

FLOW THROUGH CIRCULAR CONDUITS

Circular conduits is nothing, but, a pipe which is used to carry liquids under pressure.

When the pipe is running ~~to~~ full, the flow is under pressure. This kind of flow is known as pipe flow.

If, when the pipe is not running full, the flow is not under pressure; only atmospheric pressure exists inside the pipe. This kind of flow is termed as channel flow.

Types of flow in pipes:

The flow in pipes are classified as:

1. Viscous flow — controlled by viscosity (viscous force)
2. Non-viscous flow

Classification of viscous flow:

1. Laminar flow — Viscous force > Inertia force
2. Turbulant flow — Viscous force < Inertia force

Viscous flow (or) Laminar flow:

For this flow, following are to be determined,

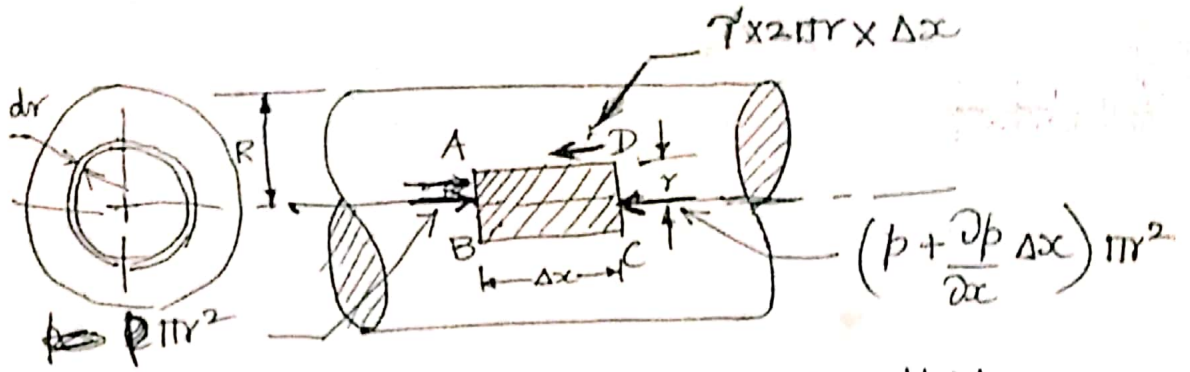
- ② * Velocity distribution across a section.
- ③ * ratio of maximum velocity to average velocity.
- ① * shear stress distribution.
- ④ * drop of pressure for a given length.

The flow through the circular pipe will be viscous or laminar, if the Reynolds number (Re) is less than 2000.

$$Re = \frac{\rho v D}{\mu}$$

ρ - Density
 v - velocity
 D - Diameter
 μ - viscosity

①



Following forces are acting on the fluid,

1. Pressure force on ^{face} AB = $p \pi r^2$
2. Pressure force on ^{face} CD = $(p + \frac{\partial p}{\partial x} \Delta x) \pi r^2$
3. Shear force, = $\gamma \times 2\pi r \Delta x$ on the surface

Since, there is no acceleration, the summation of all forces in the direction of flow should be zero

$$p \pi r^2 - (p + \frac{\partial p}{\partial x} \Delta x) \pi r^2 - \gamma \times 2\pi r \times \Delta x = 0$$

$$p \pi r^2 - p \pi r^2 - \frac{\partial p}{\partial x} \Delta x \pi r^2 - \gamma \times 2\pi r \Delta x = 0$$

$$\Rightarrow \Delta x \pi r \left[-\frac{\partial p}{\partial x} r - 2\gamma \right] = 0$$

$$\Rightarrow -\frac{\partial p}{\partial x} r - 2\gamma = 0$$

$$\downarrow \quad \downarrow$$

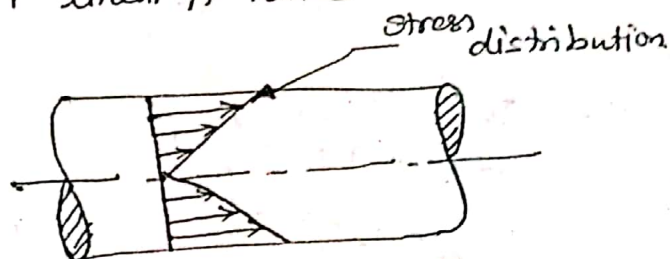
$$-2\gamma = \frac{\partial p}{\partial x} r$$

$$\gamma = -\frac{\partial p}{\partial x} \frac{r}{2} \quad \text{--- (1)}$$

$\frac{\partial p}{\partial x}$ ← pressure gradient

Shear stress distribution

From the above equation, it can be seen that 'γ' varies with 'r' linearly, hence



Velocity distribution:

