Dia. of large piston, $D = 10$ cm

∴ Area of larger piston, $A = \frac{\pi}{4} \times (10)^2 = 78.54$ cm^2

Force on small piston, $F = 80$ N

Let the load lifted $= W$.

(a) When the pistons are at the same level

Pressure intensity on small piston

$$\frac{F}{a} = \frac{80}{7.068} \text{ N/cm}^2$$

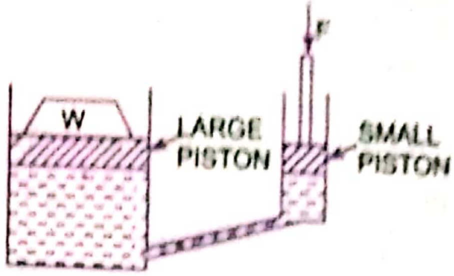


Fig. 2.5

This is transmitted equally on the large piston.

∴ Pressure intensity on the large piston

$$= \frac{80}{7.068}$$

∴ Force on the large piston

$$= \text{Pressure} \times \text{Area}$$

$$= \frac{80}{7.068} \times 78.54 \text{ N} = \mathbf{888.96 \text{ N}}. \textbf{ Ans.}$$

(b) When the small piston is 40 cm above the large piston

Pressure intensity on the small piston

$$= \frac{F}{a} = \frac{80}{7.068} \frac{\text{N}}{\text{cm}^2}$$

∴ Pressure intensity at section A-A

$$= \frac{F}{a} + \text{Pressure intensity due to height of 40 cm of liquid.}$$

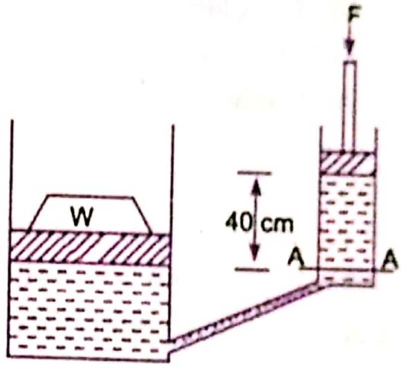


Fig. 2.6

But pressure intensity due to 40 cm of liquid

$$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2$$

$$= \frac{1000 \times 9.81 \times .40}{10^4} \text{ N/cm}^2 = 0.3924 \text{ N/cm}^2$$

∴ Pressure intensity at section A-A

$$= \frac{80}{7.068} + 0.3924$$

$$= 11.32 + 0.3924 = 11.71 \text{ N/cm}^2$$

∴ Pressure intensity transmitted to the large piston $= 11.71$ N/cm^2

∴ Force on the large piston = Pressure × Area of the large piston

$$= 11.71 \times A = 11.71 \times 78.54 = 919.7 \text{ N.}$$

▶ 2.4 ABSOLUTE, GAUGE, ATMOSPHERIC AND VACUUM PRESSURES

The pressure on a fluid is measured in two different systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and it is called gauge pressure. Thus :

1. Absolute pressure is defined as the pressure which is measured with reference to absolute vacuum pressure.

2. Gauge pressure is defined as the pressure which is measured with the help of a pressure measuring instrument, in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

3. **Vacuum pressure** is defined as the pressure below the atmospheric pressure.

The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in Fig. 2.7.

Mathematically :

(*i*) Absolute pressure

= Atmospheric pressure + Gauge pressure

or $$p_{ab} = p_{atm} + p_{gauge}$$

(*ii*) Vacuum pressure

= Atmospheric pressure − Absolute pressure.

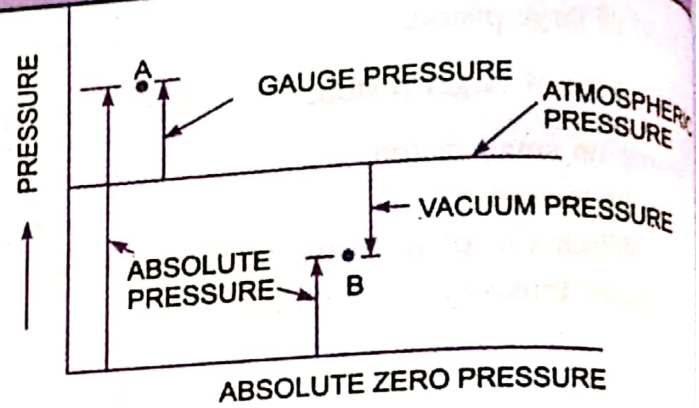


Fig. 2.7 *Relationship between pressures.*

Note. (*i*) The atmospheric pressure at sea level at 15°C is 101.3 kN/m^2 or 10.13 N/cm^2 in SI unit. In case MKS units, it is equal to 1.033 kgf/cm^2.

