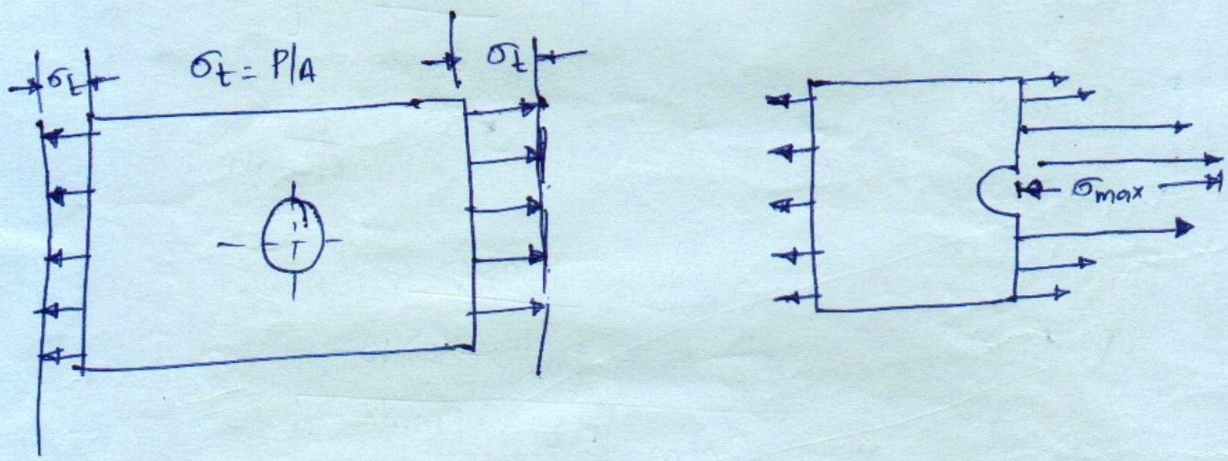
STRESS CONCENTRATION

Whenever there is a rapid change in cross section (or) discontinuity of a body, stress concentration is present. Local stresses at those sections will be more than the ~~norma~~ nominal stress. This kind of a situation is present in notches, keyways and shoulders.
It should always be tried to reduce the stress concentration.



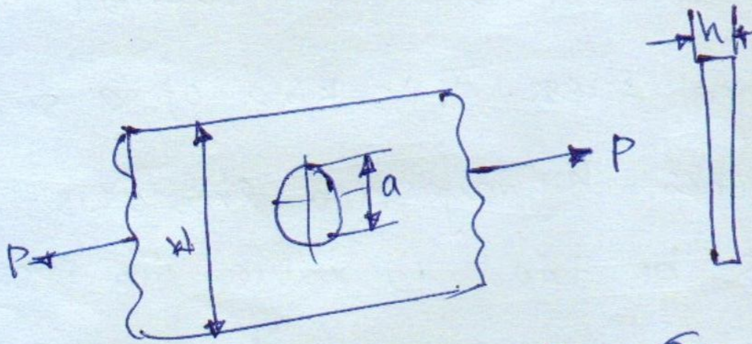
Stress concentration factor (k_t):-

Stress concentration factor k_t is defined as the ratio of the maximum stress at the change of cross section to the nominal stress.

$$\boxed{k_t = \frac{\sigma_{max}}{\sigma_0}}$$

Problems.

1) A Rectangular plate 60mm x 10mm ~~width~~ with a hole diameter 12mm is subjected to a tensile load of 12000N. Find the maximum stress induced.



$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}$$ [From DDB pg. no 7.10]

$P = 12000N$.

$W = 60mm$.

$h = 10mm$.

$a = 12mm$.

$$\sigma_{nom} = \frac{P}{(W-a)h}$$

$$\sigma_{nom} = \frac{12000}{(60-12)10}$$

$$\boxed{\sigma_{nom} = 25\ N/mm^2}$$

$a/W = 0.2 \Rightarrow$ $\boxed{K_t = 2.5}$ [From DDB pg. no 7.10].

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \Rightarrow 2.5 = \frac{\sigma_{max}}{25}$$

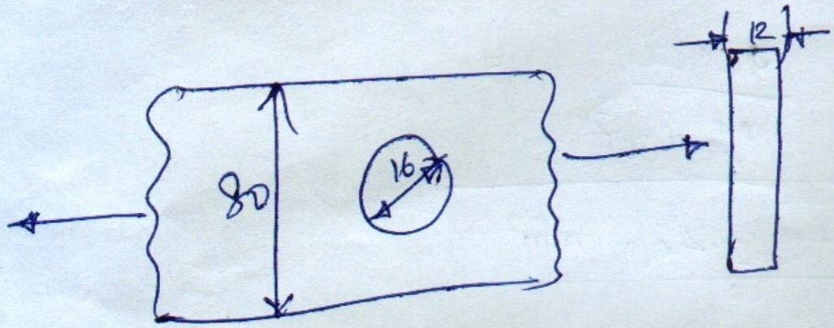
$$\boxed{\sigma_{max} = 62.5 N/mm^2.}$$

2) Taking stress concentration with account find the max stress induced when the tensile load of 20KN is applied to i) Rectangular plate of 80mm wide and 12mm thick with a hollow of dia 16mm.

ii) A stepped Shaft as diameters 60mm and 30mm with a fillet radius of 6mm.