U.S. UNIVERSITY
Engineering Faculty
1983 Group of B.Sc. 1983-1984

34

Velocity distribution across the section is obtained by substituting $\gamma = \mu \frac{du}{dy}$ in equation (1)

y is measured from the pipe wall,

$$y = R - r$$

$$\text{then } dy = -dr$$

$$\therefore \gamma = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

Substituting the above in (1)

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x} \frac{r}{2}$$

$$\frac{du}{dr} = \frac{1}{2\mu} \frac{\partial p}{\partial x} r$$

Integrating this above equation with respect to ' r '

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 + C \quad \text{--- (2)}$$

' C ' in above equation is the constant of integration and the value is obtained from the boundary condition that $r=R, u=0$.

$$0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 + C$$

$$C = -\frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

Substituting value of ' C ' in (2)

$$u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$$

$$u = -\frac{1}{4\mu} \frac{\partial p}{\partial x} [R^2 - r^2] \quad \text{--- (3)}$$

3

$$\Rightarrow \frac{1}{4\mu} \left(\frac{-\partial p}{\partial x} \right) 2\pi \left[\frac{4R^4 - 2R^4}{8} \right]$$

$$\Rightarrow \frac{1}{2 \cdot 4\mu} \left(\frac{-\partial p}{\partial x} \right) 2\pi \left[\frac{R^4}{4} \right]$$

$$\Rightarrow \frac{1}{8\mu} \left(\frac{-\partial p}{\partial x} \right) \pi R^4$$

$$Q \Rightarrow \frac{\pi}{8\mu} \left(\frac{-\partial p}{\partial x} \right) R^4 \rightarrow (5)$$

Team 5

Average velocity $\bar{u} = \frac{Q}{\text{Area}}$

$$= \frac{\pi \left(\frac{-\partial p}{\partial x} \right) R^4}{\pi R^2}$$

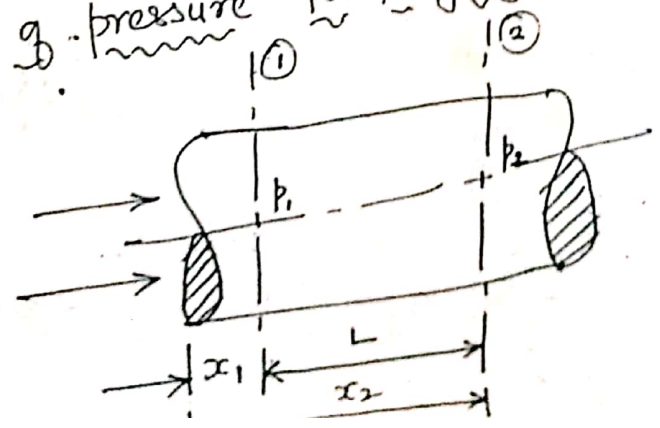
$$\bar{u} = \frac{1}{8\mu} \left(\frac{-\partial p}{\partial x} \right) R^2 \rightarrow (6)$$

Team 1

$$\frac{u_{max}}{\bar{u}} = \frac{\frac{1}{4\mu} \left(\frac{\partial p}{\partial x} \right) R^2}{\frac{1}{8\mu} \left(\frac{\partial p}{\partial x} \right) R^2} = 2$$

$$\frac{u_{max}}{\bar{u}} = 2$$

Drop of pressure for a given length (L) of a pipe:



We know that

$$\bar{u} = \frac{1}{8\mu} \left(\frac{-\partial p}{\partial x} \right) R^2$$

$$\frac{-\partial p}{\partial x} = \frac{8\mu\bar{u}}{R^2}$$

Integrating $\left(\frac{-\partial p}{\partial x} \right)$ w.r.t. x ,

$$-\int_2^1 \partial p = \int_2^1 \frac{8\mu\bar{u}}{R^2} dx$$

$$-[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2} \int_2^1 dx$$

$$= \frac{8\mu\bar{u}}{R^2} [x]_2^1$$

$$= \frac{8\mu\bar{u}}{R^2} [x_1 - x_2]$$

$$p_1 - p_2 = \frac{8\mu\bar{u}}{R^2} [x_2 - x_1]$$

$$= \frac{8\mu\bar{u}}{R^2} L$$

$$[\because x_2 - x_1 = L]$$

$$= \frac{8\mu\bar{u}}{\left(\frac{D}{2}\right)^2} L$$

$$(p_1 - p_2) = \frac{32\mu\bar{u}}{D^2} L$$

$$(p_1 - p_2) = \text{drop of pressure}$$

$$\therefore \text{Loss of pressure head} = \frac{p_1 - p_2}{\rho g} = h_f \quad [p = \rho gh]$$

$$\therefore \boxed{h_f = \frac{32\mu\bar{u}L}{\rho g D^2}} \rightarrow \text{Hagen Poiseuille Equation}$$