(*ii*) The atmospheric pressure head is 760 mm of mercury or 10.33 m of water.

Problem 2.8 *What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of 1.53×10^3 kg/m^3 if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity of mercury is 13.6 and density of water = 1000 kg/m^3.*

Solution. Given :

Depth of liquid, $Z_1 = 3$ m

Density of liquid, $\rho_1 = 1.53 \times 10^3$ kg/m^3

Atmospheric pressure head, $Z_0 = 750$ mm of Hg

$$= \frac{750}{1000} = 0.75 \text{ m of Hg}$$

∴ Atmospheric pressure, $p_{atm} = \rho_0 \times g \times Z_0$

where ρ_0 = Density of Hg = Sp. gr. of mercury × Density of water = 13.6 × 1000 kg/m^3

and Z_0 = Pressure head in terms of mercury.

∴ $$p_{atm} = (13.6 \times 1000) \times 9.81 \times 0.75 \text{ N/m}^2 \quad (\because Z_0 = 0.75)$$

$$= 100062 \text{ N/m}^2$$

Pressure at a point, which is at a depth of 3 m from the free surface of the liquid is given by,

$$p = \rho_1 \times g \times Z_1$$

$$= (1.53 \times 1000) \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

∴ Gauge pressure, p = **45028 N/m^2. Ans.**

Now absolute pressure = Gauge pressure + Atmospheric pressure

= 45028 + 100062 = **145090 N/m^2. Ans.**

▶ 2.5 MEASUREMENT OF PRESSURE

The pressure of a fluid is measured by the following devices :

1. Manometers 2. Mechanical Gauges.

2.5.1 Manometers. Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(*a*) Simple Manometers, (*b*) Differential Manometers.

2.5.2 Mechanical Gauges. Mechanical gauges are defined as the devices used for measuring the pressure by balancing the fluid column by the spring or dead weight. The commonly used mechanical pressure gauges are :

(*a*) Diaphragm pressure gauge,
(*b*) Bourdon tube pressure gauge,
(*c*) Dead-weight pressure gauge, and
(*d*) Bellows pressure gauge.

▸ 2.6 SIMPLE MANOMETERS

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer,
2. U-tube Manometer, and
3. Single Column Manometer.

2.6.1 Piezometer. It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. 2.8. The rise of liquid gives the pressure head at that point. If at a point *A*, the height of liquid say water is *h* in piezometer tube, then pressure at *A*

$$= \rho \times g \times h \ \frac{N}{m^2}.$$

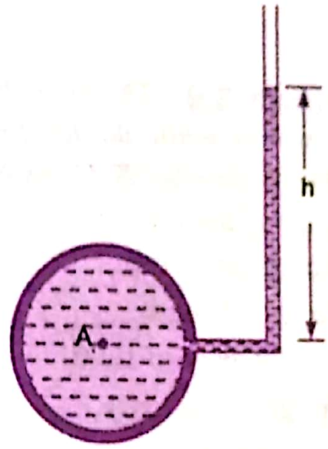


Fig. 2.8 *Piezometer.*

2.6.2 U-tube Manometer. It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in Fig. 2.9. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

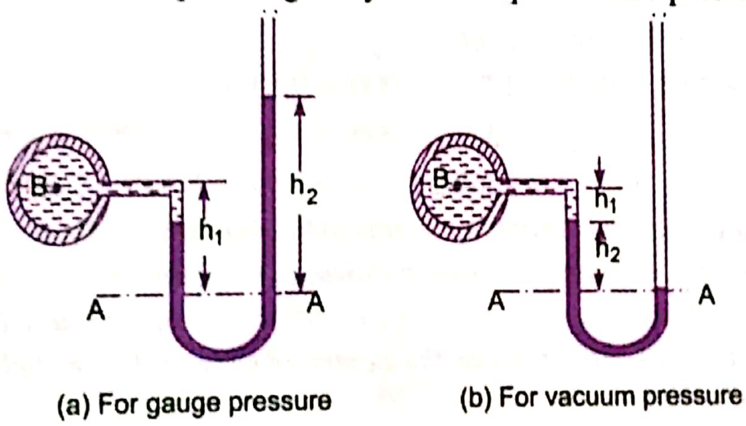


Fig. 2.9 *U-tube Manometer.*

(*a*) **For Gauge Pressure.** Let *B* is the point at which pressure is to be measured, whose value is *p*. The datum line is *A-A*.