Solution:

i) [From D.D.B pg.no 7.10]



$\sigma_{nom} = \dfrac{P}{(w-a)h}$

$a/w = 16/80 = \underline{\underline{0.2}}$

$K_t = 2.5$ [pg.no 7.10]

$\sigma_{nom} = \dfrac{20\times10^3}{(80-16)12}$

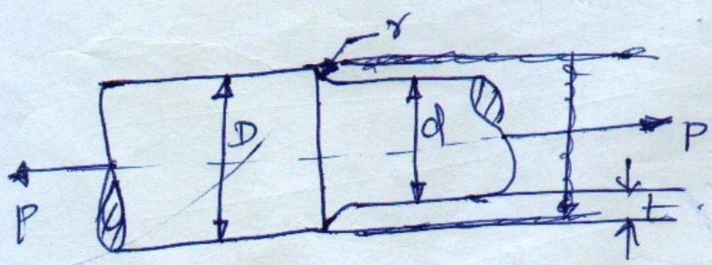
$K_t = \dfrac{\sigma_{max}}{\sigma_{nom}}$

$\boxed{\sigma_{nom} = 26.04\ N/mm^2}$

$\sigma_{max} = K_t \times \sigma_{nom}$

$\boxed{\sigma_{max} = 65.1\ N/mm^2}$

ii)



$D = 60mm.$ $d = 30mm.$ $r/d = 6/30 = \underline{\underline{0.2}}$

$r = 6mm.$

$D/d = 60/30 = \underline{\underline{2}}$

$\boxed{K_t = 1.5}$ [pg.no 7.11]

$K_t = \sigma_{max} / \sigma_{nom}$.

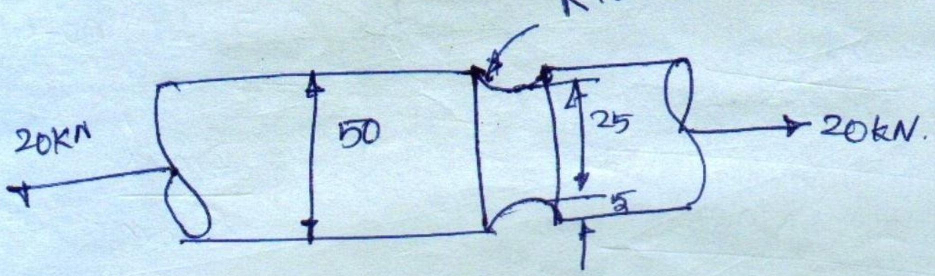
$\sigma_{max} = \sigma_{nom} \times K_t$.

$\sigma_{max} = \left[P/A \right] \times K_t$.

$$\sigma_{max} = \frac{20 \times 10^3}{\pi/4 \times (30)^2} \times 1.5$$

$$\boxed{\sigma_{max} = 42.44 \ N/mm^2}$$

3). Find the maximum stress for the given problem.



$r/d = \frac{10}{25} = \underline{\underline{0.4}}$.

$D/d = 50/25 = \underline{\underline{2}}$.

$\boxed{K_t = 1.5}$ [From pg.no 7.1]

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}}.$$

$\sigma_{max} = \sigma_{nom} \times K_t$.

$\sigma_{max} = \left[P/A \right] \times K_t$.

$$\sigma_{max} = \left[\frac{20 \times 10^3}{\pi/4 d^2} \right] \times 1.5$$

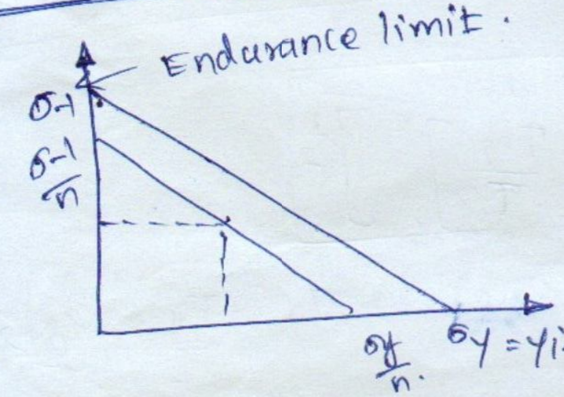
$$\boxed{\sigma_{max} = 61.12 \ N/mm^2}.$$

Design of variable loads

[From PSG DDB pg.no 7.4, 7.6]

1) Soderberg line.
2) Goodman diagram.
3) Gerber parabola.

