Homogeneous Coordinates

Computer Graphics

Homogeneous Coordinates

To represent ay 2- dimensional transformation as matrix multiplication, we represent each coordinate position (x,y) with homogeneous coordinate tuple (xh ,yh ,h).

xh = x/h

yh = y/h

- A convenient choice is to set h=1
- Each 2-D position is the represented with homogeneous coordinates
- This allows us to represent all geometric transformation equation as matrix multiplication

Translation matrix

Translatio n :

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

or

$$\mathbf{P}' = \mathbf{T}(t_x, t_y)\mathbf{P}$$

Rotation matrix

Rotation :

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

or

$$\mathbf{P}' = \mathbf{R}(\theta)\mathbf{P}$$

Scaling Matrix

Scaling :

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

or

$$\mathbf{P}' = \mathbf{S}(s_x, s_y)\mathbf{P}$$

Why we need Homogeneous Coordinates?

- One of the many purposes of using homogeneous coordinates is to capture the concept of infinity.
- If we don't use homogeneous coordinates, it would be difficult to design certain classes of very useful curves and surfaces. These curves and surfaces are very crucial in developing algorithms in computer vision, graphics, CAD, etc.
- We have seen that basic transformations can be expressed in matrix form. But many graphic application involve sequences of geometric transformations. Hence we need a general form of matrix to represent such transformations.