Let h_1 = Height of light liquid above the datum line
h_2 = Height of heavy liquid above the datum line
S_1 = Sp. gr. of light liquid
ρ_1 = Density of light liquid = $1000 \times S_1$
S_2 = Sp. gr. of heavy liquid
ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$\therefore$ $p = (\rho_2 g h_2 - \rho_1 \times g \times h_1)$. ...(2.7)

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A $= 0$

$\therefore$ $\rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore$ $p = -(\rho_2 g h_2 + \rho_1 g h_1)$. ...(2.8)

Problem 2.9 *The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gr. 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.9$

$\therefore$ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900$ kg/m^3

Sp. gr. of mercury, $S_2 = 13.6$

$\therefore$ Density of mercury, $\rho_2 = 13.6 \times 1000$ kg/m^3

Difference of mercury level, $h_2 = 20$ cm $= 0.2$ m

Height of fluid from A-A, $h_1 = 20 - 12 = 8$ cm $= 0.08$ m

Let p = Pressure of fluid in pipe

Equating the pressure above A-A, we get

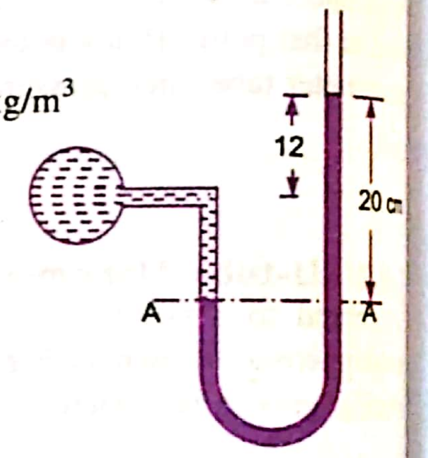


Fig. 2.10

$$p + \rho_1 g h_1 = \rho_2 g h_2$$

or $$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$= 26683 - 706 = 25977$ N/m^2 $=$ **2.597 N/cm^2. Ans.**

Problem 2.10 *A simple U-tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the centre of pipe is 15 cm below.*

Solution. Given :

Sp. gr. of fluid, $S_1 = 0.8$

Sp. gr. of mercury, $S_2 = 13.6$

Density of fluid, $\rho_1 = 800$

Density of mercury, $\rho_2 = 13.6 \times 1000$

Difference of mercury level, $h_2 = 40$ cm $= 0.4$ m. Height of liquid in left limb, $h_1 = 15$ cm $= 0.15$ m. Let the pressure in pipe $= p$. Equating pressure above datum line A-A, we get

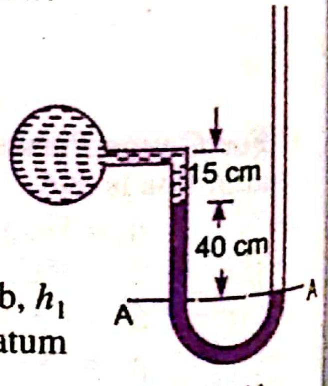


Fig. 2.11

$$\rho_2 g h_2 + \rho_1 g h_1 + p = 0$$

$$\therefore \quad p = -[\rho_2 g h_2 + \rho_1 g h_1]$$
$$= -[13.6 \times 1000 \times 9.81 \times 0.4 + 800 \times 9.81 \times 0.15]$$
$$= -[53366.4 + 1177.2] = -54543.6 \text{ N/m}^2 = \mathbf{-5.454 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 2.11 *A U-Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to atmosphere. The contact between water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to 9810 N/m², calculate the new difference in the level of mercury. Sketch the arrangements in both cases.*

Solution. Given :

Difference of mercury = 10 cm = 0.1 m

The arrangement is shown in Fig. 2.11 (*a*)

Ist Part

Let p_A = (pressure of water in pipe line (*i.e.*, at point *A*)

The points *B* and *C* lie on the same horizontal line. Hence pressure at *B* should be equal to pressure at *C*. But pressure at *B*

= Pressure at *A* + Pressure due to 10 cm (or 0.1 m) of water

$= p_A + \rho \times g \times h$

where $\rho = 1000 \text{ kg/m}^3$ and $h = 0.1$ m

$= p_A + 1000 \times 9.81 \times 0.1$

$= p_A + 981 \text{ N/m}^2$ ...(*i*)