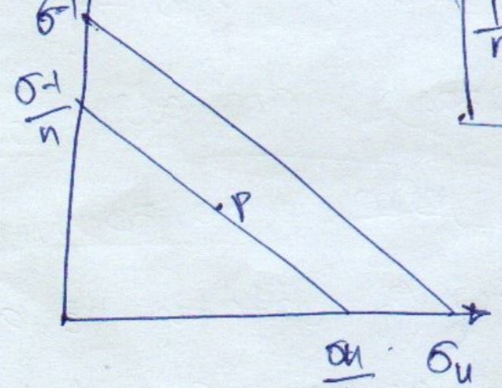
1) Soderberg line:-



$$\boxed{\frac{1}{n} = \frac{\sigma_m}{\sigma_y} + k_t \frac{\sigma_a}{\sigma_{-1}}} \; ; \; \boxed{\frac{1}{n} = \frac{\tau_m}{\tau_y} + k_t \frac{\tau_a}{\tau_{-1}}}$$

For ductile materials.

2) Goodman diagram:-

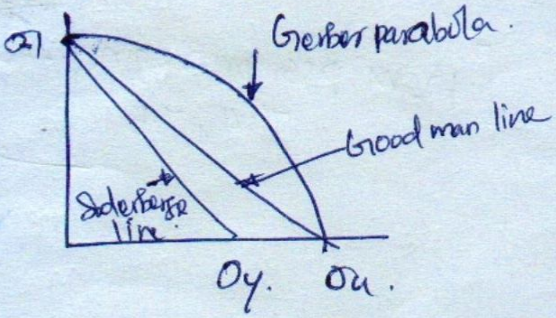


$$\boxed{\frac{1}{n} = k_t \left[\frac{\sigma_m}{\sigma_u} + \frac{\sigma_a}{\sigma_{-1}} \right]} ;$$

$$\boxed{\frac{1}{n} = k_t \left[\frac{\tau_m}{\tau_u} + \frac{\tau_a}{\tau_{-1}} \right]}$$

for brittle materials.

3) Gerber parabola



$$\boxed{\sigma_{eq} = \frac{\sigma_y}{n} = \sigma_m + k_t \frac{\sigma_a \sigma_y}{\sigma_{-1}}} \; ; \; \boxed{\tau_{eq} = \frac{\tau_y}{n} = \tau_m + k_t \frac{\tau_a \tau_y}{\tau_{-1}}}$$

$$\boxed{\frac{1}{n} = \left[\left(\frac{\sigma_{eq}}{\sigma_y}\right)^2 + \left[\frac{\tau_{eq}}{\tau_y}\right]^2\right]^{\frac{1}{2}}}$$

Max. shear theory: $\tau_y = \sigma_y/2$.

Octahedral

shear theory: $\tau_y = \sigma_y/\sqrt{3}$.

Problems ⊗⊗

1). A circular shaft made of C45 material is subjected to an axial load varying from −1000N to +2500 N, and also to a torsional moment that varies from 0 to 500 N-m. Assuming a factor of safety 1.5 and stress concentration factor as 1.5. Calculate the diameter of the shaft.

Given data:-

material C45.

Minimum load = −1000N.
Maximum load = +2500N.
Minimum torque = 0N-m.
Maximum torque = +500 N-m.
FOS (or) n = 1.5
k_t = 1.5.

Solution:-

[From DDB pg.no 1.9].

$\sigma_u = 63 - 71$ kgf/mm².

$\boxed{1\,kgf = 10N}$ ← [From PSG DDB pg.no 1.9]

$= 630 - 710$ N/mm².

$\boxed{\sigma_u = 650 N/mm^2}$ [Assume]

Yield stress σ_y = 36kgf/mm².

