> 10^7$

$C_D = \frac{0.455}{(\log_{10} Re_L)^{2.58}}$

Determine the thickness of the boundary layer at the trailing edge of smooth plate of length 4m and of width 1.5m, when the plate is moving with a velocity of 4m/s in stationary air. Also determine the total drag on one side of the plate assuming that (i) the boundary layer is laminar over the entire length of the plate and (ii) the boundary layer is turbulent from the very beginning. Take $\nu = 1.5 \times 10^{-5} \frac{m^2}{s}$ and $\rho = 1.226 \frac{kg}{m^3}$.

47

13

Given data:

$L = 4m$
$W = 1.5m$
$U = 4 \frac{m}{s}$
$\nu = 1.5\times10^{-5} \frac{m^2}{s}$
$\rho = 1.226 \frac{kg}{m^3}$

$\delta = ?$
$F_D = ?$ (i) Laminar
(ii) Turbulent

HD

$5\times10^5 = \frac{Ux}{\nu}$
$x = \frac{5\times10^5 \times \nu}{U}$
$= 1.875$

Solution:

$$Re_L = \frac{U\times L}{\nu} = \frac{4\times4}{1.5\times10^{-5}} = 10.67\times10^5 > 5\times10^5$$

Hence,

$$\delta = \frac{0.37x}{(Re_x)^{\frac{1}{5}}} = \frac{0.37x}{(Re_x)^{\frac{1}{5}}} = \frac{0.37\times4}{(10.67\times10^5)^{\frac{1}{5}}} = 0.092\,m$$

$$F_D = \frac{1}{2}\rho A U^2 \times C_D$$

If laminar, $C_D = \frac{1.328}{\sqrt{Re_L}} = 0.001286$

$$\therefore F_D = \frac{1}{2}\times1.226\times1.5\times4\times4^2\times0.001286$$
$$= 0.0757\,N$$

If turbulent, $C_D = \frac{0.072}{(Re_L)^{\frac{1}{5}}} = \frac{0.072}{(10.67\times10^5)^{\frac{1}{5}}} = 0.0045$

$$\therefore F_D = \frac{1}{2}\times1.226\times1.5\times4\times4^2\times0.0045$$
$$= 0.2638\,N$$

... flowing over a thin smooth plate of length ... boundary