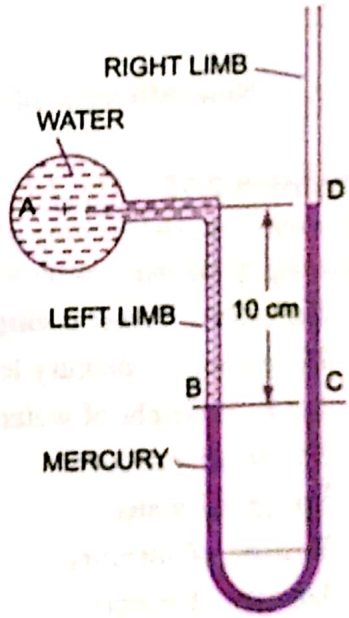


Fig. 2.11 (*a*)

Pressure at *C* = Pressure at *D* + Pressure due to 10 cm of mercury

$= 0 + \rho_0 \times g \times h_0$

where ρ_0 for mercury $= 13.6 \times 1000 \text{ kg/m}^3$

and $h_0 = 10 \text{ cm} = 0.1 \text{ m}$

$\therefore$ Pressure at $C = 0 + (13.6 \times 1000) \times 9.81 \times 0.1$

$= 13341.6$ N ...(*ii*)

But pressure at *B* is equal to pressure at *C*. Hence equating the equations (*i*) and (*ii*), we get

$$p_A + 981 = 13341.6$$
$$\therefore \quad p_A = 13341.6 - 981$$
$$= \mathbf{12360.6 \frac{N}{m^2}} \text{. Ans.}$$

IInd Part

Given, $p_A = 9810 \text{ N/m}^2$

Find new difference of mercury level. The arrangement is shown in Fig. 2.11 (*b*). In this case the pressure at *A* is 9810 N/m² which is less than the 12360.6 N/m². Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let x = Rise of mercury in left limb in cm

Then fall of mercury in right limb = x cm

The points *B*, *C* and *D* show the initial conditions whereas points B^*, C^* and D^* show the

$$= \frac{Z \times a}{A} = \frac{Z \times 0.25}{10} = \frac{Z}{40}.$$

Also fall in surface level of C

$$= \text{Rise in surface level of } B$$

$$= \frac{Z}{40}$$

The pressure of 1 cm (or 0.01 m) of water = ρgh = 1000 × 9.81 × 0.01= 98.1 N/m^2

Consider final separation level Y-Y

Pressure above Y-Y in the left limb $= 1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right)$

Pressure above Y-Y in the right limb $= 900 \times 9.81 \left(Z + h_C - \frac{Z}{40}\right) + 98.1$

Equating the two pressure, we get

$$1000 \times 9.81 \left(Z + h_B + \frac{Z}{40}\right) = \left(Z + h_C - \frac{Z}{40}\right) 900 \times 9.81 + 98.1$$

Dividing by 9.81, we get

$$1000 \left(Z + h_B + \frac{Z}{40}\right) = 900 \left(Z + h_C - \frac{Z}{40}\right) + 10$$

Dividing by 1000, we get $Z + h_B + \frac{Z}{40} = 0.9 \left(Z + h_C - \frac{Z}{40}\right) + 0.01$

But from equation (i), $h_B = 0.9\, h_C$

$$\therefore \quad Z + 0.9\, h_C + \frac{Z}{40} = \frac{39Z}{40} \times 0.9 + 0.9\, h_C + 0.01$$

or

$$\frac{41Z}{40} = \frac{39}{40} \times .9Z + .01$$

or

$$Z\left(\frac{41}{40} - \frac{39 \times .9}{40}\right) = .01 \quad \text{or} \quad Z\left(\frac{41 - 35.1}{40}\right) = .01$$

$$\therefore \quad Z = \frac{40 \times 0.01}{5.9} = \mathbf{0.0678\ m = 6.78\ cm.\ Ans.}$$

2.6.3 Single Column Manometer. Single column manometer is a modified form of a U-tube manometer in which a reservoir, having a large cross-sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs (say left limb) of the manometer as shown in Fig. 2.15. Due to large cross-sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as :