$\boxed{\sigma_y = 360 N/mm^2}$

for endurance limit [DDB pg. no 1.42]

$\sigma_{-1} = 0.36 \times \sigma_u$

$= 0.36 \times 650$

$\boxed{\sigma_{-1} = 234\ N/mm^2}$

$\tau_0 = 0.3\sigma_u$

$\tau_0 = 0.3 \times 650$

$\boxed{\tau_0 = 195\ N/mm^2}$

$\tau_{-1} = 0.22\sigma_u$

$\tau_{-1} = 0.22 \times 650$

$\boxed{\tau_{-1} = 143\ N/mm^2}$

Considering axial load

$$\sigma_{max} = P/A = \frac{2500}{\pi/4 d^2}$$

$$\boxed{\sigma_{max} = \frac{3183.098}{d^2}\ N/mm^2}$$

$$\sigma_{min} = P/A = \frac{-1000}{\pi/4 d^2}$$

$$\boxed{\sigma_{min} = \frac{-1273.239}{d^2}\ N/mm^2}$$

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_{mean} = \frac{\frac{3183.098}{d^2} - \frac{1273.239}{d^2}}{2}$$

$$= \frac{1}{d^2}\left[\frac{3183.098 - 1273.239}{2}\right]$$

$$\boxed{\sigma_{mean} = \frac{954.929}{d^2}\ N/mm^2}$$

From PSG DDB pg.no 7.6

$$\sigma_{eq} = \sigma_m + k_f \frac{\sigma_a \cdot \sigma_y}{\sigma_{-1}}.$$

Assume $q = 1$

$$k_f = 1 + q(k_t - 1)$$

$$k_f = 1 + 1(1.5 - 1)$$

$$\boxed{k_f = 1.5.}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}.$$

$$\sigma_a = \frac{3183.098 - (-1273.239)}{2d^2}.$$

$$\boxed{\sigma_a = \frac{2228.168}{d^2} \ N/mm^2}$$

$$\sigma_{eq} = \frac{954.929}{d^2} + 1.5 * \frac{2228.168 \times 360}{d^2 \times 234}$$

$$\boxed{\sigma_{eq} = \frac{6096.85}{d^2} \ N/mm^2.}$$

Considering torsional stress:-

$\tau_{min} = 0$

$\tau_{max} = ?$ $\qquad \frac{T}{J} = \frac{\tau}{r}.$

$$\tau = \frac{T}{J} \cdot r.$$

$$= \frac{16 \times T \leftarrow N\text{-}m}{\pi d^3}.$$

$$\tau = \frac{T}{J} \cdot r$$

$$\tau = \frac{T}{\frac{\pi}{32} d^{4\,3}} \times \frac{d}{2}$$

$$\boxed{\tau = \frac{16 \times T}{\pi d^3}}.$$

$$\tau = \frac{16 \times (500 \times 10^3) \leftarrow N\text{-}mm.}{\pi d^3}$$

$$\boxed{\tau_{max} = \frac{2546.47 \times 10^3}{d^3} \, N/mm^2}$$

$$\tau_{eq} = \tau_m + k_f \frac{\tau_a \cdot \tau_y}{\tau_{-1}}.$$

$$\tau_{mean} = \frac{\tau_{max} + \tau_{min}}{2}.$$

$$\tau_{mean} = \frac{2546.47 \times 10^3}{d^3} \times (1/2)$$

$$\boxed{\tau_{mean} = \frac{1.273 \times 10^6}{d^3} \, N/mm^2.} \boxed{= \tau_a}.$$

$$\tau_{eq} = \tau_m + k_f \frac{\tau_a \cdot \tau_y}{\tau_{-1}}.$$

$$= \frac{1.273 \times 10^6}{d^3} + 1.5 \frac{1.275 \times 10^6 / d^3 \times 180}{143}$$

$$\boxed{\tau_{eq} = \frac{3.68\times10^6}{d^3} N/mm^2}$$

$$\frac{1}{n} = \left[\left[\frac{\sigma_{eq}}{\sigma_y}\right]^2 + \left[\frac{\tau_{eq}}{\tau_y}\right]^2\right]^{1/2}$$

$\tau_y = \sigma_y/2$

$\tau_y = 360/2$

$\tau_y = 180 N/mm^2$

$$\frac{1}{1.5} = \left[\left[\frac{6096.85}{360\times d^2}\right]^2 + \left[\frac{3.68\times10^6}{180\times d^3}\right]^2\right]^{1/2}$$

$$\frac{1}{1.5} = \left[\frac{286.82}{d^4} + \frac{417.975\times10^6}{d^6}\right]^{1/2}$$

$$\left[\frac{1}{1.5}\right]^2 = \left[\frac{286.82}{d^4} + \frac{417.975\times10^6}{d^6}\right]^{1/2}$$

$$0.4444 = \left[\frac{286.82}{d^4} + \frac{417.975\times10^6}{d^6}\right]^{1/2}$$

using trail and error method.

Sub $d = 31$.

$0.4444 \neq 0.4712$.

Say $d = 31.4$

$0.44 \simeq 0.436$

sub $d = 32$

$0.4444 \neq 0.3895$

$\therefore$ Selected diameter is 31.4 mm

From data book pg.no 7.25

$\boxed{d = 32 mm}$..