(b)

An oil of viscosity $0.1 \frac{\text{Ns}}{\text{m}^2}$ and relative density 0.8 flowing through a circular pipe of diameter 50 mm and length 300 m. The rate of flow through the pipe is 3.5 litres/sec. Find the pressure drop in a length of 300 m and also the shear stress at the pipe wall.

(36)

S. V. SREEMANJUN
M.E., (P.H.D.)
Assistant Professor / Mechanical Engg.
SVS College of Engg., Cbe-105

Given data:

$$\mu = 0.1 \frac{\text{Ns}}{\text{m}^2}$$

$$S = 0.8$$

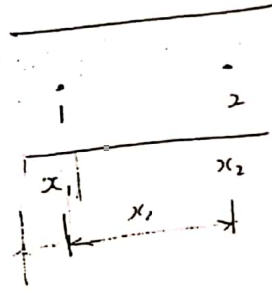
$$d = 50 \text{ mm} = 0.05 \text{ m}$$

$$l = 300 \text{ m}$$

$$Q = 3.5 \frac{\text{l}}{\text{s}} = 3.5 \times 10^{-3} \frac{\text{m}^3}{\text{s}}$$

pressure drop = $(p_1 - p_2) = ?$

shear stress = $\tau = ?$



(45)

Solution:

$$p_1 - p_2 = \frac{32 \mu \bar{u} L}{d^2}$$

$$\bar{u} = \frac{Q}{\text{Area}} = \frac{3.5 \times 10^{-3}}{\frac{\pi}{4} \times (0.05)^2} = 1.78 \frac{\text{m}}{\text{s}}$$

Team 7

$$\therefore p_1 - p_2 = \frac{32 \times 0.1 \times 1.78 \times 300}{(0.05)^2} = 683.52 \times 10^3 \frac{\text{N}}{\text{m}^2}$$

$$\tau = -\frac{\partial p}{\partial x} \frac{r}{2}$$

$r = R$ → pipe

$$-\frac{\partial p}{\partial x} = -\frac{(p_1 - p_2)}{x_2 - x_1} = \frac{p_2 - p_1}{x_2 - x_1} = \frac{p_2 - p_1}{L} = \frac{683.52 \times 10^3}{300}$$

$$= 2278.4 \frac{\text{N}}{\text{m}^3}$$

$$\tau = 2278.4 \times \frac{0.05}{2} = 28.48 \frac{\text{N}}{\text{m}^2}$$

Team 2

(7)

A fluid of viscosity $0.7 \frac{Ns}{m^2}$ and specific gravity 1.3 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as $196.2 \frac{N}{m^2}$, find (a) the pressure gradient (b) the average velocity and (c) Reynold's number of the flow.

Given data:

$$\mu = 0.7 \frac{Ns}{m^2}$$

$$S = 1.3$$

$$d = 100 \text{ mm} = 0.1 \text{ m}$$

$$\tau = 196.2 \frac{N}{m^2}$$

$$\frac{\partial p}{\partial x} = ? \quad \bar{u} = ? \quad Re = ?$$

Solution:

$$\tau = -\frac{\partial p}{\partial x} \frac{r}{2}$$

$$\frac{\partial p}{\partial x} = -\frac{\tau \times 2}{r} = -\frac{196.2 \times 2}{\frac{0.1}{2}} = -7848 \frac{N}{m^3}$$

$$\bar{u} = \frac{1}{8\mu} * \left(-\frac{\partial p}{\partial x} \right) R^2$$

$$= \frac{1}{8 \times 0.7} \times (-7848) \times \left(\frac{0.1}{2} \right)^2$$

$$= 3.50 \frac{m}{s}$$

$$Re = \frac{\rho v d}{\mu} = \frac{1300 \times 3.50 \times 0.1}{0.7}$$

$$= 650$$

$$S = \frac{\rho_l}{\rho_w} =$$

$$\Rightarrow \rho_l = S \rho_w = 1.3 \times 1000 = 1300$$