1. Vertical Single Column Manometer.
2. Inclined Single Column Manometer.

1. Vertical Single Column Manometer

Fig. 2.15 shows the vertical single column manometer. Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = Fall of heavy liquid in reservoir

h_2 = Rise of heavy liquid in right limb

h_1 = Height of centre of pipe above X-X

p_A = Pressure at A, which is to be measured

A = Cross-sectional area of the reservoir

a = Cross-sectional area of the right limb

S_1 = Sp. gr. of liquid in pipe

S_2 = Sp. gr. of heavy liquid in reservoir and right limb

ρ_1 = Density of liquid in pipe

ρ_2 = Density of liquid in reservoir

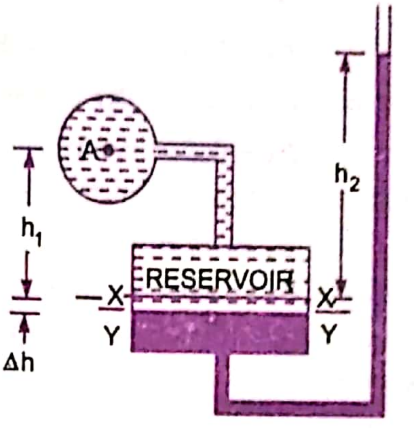


Fig. 2.15 *Vertical single column manometer.*

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore \quad A \times \Delta h = a \times h_2$$

$$\therefore \quad \Delta h = \frac{a \times h_2}{A} \qquad ...(i)$$

Now consider the datum line Y-Y as shown in Fig. 2.15. Then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in the left limb above Y-$Y = \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating these pressures, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + p_A$$

or

$$p_A = \rho_2 g\,(\Delta h + h_2) - \rho_1 g(\Delta h + h_1)$$

$$= \Delta h[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

But from equation (*i*), $\Delta h = \dfrac{a \times h_2}{A}$

$$\therefore \quad p_A = \frac{a \times h_2}{A}\,[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g \qquad ...(2.9)$$

As the area A is very large as compared to a, hence ratio $\dfrac{a}{A}$ becomes very small and can be neglected.

Then $p_A = h_2\rho_2 g - h_1\rho_1 g \qquad ...(2.10)$

From equation (2.10), it is clear that as h_1 is known and hence by knowing h_2 or rise of heavy liquid in the right limb, the pressure at A can be calculated.

2. Inclined Single Column Manometer

Fig. 2.16 shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.

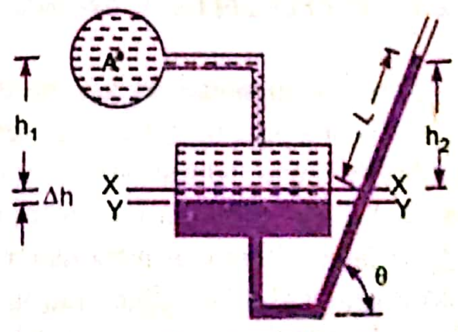


Fig. 2.16 *Inclined single column manometer.*

Let L = Length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from $X\text{-}X = L \times \sin\theta$

From equation (2.10), the pressure at A is

$$p_A = h_2\rho_2 g - h_1\rho_1 g.$$

Substituting the value of h_2, we get

$$p_A = \sin\theta \times \rho_2 g - h_1\rho_1 g. \quad ...(2.1$$

Problem 2.14 *A single column manometer is connected to a pipe containing a liquid of sp. gr. 0. as shown in Fig. 2.17. Find the pressure in the pipe if the area of the reservoir is 100 times the are of the tube for the manometer reading shown in Fig. 2.17. The specific gravity of mercury is 13.6.*

Solution. Given :

Sp. gr. of liquid in pipe, $S_1 = 0.9$

$\therefore$ Density $\rho_1 = 900 \text{ kg/m}^3$

Sp. gr. of heavy liquid, $S_2 = 13.6$

Density, $\rho_2 = 13.6 \times 1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20 \text{ cm} = 0.2 \text{ m}$

Rise of mercury in right limb,

$h_2 = 40 \text{ cm} = 0.4 \text{ m}$

Let p_A = Pressure in pipe

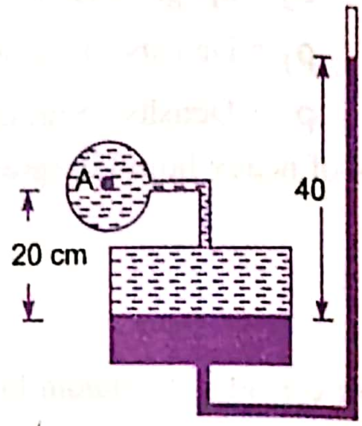


Fig. 2.17

Using equation (2.9), we get

$$p_A = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

$$= \frac{1}{100} \times 0.4[13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100}[133416 - 8829] + 53366.4 - 1765.8$$

$= 533.664 + 53366.4 - 1765.8 \text{ N/m}^2 = 52134 \text{ N/m}^2 =$ **5.21 N/cm². Ans.**

▶ 2.7 DIFFERENTIAL MANOMETERS

Differential manometers are the devices used for measuring the difference of pressures betwe two points in a pipe or in two different pipes. A differential manometer consists of a U-tube, contai ing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to measured. Most commonly types of differential manometers are :

1. U-tube differential manometer and
2. Inverted U-tube differential manometer.

2.7.1 U-tube Differential Manometer.

Fig. 2.18 shows the differential manometers c U-tube type.

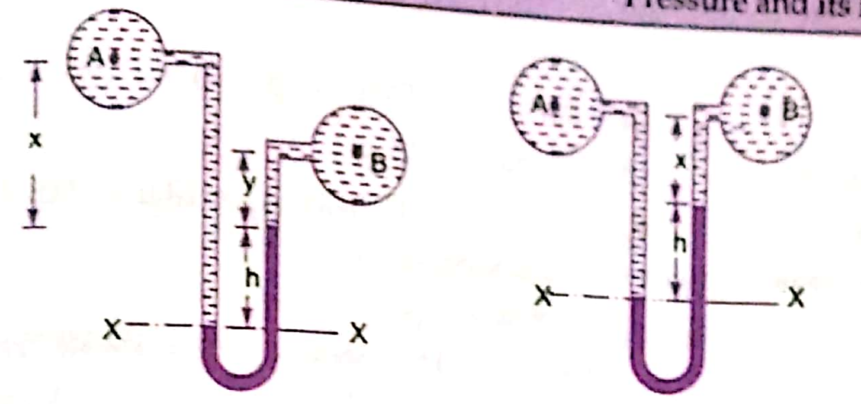


Fig. 2.18 *U-tube differential manometers.*

In Fig. 2.18 (*a*), the two points *A* and *B* are at different level and also contains liquids of different sp. gr. These points are connected to the U-tube differential manometer. Let the pressure at *A* and *B* are p_A and p_B.

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of *B*, from the mercury level in the right limb.

x = Distance of the centre of *A*, from the mercury level in the right limb.

ρ_1 = Density of liquid at *A*.

ρ_2 = Density of liquid at *B*.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at *X-X*.

Pressure above *X-X* in the left limb = $\rho_1 g(h + x) + p_A$

where p_A = pressure at *A*.

Pressure above *X-X* in the right limb = $\rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at *B*.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 g y + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x \quad \text{...(2.12)}$$

$\therefore$ Difference of pressure at *A* and *B* = $h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$

In Fig. 2.18 (*b*), the two points *A* and *B* are at the same level and contains the same liquid of density ρ_1. Then

Pressure above *X-X* in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above *X-X* in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 g x + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 g x - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1). \quad \text{...(2.13)}$$

Problem 2.15 *A pipe contains an oil of sp. gr. 0.9. A differential manometer connected at the two points A and B shows a difference in mercury level as 15 cm. Find the difference of pressure at the two points.*

Solution. Given :

Sp. gr. of oil, $S_1 = 0.9$ $\therefore$ Density, $\rho_1 = 0.9 \times 1000 = 900$ kg/m³

Difference in mercury level, $h = 15$ cm $= 0.15$ m

Sp. gr. of mercury, $S_g = 13.6$ $\therefore$ Density, $\rho_g = 13.6 \times 1000$ kg/m³

The difference of pressure is given by equation (2.13)

$$p_A - p_B = g \times h(\rho_g - \rho_1)$$

or $$= 9.81 \times 0.15\ (13600 - 900) = \mathbf{18688\ N/m^2}.\ \textbf{Ans.}$$

Problem 2.16 *A differential manometer is connected at the two points A and B of two pipes as shown in Fig. 2.19. The pipe A contains a liquid of sp. gr. = 1.5 while pipe B contains a liquid of sp. gr. = 0.9. The pressures at A and B are 1 kgf/cm² and 1.80 kgf/cm² respectively. Find the difference in mercury level in the differential manometer.*

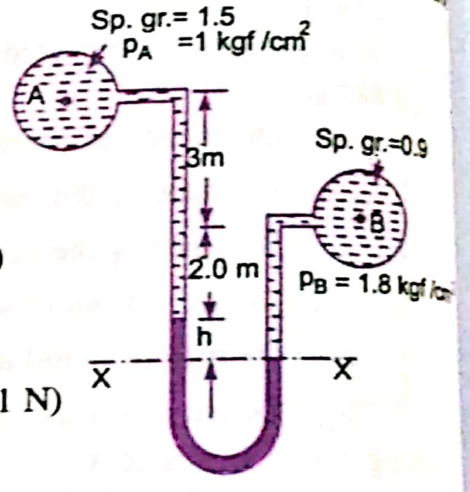


Fig. 2.19

Solution. Given :

Sp. gr. of liquid at A, $S_1 = 1.5$ $\therefore$ $\rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9$ $\therefore$ $\rho_2 = 900$

Pressure at A, $p_A = 1$ kgf/cm² $= 1 \times 10^4$ kgf/m²

$= 10^4 \times 9.81$ N/m² ($\because$ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8$ kgf/cm²

$= 1.8 \times 10^4$ kgf/m²

$= 1.8 \times 10^4 \times 9.81$ N/m² ($\because$ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000$ kg/m³

Taking X-X as datum line.

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$
$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4$$
$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$

or $$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

or $$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$

$\therefore$ $$h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1\ cm.}\ \textbf{Ans.}$$

Problem 2.17 *A differential manometer is connected at the two points A and B as shown in Fig. 2.20. At B air pressure is 9.81 N/cm² (abs), find the absolute pressure at A.*

Solution. Given :

Air pressure at $B = 9.81$ N/cm²

or $p_B = 9.81 \times 10^4$ N/m²

Density of oil $= 0.9 \times 1000 = 900 \text{ kg/m}^3$

Density of mercury $= 13.6 \times 1000 \text{ kg/m}^3$

Let the pressure at A is p_A

Taking datum line at X-X

Pressure above X-X in the right limb

$$= 1000 \times 9.81 \times 0.6 + p_B$$
$$= 5886 + 98100 = 103986$$

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times 0.1 + 900 \times 9.81 \times 0.2 + p_A$$
$$= 13341.6 + 1765.8 + p_A$$

Equating the two pressure heads

$$103986 = 13341.6 + 1765.8 + p_A$$

$\therefore$ $p_A = 103986 - 15107.4 = 88876.8$

$\therefore$ $p_A = 88876.8 \text{ N/m}^2 = \dfrac{88876.8 \text{ N}}{10000 \text{ cm}^2} = 8.887 \dfrac{\text{N}}{\text{cm}^2}$.

$\therefore$ Absolute pressure at A = **8.887 N/cm². Ans.**

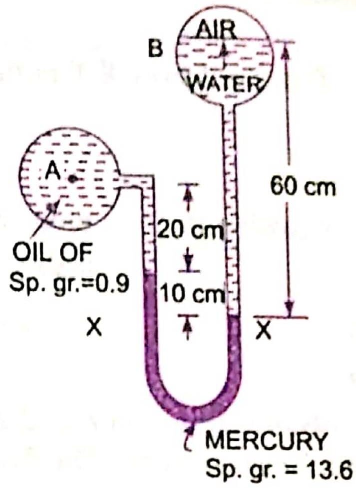


Fig. 2.20

2.7.2 Inverted U-tube Differential Manometer. It consists of an inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig. 2.21 shows an inverted U-tube differential manometer connected to the two points A and B. Let the pressure at A is more than the pressure at B.

Let h_1 = Height of liquid in left limb below the datum line X-X

h_2 = Height of liquid in right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid

p_A = Pressure at A

p_B = Pressure at B.

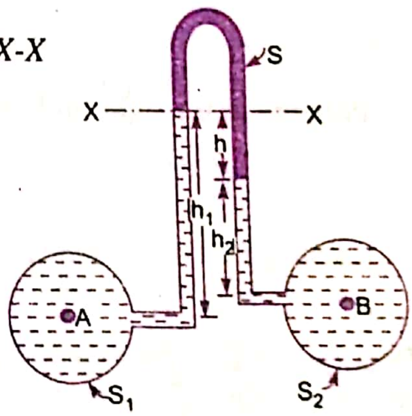


Fig. 2.21

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or $$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h. \quad ...(2.14)$$

Problem 2.18 *Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp. gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings as shown in Fig. 2.22.*

Solution. Given :

Pressure head at $A = \dfrac{p_A}{\rho g} = 2$ m of water

$\therefore$ $p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620 \text{ N/m}^2$

Fig. 2.22 shows the arrangement. Taking X-X as datum line.

Pressure below X-X in the left limb $= p_A - \rho_1 \times g \times h_1$

$= 19620 - 1000 \times 9.81 \times 0.3 = 16677 \text{ N/m}^2.$

Pressure below X-X in the right limb

$= p_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$

$= p_B - 981 - 941.76 = p_B - 1922.76$

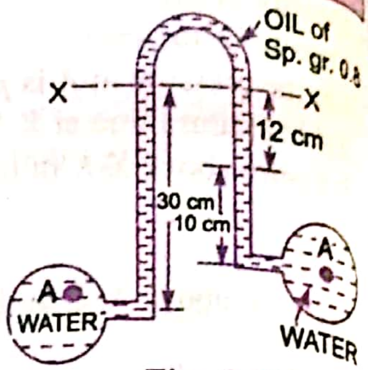


Fig. 2.22

Equating the two pressure, we get

$$16677 = p_B - 1922.76$$

or $p_B = 16677 + 1922.76 = 18599.76 \text{ N/m}^2$

or $p_B = \mathbf{1.8599 \text{ N/cm}^2}$. **Ans.**

Problem 2.19 *In Fig. 2.23, an inverted differential manometer is connected to two pipes A and B which convey water. The fluid in manometer is oil of sp. gr. 0.8. For the manometer readings shown in the figure, find the pressure difference between A and B.*

Solution. Given :

Sp. gr. of oil $= 0.8 \quad \therefore \quad \rho_s = 800 \text{ kg/m}^3$

Difference of oil in the two limbs

$= (30 + 20) - 30 = 20$ cm

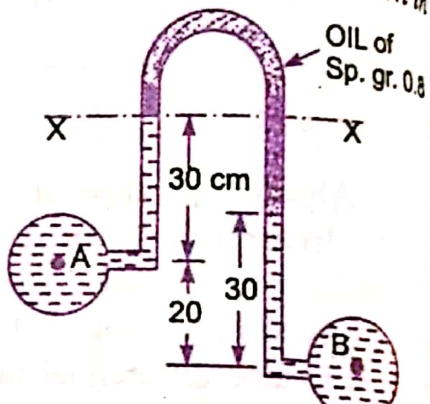


Fig. 2.23

Taking datum line at X-X

Pressure in the left limb below X-X

$= p_A - 1000 \times 9.81 \times 0$

$= p_A - 2943$

Pressure in the right limb below X-X

$= p_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$

$= p_B - 2943 - 1569.6 = p_B - 4512.6$

Equating the two pressure $p_A - 2943 = p_B - 4512.6$

$\therefore \quad p_B - p_A = 4512.6 - 2943 = \mathbf{1569.6 \text{ N/m}^2}$. **Ans.**

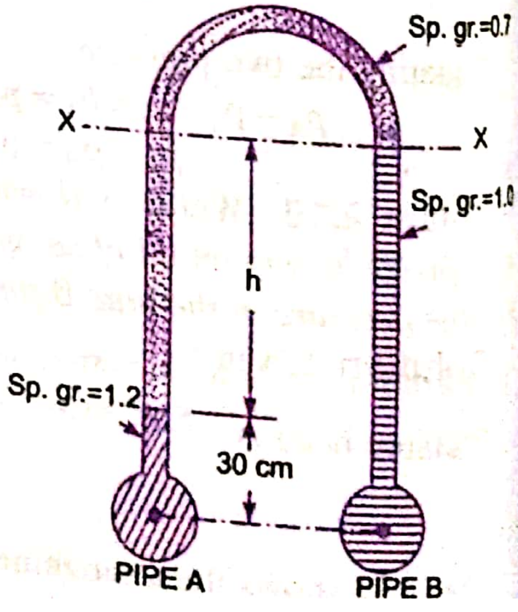
Problem 2.20 *Find out the differential reading 'h' of an inverted U-tube manometer containing oil of specific gravity 0.7 as the manometric fluid when connected across pipes A and B as shown in Fig. 2.24 below, conveying liquids of specific gravities 1.2 and 1.0 and immiscible with manometric fluid. Pipes A and B are located at the same level and assume the pressures at A and B to be equal.*

Solution. Given :

Fig. 2.24 shows the arrangement. Taking X-X as datum line.

Let p_A = Pressure at A

p_B = Pressure at B



Density of liquid in pipe A

$= \text{Sp. gr.} \times 1000$

$= 1.2 \times 1000$

$= 1200 \text{ kg/m}^2$

Density of liquid in pipe B $= 1 \times 1000 = 1000 \text{ kg/m}^3$

Density of oil $= 0.7 \times 1000 = 700 \text{ kg/m}^3$